

VIRTUAL MANIPULATIVES AND PHYSICAL MANIPULATIVES:
TECHNOLOGY'S IMPACT ON FRACTION LEARNING

By

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OVERVIEW

This dissertation examines the relative instructional efficiency of virtual fraction manipulatives and physical fraction manipulatives. More specifically, this dissertation uses a randomized experiment to determine if differences in students' knowledge of fraction magnitude exist when students learn basic fraction concepts using virtual manipulatives compared to when students learn basic fraction concepts using physical manipulatives. During the experimental study, students spent two weeks learning about fractions using different forms of manipulatives (i.e. physical or virtual), but other important variables such as the teacher, lesson plans, instructional scripts, the type of practice activities assigned to students, and the amount of time students spent practicing using manipulatives were held constant across conditions. Students completed assessments at the end of both the first and second weeks of the intervention, and the results of the assessments indicate that virtual manipulatives are at least as effective as physical manipulatives and possibly more effective. This dissertation also examines the time-efficiency of using virtual rather than physical manipulatives by tracking the number of practice activities students completed on each day of instruction and making comparisons between treatment conditions. Results indicate that when the amount of time spent practicing is held constant, students complete more practice activities using virtual rather than physical manipulatives. However, the impact on student learning of the additional practice is unclear.

This dissertation is divided into 5 chapters. Chapter I discusses the significance of the study and the specific research questions and hypotheses that were tested. It also

presents the background literature. Chapter II provides a critical review of the empirical literature and a conceptual framework, and Chapter III discusses methodology. Chapter IV summarizes the results, and Chapter V includes the discussion, limitations, recommendations for future research, and conclusions.

CHAPTER I

INTRODUCTION

Research shows that fractions are among the most difficult mathematical concepts for elementary school students to master (Behr, Harel, Post, & Lesh, 1992; Bezuk & Cramer, 1989; Moss & Case, 1999). In 1990, fewer than half of the high school seniors and only 14 percent of the eighth graders who took the NAEP Mathematics Assessment consistently demonstrated successful performance with problems involving fractions, decimals, percents, and simple algebra (Mullis, Dossey, Owen, & Phillips, 1991). In 2000, only 41% of eighth graders successfully ordered three fractions, all of which were less than 1 and in reduced form (Kloosterman & Lester, 2004). Not surprisingly, students with learning difficulties and low-achieving students face even greater struggles when trying to master fraction concepts (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Calhoon, Emerson, Flores, & Houchins, 2007; Empson, 2003; Hiebert, Wearne, & Taber, 1991). However, research indicates that manipulatives (e.g. fractions circles, fractions strips) positively impact students' conceptual and procedural understanding of fractions without impeding their ability to complete algorithmic procedures involving fractions (Cramer & Henry, 2002). Overall, students who learn any content area of mathematics using manipulatives outperform students who do not use manipulatives (Parham, 1983; Sowell, 1989; Suydam & Higgins, 1977), but unfortunately, a variety of practical and pedagogical challenges associated with using manipulatives during instruction make it difficult for teachers to implement them effectively in classrooms. As a result, students

receive far less exposure to manipulatives than the National Council of Teachers of Mathematics (NCTM) recommends for students in grades K-8 (Char, 1991; Hatfield, 1994; Hodge & Brumbaugh, 2003; National Council of Teachers of Mathematics, 2000).

Many of the practical and pedagogical difficulties associated with manipulatives may be reduced or eliminated if teachers use virtual rather than physical manipulatives during mathematics instruction (Clements, 1999; Clements & McMillan, 1996).

However, the current body of empirical research lacks adequate comparisons of virtual and physical manipulatives to determine if educators can expect students to experience similar learning gains if teachers use virtual rather than physical manipulatives to teach fractions. This dissertation addresses this gap in the literature by examining the relative instructional efficiency of virtual fraction manipulatives and physical fraction manipulatives. More specifically, this dissertation uses a randomized experiment to determine if differences in students' knowledge of fraction magnitude exist when students learn basic fraction concepts using virtual manipulatives compared to when students learn basic fraction concepts using physical manipulatives. This dissertation also examines the time efficiency of using virtual rather than physical manipulatives by tracking the number of practice activities students complete on each day of instruction and making comparisons between treatment conditions. The following sections of this chapter provide further details about the significance of this dissertation and the specific research questions and hypotheses that were tested in the experimental study. The chapter concludes with a summary of background research.

Significance

The fact that US students consistently fail to demonstrate mastery of fraction concepts on the NAEP and other standardized mathematics assessments concerns educators because of the strong link between mastery of rational number concepts and algebra learning (Kieran, 1976; National Mathematics Advisory Panel, 2008; National Research Council, 2001). In turn, algebra learning concerns educators because algebra is the “gateway” to advanced mathematics courses as well as academic science courses (Gamoran & Hannigan, 2000; Riley, 1997; Smith, 1996). Students who take both Algebra I and Geometry prior to graduating from high school are more than twice as likely to enroll in college within two years after graduation as students who do not take these courses (Riley, 1997), yet a recent survey of a nationally representative sample of algebra teachers indicated that teachers perceive their students’ overall preparation for algebra as weak (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). When asked to rate their students’ preparation for algebra in specific skill areas, teachers rated “rational numbers and operations involving fractions and decimals” as one of the three areas of weakest preparation (Hoffer, et al., 2007).

The importance of fractions to algebra learning, combined with students’ failure to demonstrate mastery of basic fraction concepts prior to enrolling in high school algebra courses, prompted NCTM (2006) to determine that mathematics instruction in the upper elementary and middle school mathematics curricula should place a particularly strong emphasis on fractions. NCTM (2006) identified three curriculum “focal points” for each grade level from pre-K to grade 8 and recommended that the three established focal points receive the most emphasis within the more comprehensive mathematics

curriculum. One of the three focal points for each of grades 3, 4, 5, and 6 explicitly mentions fractions. In addition, one of the focal points for grade 7 is “Developing an understanding of operations on all rational numbers and solving linear equations” (NCTM, 2006, p. 19). Similarly, when the National Mathematics Advisory Panel (NMAP) released its *Final Report* in 2008, the report identified “Fluency with Fractions” as one of the three “Critical Foundations for Algebra.” The topics included in the “Critical Foundations for Algebra” are not intended to represent a comprehensive mathematics curriculum, but the panel determined that they should receive primary attention and ample time in any mathematics curriculum.

Although both NCTM and NMAP recently determined that fractions should be a primary focus of instruction in the upper elementary school and middle school mathematics curricula, educators struggle to implement their recommendations in part because the past two decades of research produced few significant advances in our understanding of why students experience difficulty learning fraction concepts (Lamon, 2007). Lamon (2007) characterized the field of research on rational numbers and proportional reasoning as being at a temporary standstill, in contrast to the 1950’s when Inhelder and Piaget (1958) first drew attention to the problems students face when learning fractions and other rational number concepts by designating proportional reasoning as the hallmark of their formal operational stage of development. Additionally, in the 1970’s and 1980’s, a large body of research emerged that identified the difficulties students face when grappling with rational numbers and proportional reasoning problems (Behr, Lesh, Post, & Silver, 1983; Behr, Post, Silver, & Mierkiewicz, 1980; Bezuk, 1986; Kieran, 1980; Kieran, 1976; Lesh, Landau, & Hamilton, 1983; Lesh, Post, & Behr, 1988).

The theoretical frameworks that emerged from this body of research strongly advocate for the use of manipulatives when teaching basic fraction concepts. However, the many practical and pedagogical challenges associated with manipulatives make it difficult for teachers to effectively implement them in classrooms. As a result, teachers under-use manipulatives (Char, 1991; Hatfield, 1994; Hodge & Brumbaugh, 2003). A study that analyzed self-reports from K-6 elementary school teachers found that 70% of teachers were familiar with eight out of eleven manipulative devices included on the survey, and had access to 88% of the devices with which they were familiar, yet average use for each of the manipulative devices was rather low: in grades 4-6 the average use of each of the manipulative devices included in the survey was less than 1.3 days per month (Hatfield, 1994). Manipulative use also declined with each grade level from kindergarten through sixth grade, contrary to the recommendations of NCTM (1989; 2000).

Researchers continue to advocate for the use of manipulatives during instruction, but teachers lack the necessary tools and methods they need to effectively implement them in classrooms. Unsurprisingly, research about rational numbers and proportional reasoning appears to be at a standstill. To date, the majority of research about instruction using manipulatives only considers physical manipulatives, but Clements (1999; see also Clements & McMillan, 1996) noted many practical and pedagogical benefits to teachers using virtual manipulatives. This led him to hypothesize that computers can provide virtual representations of mathematical concepts that are just as meaningful as physical manipulatives (Clements, 1999). A comprehensive search of the literature identified only eight small-scale mathematics studies and two small-scale science studies that test this or a closely related hypothesis, and the available evidence supports Clements' hypothesis

that virtual manipulatives are at least as effective as physical manipulatives (Klahr, Triona, & Williams, 2007; Moyer, Niezgoda, & Stanley, 2005; Nute, 1997; Pleet, 1991; Reimer & Moyer, 2005; Smith, 2006; Steen, Brooks, & Lyon, 2006; Suh, 2005; Suh & Moyer, 2007; Triona & Klahr, 2003). However, all of the known mathematics studies that directly compare virtual and physical manipulatives include differences between the treatment and control conditions other than the form of manipulatives used for instruction. These differences make it impossible to completely isolate the effect of the different forms of manipulatives. In addition, other weaknesses in the research designs used in these studies threaten the internal and external validity of the outcomes. This dissertation advances the current literature about rational numbers by comparing virtual and physical manipulatives while controlling for other important variables such as the teacher, lesson plans, instructional scripts, the type of practice activities, and the amount of time spent practicing using manipulatives. In doing so, the study isolates the effect of the form of manipulatives used for instruction. Although the small sample size used in the study somewhat limits the generalizability of the results, this dissertation pilots strong research methods that can be replicated on a much larger scale.

In addition to piloting a strong research design, this dissertation makes a unique contribution to the current literature about rational numbers by assessing the time efficiency of using virtual rather than physical manipulatives to complete practice activities. While Klahr, Triona, and Williams (2007) compare the time efficiency of virtual and physical manipulatives in science, to date, no known studies compare the time efficiency of the two different forms of manipulatives in mathematics. The work of Cramer, Post, & del Mas (2002) underscores the importance of time efficiency because

the results of their large-scale comparison of a manipulative-based, experimental fractions curriculum and a non-manipulative-based, commercial fractions curriculum concluded that students need to interact with manipulatives over an extended period of time to develop the mental images necessary to think conceptually about fractions. Ericsson, Krampe, and Tesch-Romer (1993) also underscore the importance of time efficiency because they suggest that expert performance can be explained in terms of acquired characteristics resulting from extended deliberate practice. Ericsson and colleagues describe deliberate practice activities as those activities designed to improve a student's level of performance and which allow for "repeated experiences in which the individual can attend to the critical aspects of the situation and incrementally improve her or his performance in response to knowledge of results, feedback, or both from a teacher" (Ericsson, et al., 1993, p. 368). This dissertation holds the total amount of time students spent completing practice exercises and the total amount of time students spent playing games using manipulatives constant between treatment conditions. Careful practice logs were kept of the number of practice exercises students complete and the number of games students play on each day of instruction and comparisons were made between treatment conditions. The quantitative analyses of students' knowledge of fraction magnitude also include the data collected in the practice logs to determine if the amount of deliberate practice students engage in with manipulatives impacts their knowledge of fraction magnitude. The next sections of this chapter state the specific research questions investigated in this dissertation as well as the related hypotheses that were tested.

Research Questions

This dissertation examines the following research questions:

1. Are there differences in students' knowledge of fraction magnitude when they are taught basic fraction concepts using virtual manipulatives compared to when they are taught basic fraction concepts using physical manipulatives?
2. Are students able to complete more practice exercises and/or more games using virtual manipulatives than physical manipulatives?
3. Does the number of practice exercises students complete and/or the number of games students play have an impact on students' knowledge of fraction magnitude?

Hypotheses

Overall, the empirical evidence suggests that virtual manipulatives are at least as effective as physical manipulatives (Klahr, Triona, & Williams, 2007; Moyer, Niezgoda, & Stanley, 2005; Nute, 1997; Pleet, 1991; Reimer & Moyer, 2005; Smith, 2006; Steen, Brooks, & Lyon, 2006; Suh, 2005; Suh & Moyer, 2007; Triona & Klahr, 2003). One study suggests that virtual manipulatives are particularly effective for instruction about fractions (Suh, 2005). For these reasons, this dissertation hypothesizes that students who use virtual manipulatives learn as much about fraction magnitude as students who use physical manipulatives.

The available empirical evidence also suggests that virtual manipulatives are more time-efficient than physical manipulatives (Klahr, et al., 2007; Moyer, et al., 2005;

Reimer & Moyer, 2005; Steen, et al., 2006). In light of this evidence, this dissertation hypothesizes that students who use virtual manipulatives complete more practice exercises and more games than students who use physical manipulatives.

Finally, the work of Ericsson and colleagues (1993) suggests that expert performance can be explained in terms of acquired characteristics resulting from extended deliberate practice. The work of Cramer and colleagues (1997; 2002) suggests that students need to interact with manipulatives over an extended period of time to develop the mental images necessary to think conceptually about fractions. Given these findings, this dissertation hypothesizes that the number of practice exercises and games students complete has a positive impact on students' understanding of fraction magnitude.

The conceptual framework, which is displayed visually in Figure 1¹ and which is discussed in detail in Section Two of Chapter II, lends further support to these hypotheses. In turn, the background literature presented in the next sections set the stage for both the critical review of the empirical literature as well as the conceptual framework.

Background Literature

Student Thinking about Rational Numbers

In a seminal paper presented to the International Group for the Psychology of Mathematics Education, Kieran (1976) stated that to understand rational numbers,

¹ See Appendix A.

students must have adequate experience with the different interpretations of rational numbers. This is because even though many school curricula only emphasize the computational aspects of rational numbers, many of the non-computational aspects of rational numbers provide students with “face to face” confrontations with algebraic problems. For example, in learning to grapple with rational numbers, students encounter the notion of equivalence, and they learn to use mathematical properties (e.g. associative property, commutative property, etc.). Acquiring this type of understanding prior to beginning the formal study of algebra gives students control of basic conceptual understanding, and it gives students the ability to interpret and understand these concepts in the context of realistic applications.

Kieran (1976) went on to describe seven different interpretations of rational numbers that students need to experience prior to learning algebra. These interpretations include rational numbers as fractions that can be compared, added, subtracted, etc., rational numbers as equivalence classes of fractions (e.g. $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$), and rational numbers as measures or points on a number line. He created a detailed conceptual analysis of these interpretations, which included hierarchies of important sub-skills. He then proposed several kinds of research on rational number learning and analytic curriculum research.

Building upon the recommendations of Kieran, scholars involved with the Rational Number Project (RNP) launched a cohesive program for research on rational number learning that began receiving continuous funding from the National Science Foundation in 1979 (Behr, Post, & Lesh, 1981; Behr, Cramer, Harel, Lesh, & Post, 2008). Ten years after the program was launched, Bezuk & Cramer (1989) used theoretical

justifications and the results of several long-term teaching experiments to recommend that, given the complexity of fractions, teachers should allocate more time within the mathematics curriculum to developing students' understanding of fractions. They also recommended that the emphasis of instruction should shift from the development of algorithms for performing operations on fractions to the development of a quantitative understanding of fractions, and manipulatives should be used at each grade level to introduce all components of the fractions curriculum (Bezuk & Cramer, 1989). These recommendations informed the development of RNP's first experimental, manipulative-based fractions curriculum (Cramer, et al., 1997). The curriculum reflected the following understandings acquired from nearly two decades of research:

1. The opportunity to interact with multiple manipulatives enhances children's understanding of fractions.
2. Children need to interact with manipulatives over an extended period of time in order to be able to develop the mental images of fractions that lead to conceptual understanding.
3. It is important for children to talk with each other and with their teacher about their emerging understandings of fractions.
4. Curricula should emphasize conceptual understanding of fractions prior to introducing symbols and algorithmic procedures.

Empirical Evidence of the Impact of Manipulatives on Mathematics Learning

The experimental fractions curriculum developed by RNP was subsequently

compared to a commercial curriculum in a large-scale, randomized control trial with over 1600 students from 66 different classrooms (Cramer & Henry, 2002; Cramer, et al., 2002). The treatment lasted 28-30 days, and results showed that students who used the manipulative-based curriculum outperformed students who used the commercial curriculum on the posttest and retention test, as well as on four of the six subscales: concepts, order, transfer, and estimation. Furthermore, interview data showed differences in the quality of students' thinking as they solved order and estimation tasks with fractions.

In addition to the ongoing research conducted by RNP scholars that suggests students benefit from interacting with manipulatives when learning fractions, another large body of empirical research suggests that, overall, students who interact with manipulatives during mathematics instruction outperform students who do not (Parham, 1983; Sowell, 1989; Suydam & Higgins, 1977). Suydam and Higgins (1977) conducted one of the first major studies that synthesized this body of research. By reviewing research on activity-based teaching approaches in K-8 settings, which included studies on the use of manipulative materials, the authors concluded that lessons using manipulative materials have a higher probability of impacting mathematics achievement than lessons that do not use manipulative materials. Moreover, students of all ability levels, achievement levels, and socioeconomic levels appear to be likely to benefit from the use of manipulative materials during mathematics instruction.

The results of two meta-analyses conducted in the 1980's lent further support to Suydam and Higgins' findings. The first meta-analysis included the results of 64 different studies (Parham, 1983). The majority of these studies were dissertation studies conducted

during the fifteen-year period from 1965 through 1979. Parham (1983) calculated 171 effect sizes and reported an overall mean effect size of 1.0329, indicating that, on average, students in treatment groups that used manipulative materials scored in the 85th percentile on post-treatment measures of achievement while students in treatment groups that did not use manipulative materials scored in the 50th percentile. A second meta-analysis included 60 studies (Sowell, 1989). The results of Sowell's meta-analysis demonstrated that students' mathematics achievement increases with long-term exposure to concrete manipulative models, and attitudes towards mathematics improve when students receive manipulative-based instruction from teachers who are knowledgeable about their use.

The Practical Difficulties Associated with Implementing Manipulatives in the Classroom

Despite these positive findings and recommendations from NCTM (1989, 2000) that all classrooms be equipped with ample sets of manipulatives, teachers under-use manipulatives in classrooms (Char, 1991; Hatfield, 1994; Hodge & Brumbaugh, 2003). Char (1991) suggested three major difficulties associated with manipulatives that cause them to receive little or ineffective use in most classrooms. First, distributing, collecting, and reorganizing manipulatives can be time-consuming for teachers, which causes classroom management problems. Students can also be tempted to use the materials to play games rather than complete their assignments. Second, teachers struggle with structuring, monitoring, and assessing the use of manipulatives: in a typical classroom of twenty or more students, often, the teacher cannot monitor student activity and provide each child with appropriate goals. Teachers also experience difficulty providing the

appropriate amount of individualized help and relevant feedback. Finally, students often do not automatically relate manipulatives to mathematical symbols and procedures. As a result, teachers struggle to build bridges between manipulative activities and the associated symbolic procedures. The combinations of all of these factors, coupled with the fact that parents and schools expect students to learn math using written numbers, often results in teachers bypassing manipulatives in favor of more time spent learning and practicing basic facts and algorithms.

The Potential Advantages of Using Virtual Manipulatives During Mathematics Instruction

Clements (1999) acknowledged many of the same difficulties that Char (1991) noted teachers face when trying to implement manipulative-based instruction in the classroom and suggested that there are many practical and pedagogical benefits to using virtual rather than physical manipulatives during instruction. For example, virtual manipulatives are more manageable and “clean” than their physical counterparts. Virtual manipulatives are more extensible because certain constructions are easier to make with software than with physical manipulatives, and they are more flexible because students are able to change the very nature of the manipulative (i.e. size, shape, etc.). Computers can also record and replay students’ actions with virtual manipulatives and provide a link between the concrete and symbolic with feedback in a way that is not possible in a non-virtual environment. However, most practitioners and researchers believe that manipulatives are effective because they are concrete in that students can hold them in their hands. Clements challenged this singular notion of the term concrete by proposing two different types of concrete knowledge: sensory-concrete and integrated-concrete.

Sensory-concrete knowledge requires sensory materials to make sense of an idea. Integrated-concrete knowledge is built as students learn. The strength of integrated-concrete knowledge is in the combination of many separate ideas in an interconnected structure of knowledge. Following this logic, the strength of manipulatives is not in the physicality of the manipulatives themselves, rather, “good manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas” (Clements, 1999, p. 49).

Some empirical evidence exists that supports Clements’ hypothesis that virtual manipulatives are at least as effective as physical manipulatives, although the overall methodological quality of these studies is weak (Klahr, Triona, & Williams, 2007; Moyer, Niezgoda, & Stanley, 2005; Nute, 1997; Pleet, 1991; Reimer & Moyer, 2005; Smith, 2006; Steen, Brooks, & Lyon, 2006; Suh, 2005; Suh & Moyer, 2007; Triona & Klahr, 2003). In addition, several studies suggest that virtual manipulatives are more time-efficient than physical manipulatives, both because less time is spent setting up and cleaning up the materials and because students who use virtual manipulatives complete more practice exercises than students who use physical manipulatives (Klahr, et al., 2007; Moyer, et al., 2005; Reimer & Moyer, 2005). One study even suggests that instruction about fractions may be particularly well suited to virtual manipulatives (Suh, 2005). These studies will each be discussed in more detail in the next chapter. Chapter II also presents the theoretical framework that supports the hypotheses that were tested.

CHAPTER II

REVIEW OF THE LITERATURE

This chapter presents the review of the literature for this dissertation. It is divided into two sections: a critical review of the empirical literature and a conceptual framework. The critical review of the empirical literature closely examines mathematics and science studies that directly compare the achievement effects of virtual manipulatives and physical manipulatives. This section provides the empirical basis for the hypotheses tested in this study. It also draws the reader's attention to the deficiencies in the current literature and the potential for the proposed research study to expand upon the existing literature base. The conceptual framework provides the theoretical basis for the hypotheses tested in this study. Figure 1 displays the conceptual framework visually, but each component of the conceptual framework and the links between them are also described and supported with relevant literature.

SECTION ONE: CRITICAL REVIEW OF THE EMPIRICAL LITERATURE

Methods Used for Identifying Studies to Include in the Critical Review of the Empirical Literature

The researcher identified the studies included in the critical review of the empirical literature through a comprehensive search of the mathematics and science literature. Prior to beginning the comprehensive search, the researcher established a set of criteria for identifying potential empirical studies. The inclusion criteria were as follows:

1. The study was published in 1989² or later.
2. Participants in the study were enrolled in grades K-8.
3. The study examined manipulative-based mathematics or science instruction.
4. The study used quantitative or qualitative methods to make a direct comparison between virtual and physical manipulatives.
5. The results of the quantitative or qualitative analyses allowed the authors of the study to draw conclusions about how the type of manipulatives used during instruction impacted student learning.
6. Similar pedagogies were used in the virtual and physical treatment conditions.

After establishing these criteria, the researcher conducted keyword and descriptor searches of the ERIC, ProQuest, Dissertation Abstracts, and PsychInfo databases. Next, the researcher obtained hard copies or electronic copies of any study that appeared to meet at least some of the inclusion criteria. She then examined the literature review section and bibliography of each of these studies to determine if they included any other potential sources not identified through the database searches. Eight mathematics studies and two science studies met all eight of the criteria for inclusion. The next two subsections of this chapter provide a critical review of each of these studies. The first sub-

² The researcher chose this date because it corresponds to the publication of NCTM's *Curriculum and Evaluation Standards*. Students' familiarity with computers and the technical sophistication of educational software have changed so drastically since 1989 that the researcher also believed that the results of earlier studies could not be generalized to today's classrooms.

section reviews the mathematics studies, and the second sub-section reviews the science studies.

Mathematics Studies

Mathematics Study #1

The research methods used by Steen, Brooks, and Lyon (2006) to compare physical and virtual manipulatives included randomly assigning two teachers to one of two conditions: a treatment condition in which the teacher taught a group of students 1st-grade geometry concepts using virtual manipulatives or a control condition in which the teacher taught a group of students 1st-grade geometry concepts using physical manipulatives and corresponding worksheets. A total of 31 students participated in the study. Instruction in both conditions lasted for 13 days, at the end of which the researchers administered two post-test measures of achievement. The researchers subsequently compared the results of the post-test measures of achievement to the results of two pre-test measures of achievement. Although the data tables indicate that pre-treatment differences existed between the treatment groups, no statistical tests exist to determine if these differences were reliable. The statistical analyses of the post-test results show no significant differences between the treatment and control groups, but they do show that both groups improved significantly between pre-test and post-test. This suggests that the virtual manipulatives were at least as effective as the physical manipulatives.

In addition to the quantitative analyses of academic achievement, Steen, Brooks, and Lyon used qualitative methods to examine the treatment teacher's impressions and observations of student attitudes, behaviors, and interactions when using the virtual manipulatives. The treatment teacher kept a journal during the intervention, and her entries discussed the amount, type, and quality of feedback provided by the virtual manipulatives. She also discussed the students' attitudes, behaviors, and interactions using the virtual manipulatives. From the journal entries, the researchers determined that the treatment teacher perceived the amount of time saved by using the virtual manipulatives to be an important behavioral advantage of the virtual manipulatives. Her perception of the benefit of this time saved was an increased amount of time-on-task and an increased number of repetitions of a practice activity. The treatment teacher also thought the virtual manipulatives were more flexible than the physical manipulatives, and she noted that students working with virtual manipulatives went more "in-depth" into the learning than previous classes she taught with physical manipulatives.

While the results of the qualitative analyses conducted in this study are intriguing, the researchers used a weak research design to collect and analyze the qualitative data. The authors provide little information about the actual process they used to analyze the qualitative data, and they did not collect qualitative data in the control condition. The researchers also used a weak research design to conduct the quantitative analyses. For example, the quantitative analyses lack statistical tests to determine if the pre-treatment differences between students were reliable despite the fact that the results show an unequal variance between groups. Different teachers provided the instruction to the treatment and control conditions, which introduces a strong possibility of teacher effects

that vary between treatment conditions. The results section also fails to explain the relationship between the students and the teacher and to explain how the researchers assigned students to the two groups, so it is unclear whether some or all of the participants had a prior relationship with the teachers. This means that in addition to the strong possibility of teacher effects that vary between treatment conditions, the teacher effects may actually vary between students. Finally, high pre-test scores for both groups indicated that students' had significant prior knowledge of the content being taught during the intervention, making the manipulative-based instruction somewhat obsolete.

Mathematics Study #2

Like the study described in this dissertation, the study conducted by Reimer and Moyer (2005) examines the impact of virtual manipulatives on students' conceptual and procedural understanding of fractions. Interestingly, the study actually began as a teacher's action research project. The teacher, who is also the first author of the article, decided to collaborate with a researcher to make the process of inquiry more structured and formalized. Unfortunately, the fact that this study began as an action research project placed additional constraints upon the research team, which ultimately led to a weak research design. Most notably, the study lacks a true control condition. The research subjects also appear to have had significant previous exposure to the concepts taught during the intervention. However, several unique themes that warrant further investigation emerged from the qualitative analysis. The following paragraphs describe both the quantitative and qualitative analyses in more detail.

The subjects for the Reimer and Moyer study include 19 of 25 students³ enrolled in the first author's third-grade class. The subjects of the study spent one week learning to use virtual, base-10 block manipulatives and one week learning about fractions using virtual fraction manipulatives. The researchers used the first week of the experiment to introduce students to virtual manipulatives, but no data was collected during this time period. The second week of instruction lasted for four days, and the researchers collected both quantitative and qualitative data. Lessons during the second week of instruction lasted for one hour. The teacher began each lesson by introducing students to a virtual manipulative. Students then received a teacher-made worksheet that included directions for using the virtual manipulative and for completing several mathematical tasks. The content taught during the intervention had been previously taught using physical manipulatives at an earlier point during the same school year. The researchers made the decision to essentially repeat previous instruction using virtual manipulatives rather than teaching new content during the experiment because they believed it allowed them to attribute any learning that occurred between pre-test and post-test to the virtual manipulatives rather than to the effect of the series of tasks with the virtual manipulatives. The quantitative data collected during the experiment included pre-tests and post-tests of conceptual knowledge as well as pre-tests and post-tests of procedural knowledge. The researchers collected qualitative data by interviewing subjects. Both researchers interviewed each student at some point during the experiment. The researchers asked every student a series of four questions, and the answers were analyzed using a narrative analysis procedure.

³ Four students with autism and two students who missed more than three days of instruction during the experiment were excluded from the analyses.

The students' high scores on the pre-tests used for the quantitative analyses indicate significant previous exposure to the content taught during the intervention, which in turn decreases the likelihood of the researchers seeing improvement between pre-test and post-test. Unsurprisingly, the researchers found no difference between pre-test and post-test for the tests of procedural knowledge, although the tests of conceptual knowledge indicate a small but statistically significant difference between the tests of conceptual knowledge. The authors attempted to use a research design that allowed them to attribute this difference to the virtual manipulatives rather than to the effect of the series of tasks with the virtual manipulatives, but because the study lacked a true control condition, they should not have automatically assumed that the subjects gained more conceptual knowledge of fractions using virtual manipulatives than if the students had learned the same concepts using physical manipulatives for a second time. Regarding the qualitative analyses, the researchers again used a weak research design by not including a true control condition, but because all the subjects in the study previously learned the same fraction concepts using physical manipulative models, the participants' responses to questions frequently compared their current experience using virtual manipulative models to their previous experience using physical manipulative models. The students reported that they liked the immediate feedback provided by the virtual manipulatives, and they found the virtual manipulatives easier and faster to use than the physical manipulatives.

Mathematics Study #3

The mixed-methods experiment conducted by Suh and Moyer (2007) included 36 third-grade students. It appears but it is not explicitly stated that the students were drawn

from 2 intact classrooms that may or may not have been part of the same school. Students in the treatment group spent one week working with online virtual balance scales learning about algebraic relationships, while students in the control group spent one week learning the same content by working with a set of commercially available physical manipulatives. The results of the statistical analyses of the quantitative data show significant improvement between pre-test and post-test for students using both virtual and physical manipulatives, but the researchers used relatively unsophisticated methods for the statistical analyses. Pre-treatment differences between groups appear to exist, but these differences were not tested for statistical significance. Additionally, no statistical comparisons between groups on the post-test measures of achievement or the overall learning gains made during the intervention are reported.

Although Suh and Moyer fail to make quantitative comparisons between groups in their study, the authors make qualitative comparisons between groups that allow them to identify unique features of both treatment conditions. Unique features of the physical manipulatives include tactile features of the physical manipulatives, the fact that physical manipulatives appeared to provide students with more opportunities to invent solution strategies, and the fact that they required students to use more mental mathematics. Unique features of the virtual manipulatives include the explicit linking of the visual and symbolic modes, guided step-by-step support in algorithmic processes, and the immediate feedback and self-checking system that were only possible in the virtual condition. The methods used to analyze the qualitative data are not explained in detail and only moderate support for the results are given, but the findings reported are interesting nonetheless.

Mathematics Study #4

Moyer, Niezgoda, and Stanley (2005) conducted a small, exploratory study of 18 ethnically diverse kindergarten students, 12 of whom spoke a language other than English at home. The same students learned about patterns by experiencing three different treatment conditions - physical, virtual, and symbolic notation - on three separate days, and the researchers drew comparisons between the number of exercises students completed and the creativity of the elements included in the patterns they created. The researchers found that students completed more patterns in the virtual condition and that their patterns were more creative. However, the fact that the students experienced the physical condition the day before they experienced the virtual condition potentially impacted these results. The young age of the students also potentially impacted their ability to create symbolic notations. No pre-test and post-test measures of achievement and no statistical analyses of the results exist, so it remains unclear how much learning took place during the intervention.

Mathematics Study #5

Nute (1997) contributed to the dialogue about physical and virtual manipulatives by examining the type of engagement activity used during instruction (viewing manipulative presentations vs. performing a manipulation), the type of manipulative used for instruction (physical vs. virtual), time on task, efficiency of learning, and student attitudes. Twelve teachers and 241 fourth, fifth, and sixth graders participated in the study. The researcher randomly assigned participants at what appears to be the classroom level to one of seven treatment conditions. Participants assigned to the first group, which

was the control group, did not use any manipulatives during instruction and completed only traditional textbook-style assignments. Participants assigned to one of the other six conditions completed either virtual assignments or paper-pencil assignments that the researcher designed for the unit taught during the intervention. The unit included four, 20 to 24 minute lessons that focused on building and changing patterns. One-third of the participants not assigned to the control group viewed manipulative demonstrations about building and changing patterns during the lessons. Another third of the participants not assigned to the control group performed manipulations themselves during the lessons. The remaining students viewed manipulative demonstrations and performed the manipulations themselves. Within each of these three conditions, half of the students completed the virtual assignments while the other half completed the paper-and-pencil assignments. Research assistants taught all groups, but different research assistants taught the students assigned to the virtual and physical conditions. However, the study was designed to keep instruction from the research assistants to a minimum.

Data collected during the experiment included a post-test, a student attitudinal questionnaire, and a teacher attitudinal questionnaire. The results of the study indicate no significant interactive or main effect on mathematics achievement result from the type of engagement activity used during instruction or from the type of manipulative used during instruction. Additionally, no significant differences exist between the overall mathematics achievement of students who receive exposure to manipulatives and control students who receive no exposure to manipulatives. However, 4th and 5th graders in the virtual condition take less time to complete their lessons than 4th and 5th graders in the physical condition.

Unfortunately, Nute wrote up the study in a manner that is very difficult to follow. Rather than isolating the most salient findings, she attempts to simultaneously study every aspect of the physical versus virtual comparison and achieves breadth but no depth. Additionally, the lessons used during the intervention lack both depth and breadth. The research assistants taught only four lessons on subsequent days, which students completed in 20 to 24 minutes. Other weakness of the study include that Nute drew students from multiple grade levels (4th, 5th, and 6th) but assigned them to treatment groups at the classroom level. While the author attributes all the significant differences in efficiency scores to grade level, many of the statistical tests compare the treatment groups to a single control group from one grade level, and the particular grade level of the control group is not reported. Pre-test results for this group as well as the other treatment groups were also either not collected or not reported. The study finds few differences between the students in the treatment groups, all of whom received some exposure to manipulatives, and the students in the control group, all of whom received no exposure to manipulatives. Given the results of several meta-analyses that show that students who interact with manipulative models during mathematics instruction generally outperform students who do not interact with manipulatives (Parham, 1983; Sowell, 1989; Suydam & Higgins, 1977), Nute's null findings suggest the possibility that either the post-test used did not adequately measure the constructs taught during instruction, the intervention did not allow sufficient exposure to the manipulatives to impact student learning, the manipulatives used in this lesson did not add depth to instruction, or the study lacked sufficient statistical power to reject the null hypothesis. Finally, the results indicate that the "computer manipulatives only" presentation group worked significantly longer than

the “concrete manipulatives only” presentation group, but no records were kept of the number of pattern designs or alternative solutions each participant made with the manipulatives.

Mathematics Study #6

Smith’s (2006) research examined how virtual versus physical manipulative models impact algebra instruction. It also examined the impact of virtual versus physical manipulative models on the attitudes of elementary students and on time-on-task. Thirty-nine 5th-grade students in a small, rural elementary school participated in the study. Smith used a stratified random sample to randomly assign all gifted and regular education students to treatment conditions. The seven students who qualified for special education services could not be randomly assigned to treatment conditions because of scheduling constraints, so the researcher analyzed the data collected during the study both with and without the special education students. The intervention included a unit about integers and a unit about expanding polynomial functions. The research design included reversing experimental conditions between units, so students who interacted with virtual manipulative models during the first unit interacted with physical manipulative models during the second unit and vice versa. Each unit included four instructional sessions. The data collected during the study included a pre-test, two post-tests (one at the end of each week of instruction), three interest surveys, and a manipulative use questionnaire. The research assistants also kept a time-on-task record in which students were monitored every two minutes. Despite the fact that significant gains were made between pre-test and post-test, the results of the study indicate that no differences between virtual and physical

manipulatives exist. The order in which students experience the two types of manipulative models seems to affect their preferences, and overall, no major differences exist between students' attitudes towards virtual and physical manipulative models. The results of the time-on-task behavior analyses were inconclusive: during the first unit, students in the virtual condition demonstrated less on-task behaviors, but during the second unit, no statistical difference existed between the two conditions.

Smith used a stronger research design than the authors of the first five mathematics studies discussed in this critical review, but methodological weaknesses still exist. First, the small sample size makes it possible that a type II error rather than a lack of real differences between treatment conditions caused the researcher to fail to reject the null hypothesis. Taking into account that the researcher randomized students into treatment conditions, the benefit of the crossover design appears lost. In fact, the crossover design may have actually made it more difficult to detect real differences between conditions because participants interacted with each manipulative for less time than if they had been assigned to only one treatment condition. However, both groups made significant gains in learning between pre-test and post-test, which suggests that virtual manipulatives are at least as effective as concrete manipulatives during instruction about addition and subtraction of integers and expanding polynomial factors.

Mathematics Study #7

Pleet (1991) conducted a three-week intervention in which students interacted with either a commercially available computer program that integrated virtual manipulatives into instruction or a set of commercially available physical manipulatives.

The study examined whether the computer program or the physical manipulatives had a larger impact on the acquisition of transformation geometry skills and mental rotation skills in 8th-grade geometry students. The study also examined possible sex differences between treatment conditions. Pre-test and post-test data were collected and analyzed for over 560 students from 30 classes in 15 different junior high schools. Eight teachers each taught one class using the computer program and one class using the physical manipulatives. Seven teachers taught two control classes each. The study used a non-equivalent control group design, and data was analyzed using an ANCOVA. The findings suggest there is no difference between the computer program that integrates virtual manipulatives into instruction and the physical manipulatives. Additionally, the study shows no difference between treatment conditions that use virtual or physical manipulatives and control conditions that do not use manipulatives. The results of the statistical tests that examined possible sex differences between treatment conditions are inconclusive, but the boys assigned to the computer program condition significantly outperformed the boys assigned to the physical manipulative condition on one measure.

Pleet conducted one of the largest studies of this topic to date. The quality of the writing used to describe the study and to report the results is quite high, but the research design could have been improved in two important ways. First and most importantly, using a computer program designed specifically to align with the curriculum used in the physical condition and using similar instructional scripts in both conditions would have controlled for the possibility of differences between conditions other than the form of manipulatives used for instruction. Second, the researcher did not find a statistically significant difference between the treatment groups that used manipulatives and the

control groups that did not use manipulatives, which suggests that none of the manipulatives have an impact on student learning above and beyond what students experience in traditional classroom settings. These null findings suggest that the manipulatives were not well integrated into the learning activities, the manipulatives were not appropriate for the learning activities, or the assessments were not an accurate measure of the learning that transpired. Improvements to one of these three areas could have strengthened the methodology and possibly would have yielded positive results.

Mathematics Study #8

Suh (2005) used a mixed-methods approach to examine both the impact of virtual and physical manipulatives on student achievement and the representation preferences that exist between the virtual environment and the physical environment. The study included 36 third-grade students in two different classrooms in the same elementary school. To control for teacher effects, the researcher taught all lessons in both classes during the intervention. Rather than assigning students to a physical condition or a virtual condition, the researcher used a within-subjects, crossover, repeated measures design. All subjects received both treatments, and as such, the researcher used each student as his or her own comparison. To avoid any residual effects, the researcher introduced two completely different mathematics units - fractions and algebra - as the topics of study during the intervention. The students in one class experienced the fractions unit using virtual manipulatives and the algebra unit using physical manipulatives. The students in the other class experienced the fractions unit using physical manipulatives and the algebra unit using virtual manipulatives. The fractions unit focused on how to add units

with unlike denominators while the algebra unit focused on how to balance equations in algebra. When experiencing the virtual manipulative condition, students used a free set of online manipulatives and completed problems on a computer screen. When experiencing the physical manipulative condition, students used commercially available manipulatives and completed problems using a task sheet.

To assess the amount of learning that took place during the intervention, the researcher developed and administered a pre-test and post-test. The results of the quantitative analyses indicate that all students who use virtual or physical manipulatives improve significantly between pre-test and post-test, but students in the virtual condition only outperform students in the physical condition during instruction about fractions. This suggests that virtual manipulatives are particularly effective during instruction about fractions. The researcher also collected qualitative data in the form of field notes, classroom videotapes, and student interviews. The results of the qualitative analyses indicate that certain unique features in the virtual manipulative environment help guide students through the process of learning formal algorithms for adding fraction with unlike denominators. Some of these unique features include linked representations, step-by-step procedures, and immediate feedback systems.

The Suh study is stronger than most of the other mathematics studies discussed here, both in the quality of the methodology as well as the clarity of the writing. The within-subjects, crossover design allowed the researcher to use each student as his or her own control for comparisons between the algebra unit and the fractions unit. Unfortunately, the decision to a within-subjects, crossover design is also a weakness of the study because the student cannot be used as his or her own control for any

comparisons between virtual and physical manipulatives within the algebra unit or within the fractions unit because each student only experienced one of the two conditions for each unit. Since the students were assigned to treatment conditions at the classroom rather than the individual student level, there may be differences between the treatment groups other than the form of manipulatives used for instruction that explain why students in the virtual condition outperformed students in the physical condition during the fractions unit. The fact that the unique features of the virtual manipulatives include “step-by-step procedures” also suggests that there were differences between the treatment conditions other than the form of manipulatives used for instruction.

Summary of Mathematics Studies

The mathematics studies summarized in this critical review inform the proposed study by providing the empirical basis for the hypotheses presented in Chapter I. In all cases where the researchers made quantitative comparisons between virtual and physical manipulatives, the researchers find that virtual manipulatives are at least as effective as physical manipulatives. While in some cases participants in both treatment conditions failed to make significant improvements between pre-test and post-test, no studies find that participants in the physical condition outperform students in the virtual condition. Participants in the virtual condition outperformed students in the physical condition in Mathematics Study #8, but only during the unit on fractions. These results suggest that instruction about fractions may be particularly well suited to virtual manipulatives. The results of the qualitative analyses summarized in this critical review also suggest that virtual manipulatives are more time-efficient than physical manipulatives, which allows

students who use virtual manipulatives to complete more practice activities than students who use physical manipulatives. However, this dissertation is the first mathematics study to report the number of practice activities students complete on every day of instruction, and unlike any previous mathematics studies that compare virtual and physical manipulatives, this dissertation includes the number of practice activities students complete as a variable in the quantitative analyses to determine if additional deliberate practice positively impacts students understanding of basic fraction concepts.

The mathematics studies summarized in this critical review also inform the research design of the proposed study by highlighting potential threats to the validity of the results. Previous mathematics studies show that it can be difficult to find significant differences between the virtual condition and the physical condition and in some cases, it can even be difficult to find significant differences between pre-test and post-test measures of achievement. These studies highlight the importance of using sufficiently large sample sizes, pre-testing students prior to the start of the intervention to determine the level of previous exposure to the content being taught during the intervention, giving students an extended period of time to interact with the manipulatives, and ensuring that manipulatives are well integrated into the instructional content. Additionally, evidence of pre-treatment differences between participants assigned to different treatment conditions at the classroom level highlight the potential benefits of within-class random assignment. Finally, noticeable differences between treatment conditions other than the form of manipulatives used for instruction highlight the importance of controlling for other important variables with the potential to influence student learning.

Science Studies

Science Study #1

The study conducted by Triona and Klahr (2003) asks whether the presentation medium used in teaching children to design unconfounded science experiments influences learning. In the literature review, the authors pointed out that most other studies of the relative effectiveness of computer-based and non-computer based instruction in the science and mathematics literature intentionally confound the contrast between the two forms of manipulatives. In other words, with a few notable exceptions, other studies that compare physical and virtual manipulatives include differences between the two treatment conditions besides the form of the manipulatives being compared. Because of this tendency to confound the contrast between treatment conditions, Triona and Klahr took steps to ensure that instructional content and process were the same in the physical and virtual conditions so any differences in outcomes that were found could be attributed solely to the instructional medium. The following paragraphs provide more specific details about the methodology and the results of this study.

All 92 fourth and fifth graders who participated in the Triona and Klahr study learned the control of variables strategy (CVS), which includes both the rationale and procedure for setting up simple experiments (Chen & Klahr, 1999). The researchers recruited students to participate in the study by sending notices to parents and then randomly assigned students to a physical condition or a virtual condition. Students in the physical condition attended a training session in which they learned to design unconfounded experiments using physical springs and weights of different sizes.

Students in the virtual condition attended a training session in which they learned to design unconfounded experiments using virtual springs and weights of different sizes. Other variables with the potential to influence instruction including the teacher, lesson plans, instructional script, time-on-task, number and type of examples, types of questions from the teacher, and the learners' choice of how to set up different experiments, were the same across treatment conditions. Data collected during the study included a pre-test and a post-test the researchers administered on the same day as the training session. The researchers also administered a test for transfer a week after the training session. All participants in both treatment conditions worked with physical ramps during the test for transfer. Data was analyzed using a 2 (condition: physical vs. virtual) X 3 (phase: pretest and training, posttest, and transfer) factorial design. The results indicate that students learn to design unconfounded experiments equally well using physical and virtual manipulatives. Students who use either form of manipulatives also make significant gains between pre-test and post-test, and, based on the results of the test for transfer, have a similar ability to transfer learning gains to other content domains.

While the results of the Triona and Klahr study can be considered reliable given the high quality of the research methodology used by the authors, they are somewhat limited in their generalizability. The results can only be generalized to populations of students who are very similar to the participants recruited to participate in this study, and it is unclear from the results of this study whether other types of science instruction other than CVS would be equally as effective in a virtual environment as they are in a physical environment. For these reasons, Klahr, Triona, and Williams (2007) conducted a second study that used very similar methods but differed from the Triona and Klahr study in

several important ways. This second study is described in further detail in the following sub-section.

Science Study #2

Subjects in the study conducted by Klahr, Triona, and Williams (2007) engaged in a “hands-on”, discovery learning activity in which they designed and tested mousetrap cars with the goal of designing the car that would go the farthest. The primary research question of the study asked whether or not children’s knowledge gains about mousetrap cars would be different if they built virtual mousetrap cars versus physical mousetrap cars, so the researchers assigned students to either a virtual manipulative condition or a physical manipulative condition. A related question asked whether constraints on either the total amount of time given or the total number of practice exercises students complete affects learning, so students were also assigned to a condition in which they were given a fixed amount of time in which to design and test as many cars as possible or a condition in which they were given an unlimited amount of time to design and test a fixed number of cars. This made for a total of 4 treatment conditions. To track student learning during the intervention, the researchers administered knowledge assessment questionnaires before and after students interacted with the mousetrap cars. The assessments measured changes in the children’s knowledge about the features that enable a mousetrap car to go further. Students also used datasheets to record their various trials with the mousetrap cars.

To analyze the results of the study, the authors used a 2 (material: physical vs. virtual) X 2 (constraint: fixed amount of time vs. fixed amount of cars) X 2 (test phase:

pretest vs. posttest) factorial design with test phase as a within-participant factor. While the authors do not find an advantage to either the physical or virtual manipulative conditions, they find that all four treatment conditions are equally effective in producing significant gains in learners' knowledge about causal factors, in their ability to design optimal cars, and in their confidence in their knowledge. The study reports that there are no significant differences in learning outcomes for girls and boys, but children can construct and test cars much faster using virtual manipulatives than physical manipulatives. Students who use virtual manipulatives are able to construct a set number of cars in less time than students who use physical manipulatives, and students who are asked to construct as many cars as possible in a set amount of time complete and test more cars than their counterparts who use physical manipulatives.

Like the Triona and Klahr (2003) study, the Klahr, Triona, and Williams study used very strong research methodology, and the results can be considered reliable. While a larger study would be necessary to determine if the results of this study are generalizable to *all* populations of students, this study extends upon the previous work of Triona and Klahr by examining an older population of students (7th and 8th graders instead of 3rd and 4th graders) a different instructional context (discovery learning instead of direct instruction), and by randomly assigning participants to treatment conditions at the individual student level. The Klahr, Triona, and Williams study also expanded upon the previous work of Triona and Klahr by testing the effect of gender and time. Gender was not found to be a statistically significant predictor of the outcome, but time appears to be an important variable. Klahr, Triona, and Williams used quantitative methods to determine that students who use virtual manipulatives can construct and test more cars

than students who use physical manipulatives. The qualitative analyses included in the mathematics studies reviewed in the previous section suggest this would be the case, but no other study used quantitative methods to empirically test the difference in time efficiency between treatment conditions.

Summary of Science Studies

Both of the science studies included in this review give further support to the hypotheses proposed in this dissertation. Science Study #1 and Science Study #2 show that virtual manipulatives are at least as effective as physical manipulatives, and in Science Study #2, quantitative methods show that students who use virtual manipulatives construct and test cars much faster than students who use physical manipulatives. However, this dissertation extends the work of Triona and Klahr and of Klahr, Triona, and Williams by examining a different content domain and by investigating whether additional deliberate practice using manipulatives impacts student learning. This dissertation also examines an intervention that took place over a period of two weeks. In contrast, the interventions examined in Science Study #1 and Science Study #2 lasted only one day.

The science studies included in this critical review also informed this dissertation's methodology. Both science studies and especially Science Study #2 use considerably stronger methodology than any of the mathematics studies. Similar to Science Study #1 and #2, this dissertation takes measures to control for important instructional content and process variables between treatment conditions. Similar to the latter study by Klahr, Triona, and Williams, this dissertation also used within-class

random assignment and controlled for the amount of time spent practicing using the manipulatives in both treatment conditions. Note that Chapter III reports more specific details about the methodology used in this dissertation.

SECTION TWO: CONCEPTUAL FRAMEWORK

Figure 1⁴ displays the conceptual framework for this dissertation. In summary, Figure 1 shows that students begin formal instruction about basic fraction concepts with formal knowledge of whole number concepts and informal knowledge of fractions. When students first are taught basic fraction concepts, the combination of their previous formal and informal knowledge and the content of the new instruction about fractions results in student learning. When teachers integrate virtual or physical manipulatives into instruction about basic fraction concepts, the combination of previous formal and informal knowledge, the content of the new instruction, and the deliberate practice with the manipulatives results in a greater amount of student learning. Students who learn about fractions using virtual manipulatives experience certain unique environmental factors (i.e. sound effects, immediate feedback from the computer). Students who learn about fractions using physical manipulatives experience other unique environmental factors (i.e. the tactile nature of the manipulatives, delayed feedback from the teacher). There is no indication in the literature that the impact of these environmental factors on student learning is greater for students who use virtual manipulatives or for students who use physical manipulatives. However, virtual manipulatives are more time-efficient than

⁴ See Appendix A.

physical manipulatives, so students in the virtual condition complete more practice activities. The added opportunity for deliberate practice in the virtual condition positively impacts student learning. What follows is a more extensive discussion of each of the major ideas included in the conceptual framework.

Formal Knowledge of Whole Numbers and Informal Knowledge of Fractions

Whole number concepts dominate the majority of mathematics instruction in grades PreK-2. When children first begin formal instruction about fractions in the upper elementary grades, they must adopt new rules for fractions that often conflict with their well-established ideas about whole numbers (Bezuk & Cramer, 1989). For example, when students first learn how to order fractions, they learn that $\frac{1}{3}$ is less than $\frac{1}{2}$, but when students learn whole numbers, they learn that 3 is greater than 2. Fortunately, most students also have some informal knowledge of fractions, which helps them understand the inverse relationship between the size of the denominator of a fraction and the relative size of the fraction. For example, most students understand informal notions of partitioning, sharing, and measuring, and even very young children usually appreciate the idea of a “fair share” (NRC, 2001). Students’ informal knowledge of fractions can enhance their understanding of rational number concepts, but students’ informal knowledge of fractions and other rational numbers is probably less than their corresponding informal understanding of whole numbers (NRC, 2001). For these reasons, if students only learn rote procedures and algorithms during formal instruction about fractions, they gain only a limited understanding of fractions.

Formal Instruction About Basic Fraction Concepts With Manipulatives

Given students' demonstrated lack of understanding of basic fraction concepts, both NCTM (2006) and NMAP (2008) recommended that teachers emphasize conceptual understanding during instruction about fractions. In addition, NMAP recommended that teachers emphasize fluency with algorithmic procedures involving fractions. Empirical evidence suggests that using physical manipulative models (i.e. fraction strips, fraction circles) during instruction about basic fraction concepts enhances students' conceptual understanding of fractions without impeding their ability to complete algorithmic procedures involving fractions (Cramer, et al., 2002). In addition, the results of several meta-analyses suggest that students who interact with manipulatives during mathematics instruction outperform students who do not interact with manipulatives (Parham, 1983; Sowell, 1989; Suydam & Higgins, 1977). Therefore, Figure 1 shows that students who interact with manipulatives while learning basic fraction concepts outperform students who do not interact with manipulatives.

Instruction with Virtual Manipulatives Compared to Instruction with Physical Manipulatives

Clements (1999) states that most practitioners and researchers believe that manipulatives are effective because they are concrete, and by "concrete" these individuals mean that manipulatives are effective because students can hold them in their hands. Clements challenged this singular notion of the term concrete by proposing two different types of concrete knowledge: sensory-concrete and integrated-concrete. Sensory-concrete knowledge requires sensory materials to make sense of an idea. Integrated-concrete knowledge is built as students learn. The strength of integrated-

concrete knowledge is in the combination of many separate ideas in an interconnected structure of knowledge. Following this logic, the strength of manipulatives is not in the physicality of the manipulatives themselves, rather, “good manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas” (Clements, 1999, p. 49). Clements hypothesizes that virtual manipulatives provide students with representations of mathematical ideas that are just as meaningful to students as physical manipulatives. A comprehensive search of the mathematics and science literature yielded ten studies that tested this hypothesis. The combined results of these studies suggest that virtual manipulatives are at least as effective as physical manipulatives (Klahr, Triona, & Williams, 2007; Moyer, Niezgod, & Stanley, 2005; Nute, 1997; Pleet, 1991; Reimer & Moyer, 2005; Smith, 2006; Steen, Brooks, & Lyon, 2006; Suh, 2005; Suh & Moyer, 2007; Triona & Klahr, 2003). In addition, the results of one study suggested that instruction about fractions is particularly well suited to virtual manipulatives (Suh, 2005).

The Impact of Environmental Factors Unique to the Virtual and Physical Manipulative Conditions

Students who learn about fractions using virtual manipulatives experience different environmental factors than students who learn about fractions in a physical environment using physical manipulatives. For example, students who learn about fractions using virtual manipulatives may hear computer-generated sound effects and may receive immediate feedback about their responses to practice exercises. Students who learn about fractions using physical manipulatives experience the tactile nature of the manipulatives themselves and receive delayed feedback from the teacher, usually in

the form of corrected responses to practice exercises that are returned to the student one or more days after they were turned in to the teacher. The fact that previous comparisons of virtual and physical manipulatives show that virtual manipulatives are at least as effective as physical manipulatives suggests that the combination of environmental factors unique to both conditions do not impact student learning in a way that creates an advantage for students who use virtual manipulatives or for students who use physical manipulatives, although the possibility exists that certain environmental factors have a stronger impact on student learning than others.

The Impact of Deliberate Practice

Previous mathematics and science studies indicate that virtual manipulatives are more time-efficient than physical manipulatives (Klahr, et al., 2007; Moyer, et al., 2005; Reimer & Moyer, 2005; Steen, et al., 2006). For this reason, Figure 1 predicts that when time is held constant, students who use virtual manipulatives will complete more deliberate practice than students who use physical manipulatives. The work of Ericsson and colleagues (1993) shows that expert performance can be explained in terms of acquired characteristics resulting from extended deliberate practice. Ericsson and colleagues describe deliberate practice activities as those activities that were designed to improve a student's level of performance and which allow for "repeated experiences in which the individual can attend to the critical aspects of the situation and incrementally improve her or his performance in response to knowledge of results, feedback, or both from a teacher" (Ericsson, et al., 1993, p. 368). Ericsson and colleagues' theoretical framework, which they tested empirically, shows that the amount of time an individual

engages in deliberate practice is related to performance. In addition, the work of Cramer and colleagues shows that students need to interact with manipulatives over an extended period of time to develop the mental images necessary to think conceptually about fractions (Cramer, et al., 1997; Cramer, et al., 2002). Considering the theoretical and empirical work of these scholars, the hypotheses posed in this dissertation predict that deliberate practice has a positive impact on student learning. See Chapter III for a more a detailed description of the specific methodology used to test these hypotheses.

CHAPTER III

METHODS

Research Type and Research Perspective

This dissertation used a randomized experiment to assess the relative instructional efficiency of virtual fractions manipulatives and physical fractions manipulatives. The study used a quantitative perspective.

Pilot

Prior to the full implementation of the study, the researcher conducted a pilot at a private school for students with special needs located in Middle Tennessee. The school houses students in grades pre-K-12, and the typical class size for all grades is 3-6 students. Approximately 94% of students at the school are Caucasian and 72% are male. Students begin learning to use Macbook laptops in kindergarten, and by the time students reach upper elementary school, the majority of students' academic classes integrate at least 10 – 20 minutes of laptop computer time into every class period. However, most teachers use the laptops for individualized practice or supplemental activities at the beginning or end of class rather than during whole-class or small-group instruction.

A total of twenty-one 6th grade students (13 boys, 8 girls) drawn from 5 different mathematics classes participated in the pilot study. Thirteen of the 21 students tested below grade-level in mathematics at the beginning of the school year. The researcher randomly assigned students within-class to a physical manipulative condition or a virtual

manipulative condition. The pilot lasted for a total of 2 weeks; the 10 students assigned to the physical manipulative condition participated in the intervention for 5 days during the first week of the pilot and the 11 students assigned to the virtual manipulative condition participated in the intervention for 5 days during the second week of the pilot. All students completed a pre-assessment prior to the first day of instruction and a post-assessment on the last day of instruction. The pre-assessment results indicated that students assigned to the physical and virtual manipulative conditions began the study with approximately the same prior knowledge of fractions, but students assigned to the virtual manipulative condition received higher mean scores on the post-assessment. Students assigned to the virtual manipulative condition also completed more practice activities than students assigned to the physical manipulative condition.

Although the results of the pilot study are interesting in that they support the hypotheses posed in this dissertation, the primary purpose of the pilot was to field test the research methods and the instructional materials the researcher planned to use during the full implementation of the study. As such, the researcher made changes to the research methods and the instructional materials as needed during the pilot. These changes included revisions to the instructional scripts, adjustments to the pacing and timing of various aspects of the lessons, and trouble shooting of the computer program used to teach students assigned to the virtual condition. These changes potentially created slight differences between treatment conditions. Because of the potential of slight differences between treatment conditions and because the pilot included a small number of students, this dissertation includes only a brief description of the pilot, and the discussion and conclusions presented in Chapter V only consider the results of the pilot within the

context of the discussion of the generalizability of the results of the full implementation of the study. The next section of this chapter gives a detailed description of the methods used to conduct the full implementation.

Full Implementation

Participants and Setting

The full implementation of the study took place at a charter middle school in Middle Tennessee that houses students in grades 5-8. Approximately 98.9% of students in the school are African-American, and 88% of students qualify for free- and reduced-price lunch. All classes at the school are single-gender, and typical class size is 22-24 students. Approximately 62% of the students that participated in the study tested below grade level in mathematics during a recent administration of a comprehensive benchmark assessment administered by a private assessment company. The school owns a PC computer lab, and most students in the school spend at least some time in the computer lab during the week. Some classrooms at the school are also equipped with student computer stations. The amount of time students use computers during class and the type of activities they complete using computers varies quite significantly between classes, but in general, students spend very little time using computers during their regular academic classes. The students who participated in the full implementation of the study had not used computers during mathematics class at any point during the school year.

The entire 5th grade participated in the intervention, but the study sample only included students who signed an assent form and whose parents signed an informed

consent form. The original study sample included a total of 70 students, but of those 70 students, 2 students transferred to another middle school during the first week of the intervention and were dropped from the study. Another student missed 4 days of instruction during the first week of the intervention and was subsequently excluded, so the final sample included a total of 67 students (39 girls, 28 boys).

Design

Prior to the first day of the intervention, the researcher randomly assigned half of the students within each of the four 5th grade mathematics classes to a virtual manipulative condition and the other half of the students within each class to a physical manipulative condition. In cooperation with the school's administrators and the school's 5th grade teachers, the researcher reorganized the 5th grade students class schedules so that students assigned to the same treatment condition participated in the intervention in groups of 22 – 24 students⁵. Since the school groups students into single-gender classes and the school administrators expressed a strong preference for maintaining gender separation during the intervention, the researcher grouped students according to gender as well as according to treatment condition. This created a 2 (treatment: physical vs. virtual) × 2 (gender: girls vs. boys) factorial design. The Girls Virtual (n = 22) and Girls Physical (n = 17) conditions participated in the intervention in the morning before lunch while the Boys Virtual (n = 12) and Boys Physical (n = 16) conditions participated in the intervention in the afternoon after lunch.

⁵ Although students participated in the intervention in groups of 22 – 24 students, the researcher only collected data for students' with the appropriate informed consent forms. As a result, the "n" for each treatment group ranges from 12 – 22 students

Procedures

To control for possible teacher effects, the researcher acted as the teacher in all 4 of the treatment conditions and used instructional scripts during all lessons. The students' regular classroom teacher remained in the classroom during all lessons, and an educational assistant attended approximately 50% of the afternoon lessons with the Boys Virtual and Boys Physical conditions⁶. The students' regular classroom teacher and the educational assistant helped the researcher with various aspects of classroom management but did not participate in instruction. The full implementation of the study lasted 11 days. On Day 0 of the study, which took place right before the Thanksgiving holiday, the researcher visited all 4 of the 5th grade mathematics classes at the site school to introduce the study to the students and to administer a pre-assessment of students' knowledge of 5th grade fractions concepts. On Day 1 of the study, which took place right after the Thanksgiving holiday, students began learning basic fractions concepts using either physical or virtual manipulatives. Instruction lasted for 9 consecutive days, and on Day 10 of the intervention, students completed 2 post-assessments. Students also completed an assessment on Day 5. The following sections give additional details about the procedures specific to each treatment condition.

Physical Manipulative Condition

The researcher used instructional scripts during all lessons taught to students assigned to the physical manipulative condition. The instructional scripts closely aligned

⁶ An educational assistant is usually present during all classes at the school site, but the school experienced some unexpected staffing issues during the intervention that prevented the administration from providing an assistant in all of the classes taught during the study.

with the first 10 lessons of a commercially available, manipulative-based fractions curriculum. The complete commercial fractions curriculum included 30 lessons. The researcher chose this particular fractions curriculum for the study because a well-known and well-respected math educator designed the curriculum specifically for struggling students similar to those that participated in the study, the curriculum fully integrates manipulatives into both instruction and practice, and it assumes no prior knowledge of fractions. The curriculum is also being used in all 50 states and in several school districts in Middle Tennessee. The only major differences between the instructional scripts designed by the researcher for the study and the teacher's manual that accompanied the commercial curriculum related to the research methods used in the study. For example, the research methods used in the study called for the researcher to hold the total amount of time students spent completing practice activities with the manipulatives constant across treatment conditions, so the researcher added time limits to the instructional scripts for all of the practice activities.

Students assigned to the physical manipulative condition received instruction in a regular mathematics classroom at the school site. The researcher provided all the materials necessary to complete the intervention, which included pencils, workbooks, scissors, and enough colored paper strips for each student to construct a fractions kit. Students participated in the following instructional activities during the intervention:

1. Teacher-led, whole-class instruction – The researcher facilitated whole-class instruction using instructional scripts. The dialogue of the instructional scripts closely mirrored the dialogue included in the teacher's manual that accompanied

- the commercial curriculum. The researcher used a set of magnetic fractions strips to demonstrate concepts to the students. She asked students questions and reviewed as necessary based on the accuracy of student responses. The researcher also periodically responded to questions generated by students.
2. Construction of a fractions kit – Students constructed a set of fractions manipulatives using scissors, strips of colored paper, and a pencil. The fractions kit only included representations of a whole, halves, fourths, eighths, and sixteenths⁷.
 3. Fractions games – Students played 2-player games using the fractions kits and a special fractions cube⁸. The researcher set a time limit of between 6 and 10 minutes on each day the students played games and asked the students to complete as many rounds of the game as possible within that time limit.
 4. Practice exercises – Students used manipulatives to complete practice exercises. The researcher set a time limit of between 7 and 10 minutes on each day the students completed practice exercises and asked students to complete a set of problems in a workbook from the commercial curriculum. The researcher also created a set of additional worksheets with similar problems as the workbook. Students who completed all of the workbook problems within the time limit

⁷ In the commercial curriculum used in this intervention, thirds, sixth, and twelfths are introduced during the third week of instruction. As was previously mentioned, instruction during the full implementation lasted for only two weeks.

⁸ It is important to note that the use of a fractions cube introduced an element of chance into the fractions games. As a result, the students in the class who knew the most fractions content did not necessarily play the most games and did not necessarily win the most games.

received an additional worksheet to complete. Students who completed all of the problems on the first additional worksheet received a second additional worksheet to complete. Students received up to 4 additional worksheets. The researcher corrected the workbooks and worksheets and returned them to the students within 2 days.

Virtual Manipulative Condition

The researcher facilitated instruction in the virtual manipulative condition using Macbook laptops and a software program designed specifically for this dissertation study. Rather than using a fractions kit made out of scissors, pencils, and colored paper, students in the virtual manipulative condition simulated the construction of a fractions kit on the computer and then used the virtual fractions kit during the rest of the intervention. Rather than using a set of magnetic fractions strips when demonstrating concepts to students, the researcher used a computer and a projector to demonstrate concepts to students using a virtual fractions kit. Since the software program used in this dissertation study was intentionally designed to align with instruction in the physical condition, the researcher facilitated instruction in the virtual manipulative condition using scripts that aligned very closely with the scripts used to facilitate instruction in the physical condition. The only differences between the instructional scripts used in the physical and virtual manipulative conditions related to the directions given to students. For example, a portion of the script used in the physical manipulative condition on Day 2 called for students to look up vocabulary words in the glossary in the back of their workbooks and then called for the researcher to choose students to read the definitions out loud to the whole class. A portion of the script used in the virtual manipulative condition on the same day called for

students to use their mouse to click on the glossary icon located at the bottom of their computer screen and to click on the appropriate words within the vocabulary list. The script then called for the researcher to choose students to read the definitions out loud to the whole class.

Students participated in the same type of activities in the virtual manipulative condition as students experienced in the physical manipulative condition, but the activities in the virtual manipulative condition differed from the physical manipulative condition in the following ways:

1. Teacher-led, whole-class instruction – The researcher projected a virtual fractions kit onto a screen and used the virtual fractions kit to demonstrate concepts to the students.
2. Construction of a virtual fractions kit – Students constructed a virtual fractions kit by simulating the actions of folding, cutting, and labeling paper fractions strips.
3. Fractions games – Students used a computer interface with a virtual fractions cube and a virtual fractions kit to play 2-player games.
4. Practice Exercises – Students used virtual manipulatives to complete practice exercises on their laptops. Note that the content, sequence, and maximum number of practice exercises as well as the type of feedback given to students were held constant between treatment conditions. However, the virtual manipulative condition differed from the physical manipulative condition in that

the computer provided students with immediate rather than delayed feedback about the accuracy of their responses.

Data Collection

The researcher administered 4 paper-and-pencil assessments during the study. When correcting the assessments, the researcher marked each question correct or incorrect and tallied the total number of correct responses. In addition to the 4 assessments, the data collected during the study included practice logs. The following sections provide additional details about the methods used to collect the assessments and the practice logs. Sample questions from all of the assessments can be found in Appendix B.

Fractions Probe (Pre-Assessment)

Prior to the first day of the intervention, the researcher administered a pre-assessment to determine students' prior knowledge of 5th grade fractions content. The researcher designed the paper-and-pencil assessment using software provided by a private assessment company that contracts with schools nationwide to measure and improve student achievement and to predict students' performance on state exams. The assessment company periodically administers comprehensive benchmark assessments to the entire student body at the school site where the study took place, but individual teachers at the school can create diagnostic "probes" focused on a specific content area at any time. The probes draw questions from a testbank of validated, multiple-choice assessment items. The fractions probe created by the researcher for this study included 20 multiple-choice questions drawn from the 5th grade testbank. The questions tested students' knowledge of

estimation of fractions and decimals, addition and subtraction of fractions and mixed numbers, equivalent representations, and comparing fractions.

Day 5 Assessment

On Day 5 of the intervention, all students completed a paper-and-pencil assessment of the content taught during the first week of the intervention. The 12-question assessment covered content taught during the first week of instruction and was drawn directly from the commercial fractions curriculum used during the intervention. Students were allowed to use their physical or virtual fractions kits during the assessment.

Day 10 Assessment

On Day 10 of the intervention, all students completed a 20-question, paper-and-pencil assessment of the content taught during the second week of the intervention. This assessment was also drawn directly from the commercial fractions curriculum used during the intervention. Students were allowed to use their physical or virtual fractions kits during the assessment.

Fractions Probe (Post-Assessment)

The researcher administered the fractions probe for a second time on Day 10 to determine if the manipulative-based fractions intervention impacted students' ability to answer multiple-choice questions about fractions similar to the questions 5th grade students in Tennessee encounter on state exams. Students were not allowed to use manipulatives to answer questions on the Fractions Probe, but note that the fact that

students were not allowed to use manipulatives on the Fractions Probe is just one reason why it was considered a test for transfer. Questions on the Fractions Probe were also asked in a different format than the students had been exposed to during the previous two weeks of instruction, and the test included fractions with denominators other than the denominators students learned using the fraction kits.

Practice Logs – Physical Condition

The commercial fractions curriculum used for this study included two types of practice activities: games and practice exercises. In the physical manipulative condition, students recorded the outcome of each game on a scorecard. At the end of each day in which students played games, the researcher used the scorecards to tally the total number of games played by each student and then recorded the data in a practice log. Students used workbooks and worksheets to complete practice exercises. At the end of each day in which students completed practice exercises, the researcher corrected the workbooks and worksheets and then tallied the total number of practice exercises completed by each student. The researcher also recorded this data in the practice log.

Practice Logs – Virtual Condition

At the beginning of each class session in the virtual manipulative condition, students logged-in to the fractions program used for instruction. Once students logged-in, the computer kept a running tally of the number of games played by each student and the total number of practice exercises completed by each student. The researcher downloaded this data into the practice log every 3-4 days

Data Analysis

Database

The researcher compiled the data collected during the study into a STATA database. The database included a unique ID for each student, a dummy variable that indicated treatment condition, a dummy variable that indicated gender, the raw data for each assessment, the raw data from the practice logs, a tabulation of the total number of practice exercises completed overall and during each week of the intervention, and a tabulation of the total number of games completed overall and during each week of the intervention.

Missing Data

The practice log data included in the STATA database has missing observations for 4 students who were absent for one or two days on instruction during the first week of the intervention and 2 students who were absent for one or two days of instruction during the second week of the intervention. None of the students who were absent during the first week of the intervention were absent during the second week of the intervention. In addition, assessment data is missing for eleven students on Day 0⁹, two students on Day 5, and one student on Day 10. The researcher chose to impute the missing data in the database using the “ice” program in STATA. The ice program in STATA utilizes the expectation-maximization algorithm for Maximum Likelihood Estimation recommended

⁹ Day 0 of the intervention was the last school day before the Thanksgiving holiday, and as a result, there were a higher number of absences on Day 0 than on any other day during the intervention.

by Schafer and Graham (2002). When less than 20% of data is missing at random, simulation studies indicate that imputation leads to the same conclusions as case-wise deletion (Schafer & Graham, 2002). The missing pre-assessment scores were estimated from all non-missing data from the study as well as from the scores from a recent benchmark mathematics assessment administered by a private assessment company.

Other Data Problems

The students assigned to the Boys Physical condition arrived late to class on Day 4. The instructional scripts for that day called for the students to participate in whole-class instruction about a new fractions concept, to complete a set of practice exercises, and to play the same fractions game that the researcher introduced to the students on Day 3. Since the researcher did not have time to complete all 3 activities and the whole-class instruction and practice exercises were the more essential components of instruction for that day, the researcher chose to skip the fractions game in the Boys Physical condition on Day 4. On Day 7, a discipline problem arose during the Girls Physical condition that caused the researcher to run short of time at the end of class. No students in the Girls Physical played the fractions game on Day 7, but all students participated in whole-class instruction and completed practice exercises, which again were the more essential components of instruction for that particular day. Students played fractions games during class on Day 3, Day 4, Day 6, Day 7, Day 8, and Day 9, but since all the students in one treatment did not have the opportunity to play games on Day 4 and Day 7, the researcher dropped the game data for all students on those days when analyzing the second research

question. However, the researcher included all the available game data in the analysis of the third research question.

Statistical Tests

The researcher used ANCOVA models to analyze the research questions posed in the first chapter of this dissertation. All of the statistical models included the students' scores on the pre-assessment Fractions Probe as a covariate. The inclusion of a covariate with a strong correlation with the outcome variables increased the sensitivity of the tests of main effects and interactions by reducing the error terms. Chapter IV presents a summary of the results of each of the statistical tests used to analyze the research questions posed in the first chapter of this dissertation.

CHAPTER IV

RESULTS

As stated in Chapter I, this dissertation uses a randomized experiment to determine if differences in students' knowledge of fraction magnitude exist when students learn basic fraction concepts using virtual manipulatives compared to when students learn basic fraction concepts using physical manipulatives. This dissertation also examines the time efficiency of using virtual rather than physical manipulatives to learn fractions by tracking the number of practice activities students complete on each day of instruction and making comparisons between treatment conditions. Chapter IV begins by examining students' knowledge of fraction magnitude prior to the start of the intervention by reporting the results of the Fractions Probe given on Day 0. It then examines the first research question posed in Chapter I by reporting the results of the Day 5 and Day 10 assessments as well as the results of the Fractions Probe given at post-assessment. Next, it examines the second research question posed in Chapter I by analyzing the data collected in the practice logs (i.e. the number of practice exercises and the number of games). Finally, it examines the third research question posed in Chapter I by including the number of practice exercises and the number of games as covariates in further analyses of the Day 5 and Day 10 assessments.

Pre-Assessment

To determine students' knowledge of fraction magnitude prior to the start of the intervention, the researcher administered a Fractions Probe on Day 0. The results of the 20-question pre-assessment showed that most students began the intervention with at least some prior knowledge of fractions, but the majority of students fell short of demonstrating mastery of the 5th grade fractions concepts they are likely to encounter on state assessments ($M = 7.02$, $SD = 3.29$). Students in the physical manipulative condition ($M = 6.93$, $SD = 3.83$) and virtual manipulative ($M = 7.11$, $SD = 2.71$) condition demonstrated similar prior knowledge of fractions at pre-assessment, $F(1, 52) = 0.13$, $p < 0.67$, but the boys ($M = 8.33$, $SD = 2.68$) scored significantly higher on the fractions probe than the girls ($M = 6.03$, $SD = 3.39$), $F(1, 52) = 7.36$, $p < 0.01$, $d = 0.74$.

Research Question #1

1. Are there differences in students' knowledge of fraction magnitude when they are taught basic fraction concepts using virtual manipulatives compared to when they are taught basic fraction concepts using physical manipulatives?

The researcher examined Research Question #1 using three separate ANCOVA models that included the results of the Day 5, Day 10, and Fractions Probe (Day 10) assessments as outcome variables. Chapter I of this dissertation hypothesizes that students who use virtual manipulatives learn as much about fraction magnitude as students who use physical manipulatives, and the results of the post-assessments support

this hypothesis: students assigned to the virtual condition received higher mean scores on all of the post-assessments. The boys also received higher mean scores than the girls. While the differences between manipulative treatment conditions and between genders were not all significant, the Fractions Probe given at pre-assessment was a significant predictor of the outcome in all of the ANCOVA models (all $ps < .01$). The following sections summarize the results of each of the post-assessments. Additionally, Table A1 presents a side-by-side comparison of the mean and standard deviation of each assessment by manipulative treatment condition and by gender.

Day 5 Assessment

At the end of the first week of the intervention, students completed a 12-question, paper-and-pencil assessment of the content taught during the first 5 days of the intervention. Students assigned to the virtual manipulative condition ($M = 7.47$, $SD = 4.16$) scored marginally higher than students assigned to the physical manipulative condition ($M = 6.93$, $SD = 3.83$), but when controlling for students' pre-assessment scores on the Fractions Probe, the main effect for manipulative treatment condition was not statistically significant, $F(1, 62) = 1.54$, $p < .22$, $d = 0.16$. There was a main effect for gender, $F(1, 62) = 4.80$, $p < .03$, $d = 0.83$, but no interaction effect between manipulative treatment condition and gender, $F(1, 62) = .50$, $p < .48$.

Day 10 Assessment

At the end of the second week of the intervention, students completed a 20-question, paper-and-pencil assessment of the content taught during the second week of

the intervention. In contrast to the Day 5 assessment, the Day 10 assessment showed a statistically significant main effect for manipulative treatment condition, $F(1, 62) = 4.41$, $p < .04$. Students assigned to the virtual condition answered an average of 1.78 more questions correctly on the Day 10 assessment than students assigned to the physical condition ($d = 0.31$). Also in contrast to the Day 5 assessment, the difference between boys and girls was not statistically significant, $F(1, 62) = .64$, $p < .43$, $d = 0.51$. The interaction between manipulative treatment condition and gender was not significant, $F(1, 62) = .90$, $p < .35$.

Fractions Probe (Day 10)

The mean score for all students on the 20-question Fractions Probe administered on Day 10 ($M = 7.57$, $SD = 4.10$) was only .55 questions higher than the mean score for all students on the Fractions Probe administered on Day 0 ($M = 7.02$, $SD = 3.29$). This suggests that students were not able to transfer the knowledge gained during the two-week, manipulative-based fractions intervention to the type of questions about fractions that students in Tennessee encounter on state exams. Controlling for students' scores on the pre-assessment, no significant main effects were found for either manipulative treatment condition, $F(1, 62) = .79$, $p < .38$, $d = 0.07$, or gender, $F(1, 62) = .17$, $p < .67$, $d = 0.54$ and the interaction effect was not significant, $F(1, 62) = .01$, $p < .92$.

Research Question #2

2. Are students able to complete more practice exercises and/or more games using virtual manipulatives than physical manipulatives?

To answer Research Question #2, the researcher analyzed a set of ANCOVA models that included tabulated data from the practice logs as outcome variables. The number of practice exercises and the number of games were treated as separate outcome variables. Chapter I of this dissertation hypothesizes that students who use virtual manipulatives complete more practice activities than students who use physical manipulatives, and the raw data collected in the practice logs supports this hypothesis: students assigned to the virtual condition complete more practice exercises and play more games than students assigned to the physical condition (see Table 1A). However, since the commercial curriculum used during the intervention calls for students to complete assessments every 5 days and the content of the Day 5 and Day 10 assessments¹⁰ reflect related but distinctly different learning objectives for each of the two weeks of instruction, the data from the practice logs is analyzed overall (i.e. totaled across all 10 days of the intervention) and by week (i.e. totaled separately across the first 5 days of the intervention and the second 5 days of the intervention). The next sections of this chapter summarize the most salient findings. Since there are no significant differences between boys and girls on the number of practice exercises (all $ps > .57$) or the number of games (all $ps > .13$), the main effects for gender are not reported.

Practice Exercises

Students assigned to the virtual manipulative condition ($M = 77.67$, $SD = 18.93$) complete significantly more practice exercises overall than students assigned to the physical condition ($M = 53.90$, $SD = 24.00$), $F(1, 56) = 16.03$, $p < .00$, $d = 1.10$.

¹⁰ Appendix B includes sample questions from the Day 5 and Day 10 assessments.

However, virtually no difference between the virtual manipulative condition ($M = 24.61$, $SD = 16.93$) and the physical manipulative condition ($M = 24.47$, $SD = 9.80$) exists during the first week of the intervention, $F(1, 58) = .01$, $p < .92$, $d = -0.01$. The opposite is true during the second week of the intervention. Students assigned to the virtual manipulative condition ($M = 51.34$, $SD = 15.26$) complete a higher mean number of practice exercises than students assigned to the physical manipulative condition ($M = 28.58$, $SD = 13.77$), and the difference between manipulative treatment conditions is highly statistically significant, $F(1, 60) = 32.49$, $p < .00$, $d = 1.57$. However, the interaction between treatment and gender is not significant overall or during either week of the intervention (all $ps > .10$)

Games

The main effect¹¹ for manipulative treatment condition is significant for the number of games students play overall and for the number of games students play during the first and second week of the intervention (all $ps < .01$). Students in the virtual condition play more games than students in the physical condition overall and during each week of the intervention (see Table A1). The interaction between manipulative treatment condition and gender is also significant overall and during each week of the intervention (all $ps > .01$). This suggests that the effect of manipulative treatment condition varies between genders even though the overall difference between boys and girls is not

¹¹ As was mentioned in Chapter III, the data reported here reflects results for only 1 of the 2 days in which students played games during the first week and only 3 of the 4 days in which students played games during the second week.

statistically significant. Note that the boys ($M = 15.83$, $SD = 7.02$) included in the sample play fewer games than the girls ($M = 20.01$, $SD = 6.40$).

Research Question #3

3. Does the number of practice exercises students complete and/or the number of games students play have an impact on students' knowledge of fraction magnitude?

To answer Research Question #3, the number of practice exercises and the number of games students completed during the first or second week of the intervention were included as covariates in the ANCOVA models from Research Question #1 that analyzed the results of the Day 5 and Day 10 assessments. The results of the analyses of the data collected during the first week of the intervention indicates that the number of practice exercises students complete during Week 1 is a significant predictor of the outcome of the Day 5 assessment ($p < .05$) in that the students who complete more practice exercises receive higher scores on the Day 5 assessment. However, the number of games students play during Week 1 does not impact the results of the Day 5 assessment ($p < .90$). During the second week of the intervention, the number of practice exercises students complete and the number of games students play are not significant predictors of the outcome of the Day 10 assessment (both ps are $< .65$). The results of these analyses and of the analyses of Research Question #1 and #2 are interpreted in Chapter V.

CHAPTER V

DISCUSSION

This dissertation examines the relative instructional efficiency of physical and virtual fraction manipulatives in terms of the differences in students' knowledge of fraction magnitude that exist when students learn basic fraction concepts using different forms of manipulatives and in terms of time efficiency. The researcher randomly assigned students within-class to either a physical manipulative condition or a virtual manipulative condition. Students spent a total of 2 weeks learning basic fraction concepts using the different forms of manipulatives. Assessments were administered at the end of each week of instruction, and the numbers of practice activities students completed on each day of instruction were recorded in practice logs. The assessments and the data collected in the practice logs were then analyzed using ANCOVA models. The next section of this chapter interprets the results of the statistical analyses reported in Chapter IV. Chapter V also discusses the limitations of this dissertation study, suggests directions for future research, and presents the conclusions.

Interpretation of Results

The Impact on Achievement of Virtual and Physical Manipulatives

The results of the post-assessment data collected in this study support Clements' (1999, 1996) hypothesis that computers can provide students with virtual representations

of mathematical concepts that are just as meaningful as physical manipulatives. They also align with the results of previous empirical mathematics and science studies that compare virtual and physical manipulatives (Klahr, Triona, & Williams, 2007; Moyer, Niezgod, & Stanley, 2005; Nute, 1997; Pleet, 1991; Reimer & Moyer, 2005; Smith, 2006; Steen, Brooks, & Lyon, 2006; Suh, 2005; Suh & Moyer, 2007; Triona & Klahr, 2003). While only the results of the Day 10 assessment showed a statistically significant difference between the virtual and physical manipulative conditions, students assigned to the virtual manipulative condition achieved higher mean scores than students assigned to the physical manipulative condition on all 3 of the post-assessments administered in this study. Clements hypothesized no difference between virtual and physical representations, but these results suggest that virtual manipulatives may actually be *more* effective than physical manipulatives.

Time Efficiency

The results of the analyses of the data collected in the practice logs overall (i.e. across both weeks of the intervention) and during the second week of the intervention support the results of previous qualitative mathematics studies and a quantitative science study that suggest virtual manipulatives are more time-efficient than physical manipulatives (Klahr, et al., 2007; Moyer, et al., 2005; Reimer & Moyer, 2005; Steen, et al., 2006). When the data from the practice logs is summed across both weeks of the intervention and when the data from the second week is examined independently of the first week, the differences between means are all positive in favor of the virtual condition, and the main effects of manipulative treatment condition are all highly statistically

significant. However, the mean results of the number of practice exercises students complete and the number of games students play are slightly lower for students in the virtual condition during the first week of the intervention than they are for students in the physical condition. The fact that the difference between treatment conditions is only apparent when the data from the practice logs is summed across both weeks or when the data from the second week is analyzed independently from the first week is not surprising considering that the students assigned to the virtual manipulative condition went through a period of adjustment during the first week of the study learning to navigate the different interfaces of the computer program. The students who participated in the full implementation of the study used PC computers in school but had never used computers during their 5th grade mathematics classes, and some of the students had never used a laptop. The scrolling and clicking functions of the touchpad mouse built into the Macbook laptops used during the intervention proved difficult for some of the students during the first week, but by the second week of the intervention, the majority of the students had become facile with the mouse functions and the computer interface. For these reasons, longer studies that consider the time efficiency of virtual compared to physical manipulatives are more likely to show results similar to the second week of this study than the first week of this study.

The Impact of Additional Practice

Even though the results of this study show that students who use virtual manipulatives complete more practice activities than students who use physical manipulatives, the impact of the additional practice on students' knowledge of fraction

magnitude is not clear. The number of games students complete is not significant in the analysis of either the Day 5 or the Day 10 assessments. This is most likely explained by the fact that the type of games students played during the intervention involved rolling a fractions cube that resembled a six-sided die. The fractions cube introduced an element of chance into the games that affected the duration of the games. During the intervention, the researcher noticed that students with little knowledge of fractions sometimes played a game that ended quickly and that students with substantial knowledge of fractions sometimes played a game that took longer to complete. The researcher also noticed that the amount of learning gained from playing a large number of short games versus playing a small number of long games is probably similar. Therefore, the number of games students played during the intervention is not a good indicator of students' mastery of the content learned during the intervention. In addition, the results of the analyses of Research Question #3 may be explained by the fact that students did not receive structured feedback while playing the games, and feedback plays an important role in Ericsson et al.'s (1993) impact of the deliberate practice.

Drawing on the work of Ericsson et al. that explained the impact of deliberate practice and the work of Cramer and colleagues (2002) that suggested students need to interact with manipulatives over an extended period of time, this dissertation hypothesizes that additional practice has a positive impact on students' post-assessment results. Although the results presented in Chapter IV show the effect of playing additional games is null, the results of the other analyses of the Day 5 and Day 10 assessments provide some evidence in support of this hypothesis. The number of practice exercises students complete is a significant predictor of the outcome during the first week of the

intervention in that students who complete more practice exercises on average receive higher scores on the Day 5 assessment. During the second week of the intervention, there is no longer a significant predictive relationship between the number of practice exercises students complete and their scores on the Day 10 assessment. Further analysis over a longer period of time is needed to determine the student learning impact of additional deliberate practice using manipulatives.

Gender Effects

Differences between genders existed at pre-assessment in that the boys received higher mean scores on the pre-assessment than the girls. Controlling for pre-assessment scores, the boys significantly outperformed girls on the Day 5 assessment but not on the Day 10 assessment. The main effect of gender was not significant in any of the analyses of the data included in the practice logs, and the raw data shows that neither gender consistently outperforms the other in terms of the number of practice activities completed. In previous comparisons of virtual and physical manipulatives, few studies reported the effects of gender. Pleet (1991) found that the results of the overall analyses of gender effects were inconclusive, but the boys in the Pleet study outperformed girls on one measure. Klahr et al. (2007) found no significant differences between boys and girls on any of the outcome measures except for a measure of boys' and girls' confidence at pretest and posttest. Given the weak findings of previous studies, this dissertation made no specific hypotheses about the difference between boys and girls. While further quantitative or qualitative analyses of gender effects may uncover more reliable differences between genders, the results of previous research and of this dissertation

suggest that these gender difference do not have a strong impact on the learning gains associated with manipulative-based instruction.

Transfer

On average, students' scores on the Fractions Probe improved very little between pre-assessment and post-assessment. The Fractions Probe included 20 multiple-choice questions similar to the type of questions students in Tennessee encounter on high-stakes assessments, and since students are not allowed to use manipulatives during standardized assessments, students did not use manipulatives during either administration of the Fractions Probe. The fact that students in both manipulative treatment conditions failed to make significant improvements between pre-test and post-test suggests that two weeks of either form of manipulative-based instruction about basic fraction concepts does not impact students' ability to answer high-stakes assessment questions about fractions. These null findings were not entirely unexpected given the fact that the commercial curriculum used for instruction during the intervention includes a total of 30 lessons about fractions and the students who participated in the study experienced only 10 lessons. It was also not surprising given Cramer et al.'s (2002) findings that students need to interact with manipulatives over an extended period of time to construct the type of mental images necessary to think conceptually about fractions¹². Considering that both NCTM (2006) and NMAP (2008) identify fractions as one of the key areas for emphasis in the upper elementary mathematics curriculum, extra time spent using fraction manipulatives is likely to have a positive impact on students' success in higher level

¹² The experimental curriculum tested by Cramer et al lasted for 28-30 days in all classes.

mathematics courses. The problem that arises is that teachers in Tennessee have a limited amount of time to cover all the mathematics concepts students included on the state assessments, and the 5th grade state assessments usually ask less than 5 questions about fractions. This creates a time-conflict for teachers that educators should consider when revising the Tennessee state assessments. It also adds emphasis to the time efficiency advantages of virtual manipulatives.

Generalizability

As discussed briefly in Chapter III, the results of the pilot align with the results of the full implementation in that the students assigned to the virtual manipulative condition during the pilot completed more practice activities and received higher mean scores on the Day 5 assessment than students assigned to the physical manipulative condition. The small “n” used during the pilot and the fact that slight differences between manipulative treatment conditions may exist mean that the results of the pilot study should not be interpreted independently of the full implementation. However, the results of the pilot speak to the generalizability of the results of the full implementation. The students who participated in the pilot were drawn from the 6th grade at a private school for students with special needs. The student population at the school site is primarily white, typical class is 3-6 students, and the school has a 1:1 laptop to student ratio. In contrast, the full implementation took place at a public charter school where close to 90% of the student population qualifies for free- or reduced-price lunch. The students who participated in the full implementation were drawn from the 5th grade, and while the majority of students tested below grade-level in mathematics, only a few had diagnosed special needs.

Students at the school are primarily African-American and typical class size at the school is 22-24 students. The school owns a PC computer lab, but most students spend little time using computers during instruction in their regular academic classes. The fact that the students who participated in the pilot and the students who participated in the full implementation differ in many ways yet the results of both studies closely align suggests that the results of the full implementation are generalizable to more than one population of students. The fact that these results appeared at the end of one week of instruction during the pilot compared to at the end of two weeks of instruction during the full implementation also suggests that students with more previous exposure to the type of computers used during the intervention and with more previous exposure to computer-based instruction in general adapt more quickly to virtual manipulatives

Limitations

Although the results of the pilot study align with the results of the full implementation, which suggests that the results reported in this dissertation are generalizable to more than one population of students, it cannot be assumed that the results reported here generalize to *all* populations of students. Most notably, this dissertation gives no indication of how virtual and physical manipulatives might impact the learning of average and high achieving students. It also gives no indication of how the results would generalize to other content domains. Previous research by Suh (2005) suggests that virtual manipulatives are particularly effective for instruction about fractions, which may at least partially explain the positive findings in this dissertation.

Virtual manipulatives may show less of an advantage over physical manipulative in other content domains of mathematics.

The above paragraph discusses a threat to the external validity of the study, but there are also a few minor threats to the internal validity of the study. The researcher used scripts during all lessons in both treatment conditions and made every attempt to control for differences between treatment conditions, but this dissertation does not include fidelity checks. Fidelity checks conducted by at least 2 outside observers would have reduced the threat of bias towards one of the treatment condition. Even without fidelity checks, a noticeable difference between the virtual and physical manipulative conditions exists. Chapter III noted that students in the Boys Physical condition did not play the games included in the instructional scripts on Day Four and that students in the Girls Physical conditions did not play games on Day Seven. The fact that students in the physical manipulative condition had fewer opportunities to practice what they learned about fractions may explain some of the differences in performance on the Day 5 and Day 10 assessments. In a related manner, the fact that the means and standard deviations reported in Table A1 and the game data included in the ANCOVA models reported in Chapter IV include counts from only 4 of the 6 days in which students actually played games threatens the internal validity of the results of the analyses of the game data. These results should only be interpreted with caution.

Students experienced the intervention in groups of 22-24 students, which introduces a nesting problem at the classroom level. Students were randomized within-class into manipulative treatment conditions prior to the start of the intervention, and while this eliminates the threat of pre-treatment differences between manipulative

treatment conditions, the ANCOVA models used in this study cannot account for nesting once the study begins. Although the threat is minimal given the steps that were taken to control for differences between treatment conditions, without a much larger number of participants and a larger number of groups more complex statistical techniques such as hierarchical linear modeling cannot be used to account for nesting.

Finally, this dissertation pilots strong research methods that could be replicated at a much larger scale, but it was never intended to be a fully powered study. Many of the null findings could prove to be statistically significant in larger-scale implementations.

Conclusions and Directions for Future Research

The most obvious direction for future research based on the results of this dissertation is a large-scale study that compares the impact on student learning of virtual and physical manipulatives when students are learning basic fraction concepts and that compares the relative time efficiency of both forms of manipulatives. Rather than tracking students' progress for just 10 days, future research should track students' progress for the duration of an entire unit of study about fractions to see if the differences between treatment conditions increase, decrease, or remain static over time and to see if students are able to transfer the knowledge gained during manipulative-based learning to the types of questions students encounter on standardized assessments. A larger study should include more than one grade level, and would ideally make comparisons between forms of manipulatives as well as between manipulative-based and non-manipulative based learning in virtual environments.

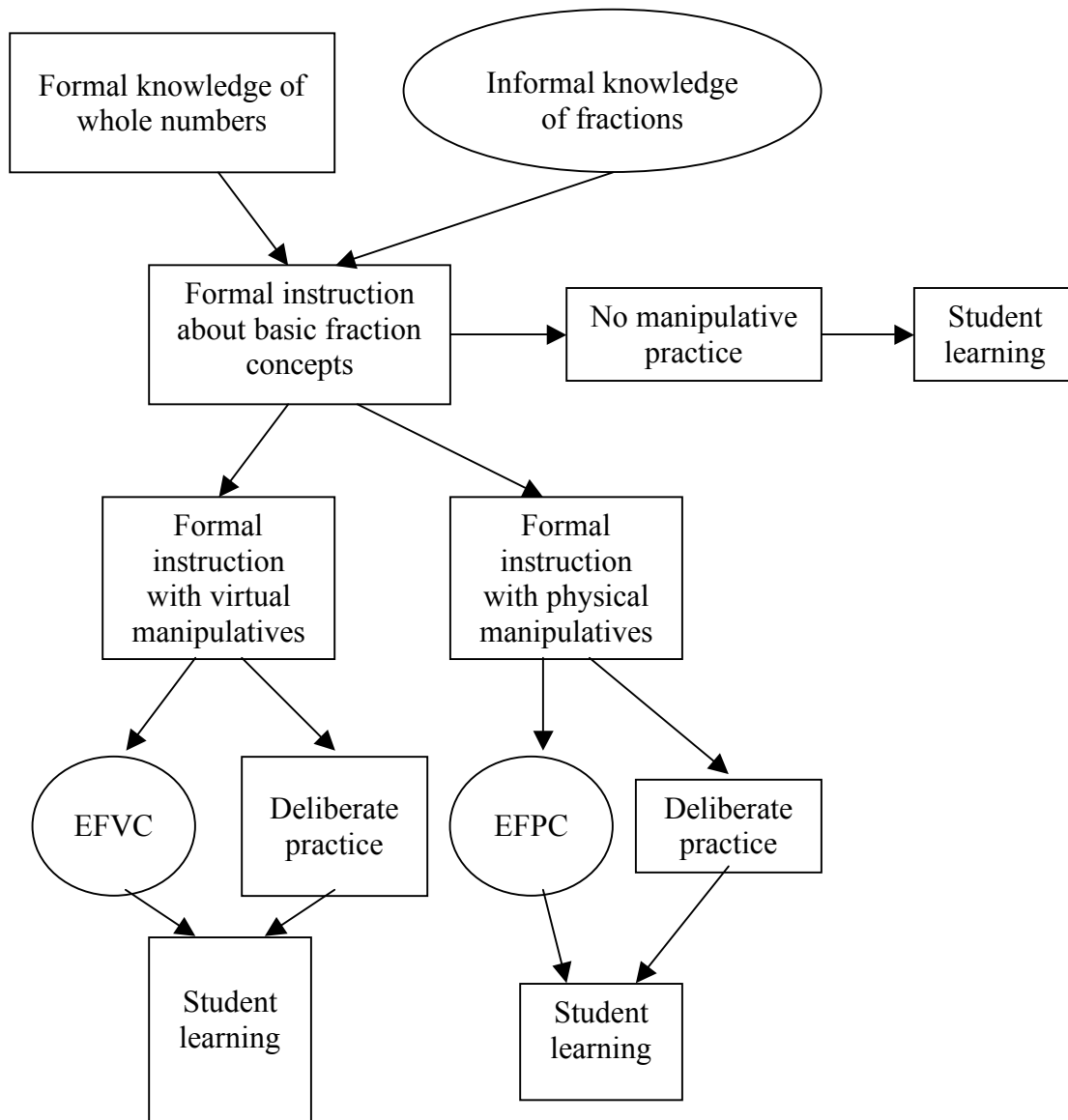
Although the most obvious direction for future research is a large-scale comparison of virtual and physical manipulatives, it is not necessarily the area of research with the greatest potential to impact learning. The area of research with the greatest potential to impact learning is further exploration of the capabilities of virtual manipulatives as well as further exploration of technology-based instruction about fractions that is intentionally designed to exceed what is possible with physical manipulatives. For example, virtual manipulatives can be designed to include links to other manipulatives and to other representations (e.g. a set of fractions strips can be linked to a set of fractions circles as well as a number line), and they can be designed to disappear and reappear based on students' performance on practice activities. Technology-based instruction about fractions can include dynamic feedback and differentiated practice based on the results of formative assessments.

The reality of classrooms today is that after more than 3 decades of high-quality research about physical manipulatives and multiple recommendations from NCTM (1989, 2000) that teachers include manipulatives in mathematics instruction, most teachers in the upper elementary grades rarely use physical manipulatives because they are practically and pedagogically difficult to implement in classrooms. In static comparisons of virtual and physical manipulatives such as the one discussed in this dissertation, instruction using virtual manipulatives is intentionally designed to closely mirror instruction using physical manipulatives in order to isolate the effect of the form of manipulatives. As such, the potential of virtual manipulatives to overcome the practical and pedagogical difficulties associated with physical manipulatives is constrained. Although this dissertation falls short of determining the magnitude of the

difference between virtual and physical manipulatives, it adds to a growing body of literature that indicates students learn at least as much using virtual manipulatives as they learn using physical manipulatives. Knowing that it is unlikely that there are negative learning gains associated with using virtual rather than physical manipulatives, researchers should concentrate on designing experiments that test the boundaries of what is possible with this technology.

APPENDIX A

FIGURE 1



EFVC: Environmental factors unique to the virtual condition
EFPC: Environmental factors unique to the physical condition

Table 1
Means of Outcome Measures by Manipulative Treatment Condition and Gender

Measure	Physical	Virtual	Diff.	Cohen's <i>d</i>	Girls	Boys	Diff.	Cohen's <i>d</i>
Assessments								
Fractions Probe (Day 0)	6.93 (3.83)	7.11 (2.71)	0.18	0.05	6.03 (3.39)	8.33 (2.68)	2.30***	0.74
Day 5	6.82 (3.97)	7.47 (4.16)	0.65	0.16	5.78 (4.06)	8.93 (3.32)	3.15**	0.83
Day 10	12.72 (5.73)	14.5 (5.79)	1.78**	0.31	12.42 (6.15)	15.29 (4.89)	2.87	0.51
Fractions Probe (Day 10)	7.42 (4.44)	7.71 (3.80)	0.29	0.07	6.67 (4.03)	8.82 (3.93)	2.15	0.54
Practice Exercises								
Total	53.90 (24.00)	77.67 (18.93)	23.77***	1.10	64.74 (26.42)	66.74 (22.52)	2.00	0.08
Week 1	24.61 (16.93)	24.47 (9.80)	-0.14	-0.01	22.06 (11.32)	27.85 (15.90)	5.79	0.43
Week 2	28.58 (13.77)	51.34 (15.26)	22.76***	1.57	41.11 (20.32)	38.04 (15.74)	-3.07	-0.17
Games								
Total	14.84 (5.9)	21.07 (6.55)	6.23***	1.00	20.01 (6.40)	15.83 (7.02)	-4.18	-0.63
Week 1	3.40 (1.91)	4.69 (2.16)	1.29***	0.63	4.34 (1.89)	3.70 (2.38)	-0.64	-0.3
Week 2	11.79 (5.46)	16.07 (5.94)	4.28***	0.75	15.68 (5.88)	11.8 (5.69)	-3.88	-0.67

Note. The values reported in parentheses represent standard deviations.

P-values are derived from 2x2 factorial ANCOVA models that include manipulative condition, dummy variables, an interaction term, and a covariate.

* $p < .10$ ** $p < .05$ *** $p < .01$

APPENDIX B

ASSESSMENTS

Sample Questions

Day 5 Assessment

1) How many $\frac{1}{8}$ equal $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

_____ eighths is equal to four-fourths

$$\underline{\hspace{2cm}} = \frac{4}{4}$$

2) How many $\frac{1}{16}$ equal $\frac{1}{8}$

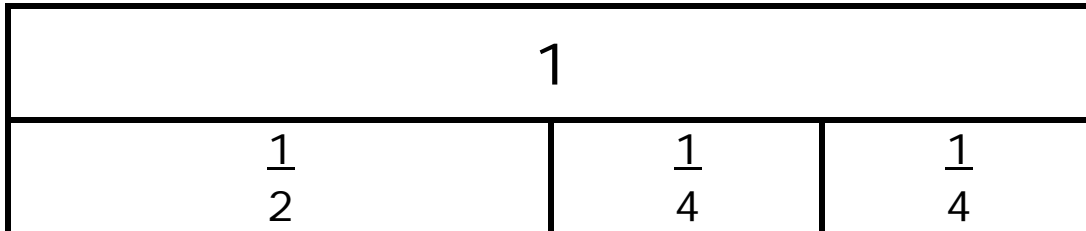
_____ sixteenths is equal to one-eighth

$$\underline{\hspace{2cm}} = \frac{1}{8}$$

Day 5 Assessment

Write equations that match the fraction pieces.

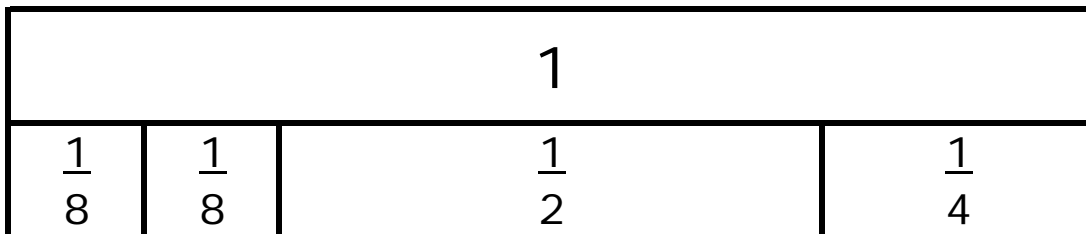
3)



Equation:

Shorten the equation:

4)



Equation:

Shorten the equation:

Day 10 Assessment

Directions: Write $<$, $>$, or $=$

1) $\frac{1}{4}$ $\frac{1}{2}$ 3) $\frac{2}{2}$ $\frac{4}{4}$

2) $\frac{3}{8}$ $\frac{1}{4}$ 4) $\frac{6}{16}$ $\frac{4}{8}$

Directions: Write the numerator or denominator that makes the fraction equivalent

5) $\frac{2}{8} = \frac{\quad}{4}$ 6) $\frac{1}{4} = \frac{4}{\quad}$

7) $\frac{16}{16} = \frac{\quad}{8}$ 8) $\frac{\quad}{16} = \frac{1}{2}$

Fractions Probe

1) What fraction would be equivalent to

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$$

- A. $\frac{4}{3}$
- B. $\frac{6}{8}$
- C. $\frac{1}{2}$
- D. $\frac{3}{12}$

2) Which of the following is equivalent to $\frac{3}{4}$?

- A. $\frac{6}{12}$
- B. $\frac{9}{12}$
- C. $\frac{9}{16}$
- D. $\frac{12}{20}$

3) What would be another equivalent fraction that would belong in the set below?

($\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, ___)

- A. $\frac{4}{5}$
- B. $\frac{4}{6}$
- C. $\frac{4}{7}$
- D. $\frac{4}{8}$

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