MANAGING CONDITION VARIABILITY IN REMANUFACTURING

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CHAPTER I

OPTIMAL ACQUISITION AND SORTING POLICIES FOR REMANUFACTURING

1: Introduction

In remanufacturing, used products can range in condition from slightly used with only minor

cosmetic blemishes to significantly damaged and requiring extensive rework. For a

remanufacturer, one of the critical operational decisions is the establishment of a sorting policy –

given variable condition, which used products should be remanufactured and which should be

scrapped? This important aspect of remanufacturing operations has received limited attention in

the literature. In this paper, we extend earlier work by deriving and analyzing optimal sorting

policies in the presence of used product condition variability. Our results, while motivated by

the remanufacturing industry, are applicable to other manufacturing organizations that face

variable raw material condition.

In this paper, the term "remanufacture" refers to restoring a used product to acceptable

condition for resale. Other equivalent terms common in literature and practice are "recondition"

and "refurbish." Our analysis of sorting policies considers two possible dispositions of the used

product – remanufacture or scrap – which are common in remanufacturing. Although some

remanufacturing environments include other potential product dispositions, such as recovery of

components, we do not consider these options in this study.

We analyze production to meet both deterministic and uncertain demands. In either case,

remanufacturing follows a "make from stock" (Fleischmann et al. 2004) model, where used

products are acquired and available as needed to meet remanufacturing needs. A common make from stock situation is that of an independent remanufacturer who obtains used products from third party brokers. Consider, for example, the cellular phone industry (Guide et al. 2003b), in which used products are purchased from brokers as needed to fulfill specific demands.

When condition is variable, some of the units acquired are likely to be too costly to remanufacture, and scrapping may be the appropriate disposition decision for these units. This implies that the number of used products acquired should be greater than the number of remanufactured products required. As the acquisition amount is increased, sorting can be made more stringent – only products with lower remanufacturing costs are actually remanufactured. Thus, the sorting policy should be driven by how many "extra" used products are acquired. Of course, the cost of acquiring more used products offsets (to some extent) the remanufacturing cost savings enabled by increased selectivity. The interaction of these two effects should drive the acquisition amount and corresponding sorting policy. In this paper, we analyze acquisition/sorting policies using a total cost model that incorporates used product condition variability. We show the existence of unique optimal policies which minimize average total costs for a remanufacturer.

The rest of this paper is organized as follows. In section 2, we review the relevant research on remanufacturing. In section 3, we describe the impact of acquisition and sorting policies on average remanufacturing costs and examine policies to meet a fixed demand. In section 4, we present results for the special case in which used product acquisition costs are linear. In section 5, we extend these results to the case of uncertain demand. Section 6 summarizes the contribution of the analysis and suggests directions for future research.

2: Remanufacturing Literature

While remanufacturing activities are often motivated by environmental concerns or demands from customers or government authorities (Thierry et al. 1995, Toktay et al. 2000, Field 2000, Fleischmann et al. 2001, Ginsberg 2001, Seitz and Peattie 2004, van Nunen and Zuidwijk 2004), the processing of returns has increasingly been viewed not simply as a cost of doing business (Padmanabhan and Png 1997), but as a profitable business model and source of competitive advantage (Dowlatshahi 2000, Klausner and Hendrickson 2000, Stock et al. 2002, Andel and Aichlmayr 2002). In fact, remanufacturing is a $50 billion/year industry in the U.S. alone (Corbett and Kleindorfer 2001, Majumder and Groenevelt 2001). Realizing the potential for profitable remanufacturing, many independent businesses have emerged to exploit specific remanufacturing opportunities (Wells and Seitz 2004).

Reverse logistics/remanufacturing has many similarities to its traditional forward logistics/manufacturing counterpart. At the most basic level, both involve supply, production and distribution. The major difference between the two involves the supply side (Fleischmann et al. 1997, Fleischmann 2001). In a remanufacturing system, supply is largely exogenous, and the timing, quantity, and quality of supply are much more uncertain than in traditional production-distribution systems. A significant consequence of this uncertainty is the inclusion of an inspection stage and a corresponding system of quality-dependent routing of supply in a reverse logistics network (Trebilcock 2002). With rare exceptions, traditional supply chains do not include such a focus on supply quality.

Many studies have acknowledged the problem of variable supply quality for remanufacturing systems (Bloemhof-Ruwaard et al. 1999, Guide and Jayaraman 2000, Fleischmann et al. 2000, Stanfield et al. 2004). Guide (2000) discusses the additional complexity

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in scheduling and planning caused by the high variability of remanufacturing processing times, and his survey of managers provides insights into how these issues can be addressed. Toktay et al. (2000) incorporate random processing times for the returned products into their queuing network model of a closed-loop system. Guide et al. (2000) highlight the operational concern caused by highly variable processing times, which are a function of returned product condition. The authors point to the need for remanufacturing firms to estimate used product condition to determine the appropriate disposition. This assessment of condition is an important step in determining the optimal recovery action (Bloemhof-Ruwaard et al. 1999, Van Nunen and Zuidwijk 2004, Blackburn et al. 2004). Several techniques for condition assessment and grading have been described in recent publications (Krikke et al. 1999, Rudi et al. 2000).

Simulation has been used by some researchers to model variable used product condition in remanufacturing operations. Fleischmann (2001) builds a simulation model which incorporates uncertainty in the quality of returned products and uses this model to evaluate several reverse logistics configurations. Humphrey et al. (1998) use a simulation which incorporates variable repair requirements to analyze a reverse logistics network for a U.S. Army depot. Guide and Srivastava (1997) incorporate stochastic processing times into their simulation model to evaluate various order release strategies in a naval aviation depot, and they acknowledge the high level of variation in work content that can be present in the remanufacturing setting.

Several models have been proposed for managing the problem of variable used product quality. Klausner and Hendrickson (2000) show how incentives can be used to increase the quality of returned goods. They assert that an increase in returned goods quality will lead to a higher remanufacturing yield. Guide and Van Wassenhove (2001) propose a model to manage

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the quality of cellular phone returns to help reduce variation in processing times. Guide et al. (2003b) expand the analysis in Guide and Van Wassenhove (2001) to include the consideration of optimal acquisition policies and pricing. They point out the limited amount of research that has addressed areas such as used product acquisition, testing, and disposition. Their research focuses on the criticality of product acquisition management in maximizing profits for a remanufacturer, and it highlights the need to move beyond management intuition in managing these processes. Guide et al. assume that used products are acquired from third-party brokers who have sorted the products into condition categories, where remanufacturing cost for a given category is known.

In contrast to Guide et al., we take remanufacturing cost of acquired products to be unknown. This reflects many potential situations, including the case in which sorting does not eliminate remanufacturing cost variability and the case where third party brokers offer only unsorted products to the remanufacturer. In addition, our analysis could help remanufacturers quantify the price they are willing to pay brokers for the service of sorting.

Several recent papers have presented mathematical models of remanufacturing operations which do not incorporate variable used product condition into the analysis. In their 2004 paper, Savaskan et al. mention the growing interest in research that considers the quality uncertainty in return flows. They analyze three collections options and discuss the conditions under which different collections processes and supply chain structures are appropriate. However, their model assumes homogeneous quality of returned products for each collection option and that all returned products are remanufactured for resale (100% yield) at a fixed unit remanufacturing cost. Krikke et al. (1999) also acknowledge variability in returned product quality, but their MILP formulation of a reverse logistics network uses a constant unit cost of remanufacturing.

Jayaraman et al. (1999) and Majumder and Groenevelt (2001) also model unit remanufacturing cost as a constant.

3: Acquisition and sorting policies for deterministic demand

We develop a model which explicitly considers variable used product condition and use this model to examine how acquisition and sorting decisions affect remanufacturing costs. In this analysis, we make the following assumptions, similar to Guide et al. (2003b): perfect testing, no capacity constraints, and no fixed costs. In our context, perfect testing means that remanufacturing cost depends on condition, but is known at the time of sorting. These assumptions result in a model that allows us to accurately depict the problem without trivializing it.

Our research is motivated by an independent remanufacturer who serves both the imaging supplies and cellular telephone markets. In many of these markets, the supply of used items from third party brokers is essentially unlimited. The acquired products have not been sorted, and their condition is highly variable. When the unsorted items arrive at the remanufacturing facility, the firm must sort each item and assign it to one of two categories – remanufacture or scrap. Scrap items are disposed of at negligible cost, and the remaining items are immediately processed. The result is a remanufacturing "yield," defined as the percentage of the used products acquired which are actually remanufactured. The firm acknowledges that increasing selectivity in its sorting (i.e. accepting fewer products for remanufacture) will have two effects: 1) yield will decrease, requiring the acquisition of more used products to meet a given demand, and 2) remanufacturing cost will decrease, since the used products sorted for remanufacture will be, on average, in better condition. Decreasing sorting selectivity will have opposite effects.

Thus, the firm must make two related decisions given demand or demand forecast – how many used items to acquire and how selective to be during sorting. In this research, we develop a quantitative model for determining optimal acquisition and sorting policies for a single period. Many remanufactured products, such as cell phones, have relatively short life cycles. For these products, a single-period model is reasonable given that future demand is not guaranteed. We first analyze the single-period problem with deterministic demand and then extend our analysis to the stochastic demand setting.

Given that some used products might not be remanufactured, it follows that a firm facing a demand Q should acquire P used products, where P>=Q. As each product is processed, it is sorted into one of two categories – remanufacture or scrap – such that Q products are remanufactured and (P-Q) are scrapped (see Figure 1).
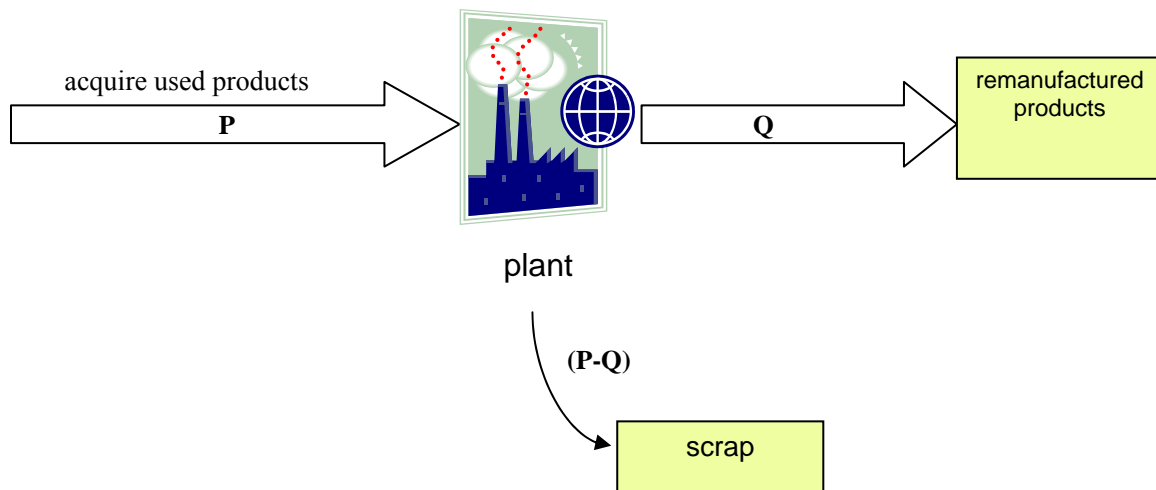
Figure 1: Acquiring and sorting used products

Thus, this remanufacturing problem has a single decision variable, P. To translate a given P into a specific sorting policy, we use the distribution of condition of the used products,

where *condition is defined as cost to remanufacture*. Clearly, remanufacturing cost can be assumed to increase as product condition worsens. While several other representations of used product condition are possible – for example, time required for remanufacture – we use remanufacturing cost in this model. Note that when Q remanufactured products are needed and P used products are acquired, the required yield, α, – that is, the fraction of used products that must be successfully remanufactured – is defined as follows:

$$\alpha = Q/P$$

Then for any acquisition amount P, sorting policy should be set such that the expected yield from remanufacturing equals α. We assume that the cumulative distribution of used product condition is known exactly, and denote this distribution as G(.) (the corresponding density function is denoted as g(.)). Given G(.), we define the "cutoff" cost – that is, the maximum cost acceptable to justify remanufacturing – to be some remanufacturing cost t such that:

$$G(t) = \alpha, \text{ or } G(t) = Q/P$$

The sorting policy is defined by the value of t: products with remanufacturing cost above t are scrapped, and those with cost below t are remanufactured (note that both α and t are functions of the single decision variable P). An important simplifying assumption of this model is that the actual yield (that is, the number of used products remanufactured) for a given t is deterministic. This allows us to gain interesting insights into this important problem in remanufacturing, and we discuss the implications of relaxing the deterministic yield assumption in section 6.

We define used product condition as being some point on a continuum, and each product's condition (cost to remanufacture) is determined during the sorting process. Figure 2

demonstrates the relationship between α and average remanufacturing cost and illustrates how

the cutoff condition is determined given a desired yield of α=80%.
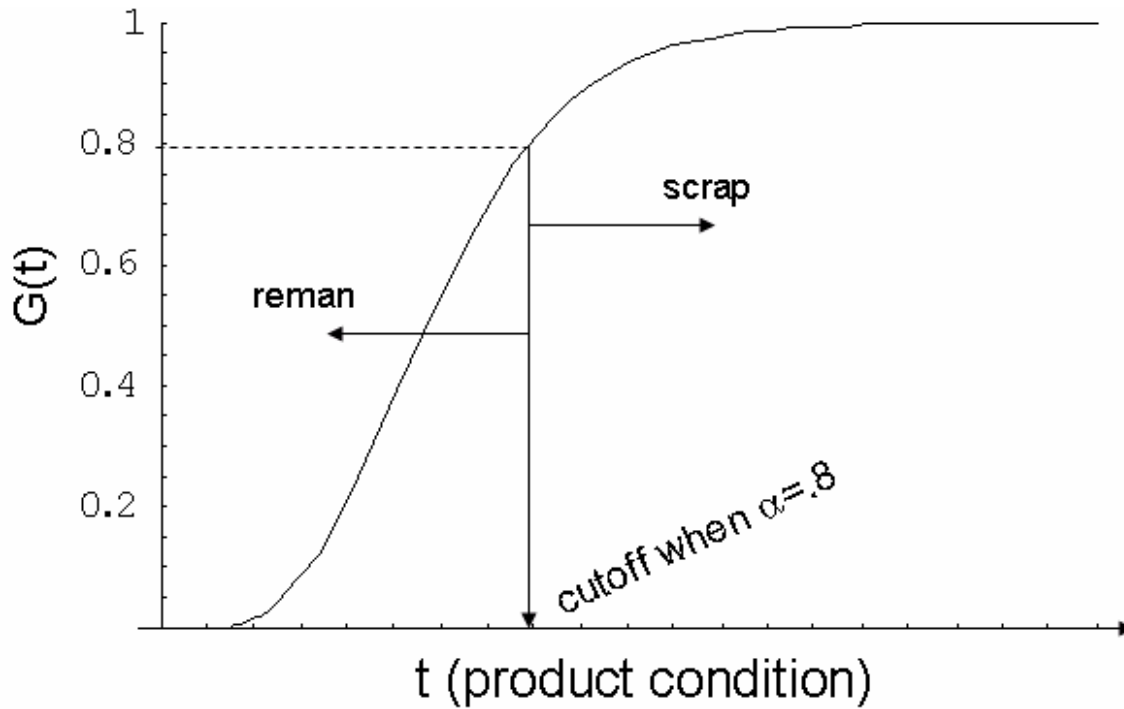
Figure 2: Translating required yield into sorting policy

Since sorting policy must be set such that G(t)=α, as α decreases (or, equivalently, as P increases

for a fixed Q), a firm is able to be more selective when processing used products – higher-cost

items are not remanufactured. Therefore, those products that are selected for remanufacture have

lower remanufacturing cost as α decreases.

Given Q and G(.), we can express sorting policy t as a function of P:

$$t = G^{-1}(\alpha) = G^{-1}\left(\frac{Q}{P}\right) \tag{1}$$

Since only the Q used products with a cost of t or less will be remanufactured, we have the

following expression for the total remanufacturing cost as a function of P:

$$R(P) = Q \frac{\int_0^t xg(x)dx}{\int_0^t g(x)dx}$$

which simplifies to:

$$R(P) = P\int_0^t xg(x)dx \qquad (2)$$

Remanufacturers also incur acquisition costs, which we define as all costs to acquire, transport, and sort used products. Adding an acquisition cost function z(.) to (2) gives us the following expression for the total cost, defined as acquisition plus remanufacturing cost, of acquiring P and remanufacturing Q products:

$$TC(P) = z(P) + P\int_0^t xg(x)dx \qquad (3)$$

Although most remanufacturing studies assume linear acquisition costs, we start by examining the case in which acquisition costs are nonlinear. In these cases, we assume increasing marginal cost (convex increasing cost) of used products because of scarcity; Guide et al. (2003b) make the same assumption in their analysis. We assume that z(.) in (3) can be any non-negative convex increasing acquisition cost function.

To prove that TC(P) is convex on [Q,∞), and therefore is minimized at a single critical number P*, we first prove the following proposition:

PROPOSITION 1. For a given production amount, average remanufacturing cost is a convex monotonically decreasing function of P, the number of used products acquired.

Proof: see Appendix A.

Proposition 1 holds for all continuous distributions of remanufacturing cost. Given Proposition 1 and the fact that (3) is simply the sum of two terms which are convex in P on $[Q,\infty)$, we can state the following:

PROPOSITION 2. TC(P) is convex in P on $[Q,\infty)$, therefore given any convex acquisition cost function, there is an optimal acquisition amount P* (and corresponding optimal sorting policy as defined by evaluating (1) at P*) which will minimize total average costs to meet a fixed demand, Q.

4: Results for linear acquisition costs

We now examine the special case in which the acquisition cost function is linear. Linearity is a reasonable assumption in many remanufacturing environments, particularly when the market is large and well-defined. Klausner and Hendrickson (2000) provide a detailed justification, grounded in data from a German remanufacturer, of the use of a constant unit cost of acquired products in their model, and linear acquisition, transportation, and handling costs are commonly assumed in the literature (e.g., Majumder and Groenevelt 2001, Fleischmann et al. 2001, Savaskan et al. 2004).

Given linear acquisition costs, total cost is a convex function of P on $[Q,\infty)$ as shown in Figure 3.
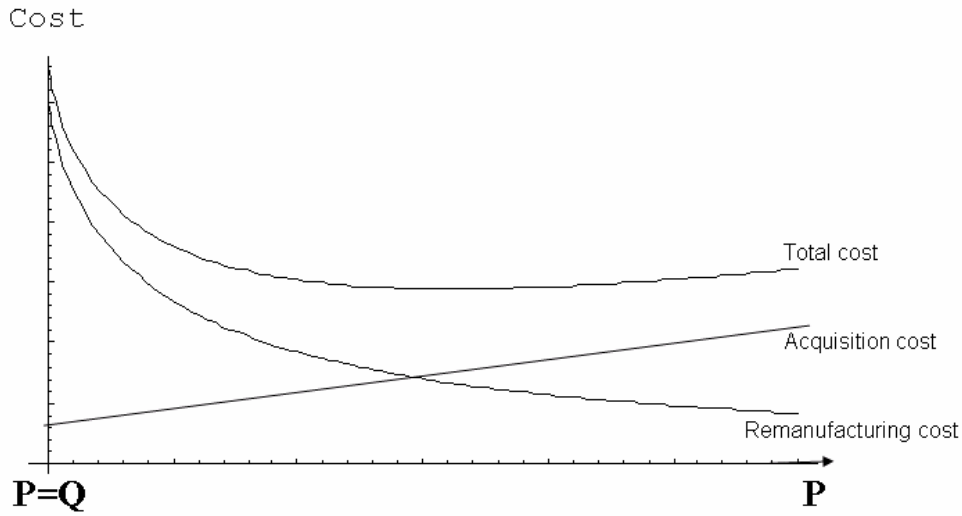
Figure 3: Total cost with linear acquisition costs

Let u = unit acquisition cost. Then the total cost of acquiring P used products at unit cost u and remanufacturing Q of those products is expressed in (4).

$$TC(\alpha) = uP + P\int_0^t xg(x)dx \tag{4}$$

Since $\alpha = Q/P$, (4) can be rewritten as a function of α:

$$TC(\alpha) = \frac{uQ}{\alpha} + \frac{Q\int_0^t xg(x)dx}{\alpha} \tag{5}$$

We then have the following proposition:

PROPOSITION 3. When acquisition costs are linear, the average total cost per unit is separable from the production amount, therefore the optimal value of α does not depend on Q.

Proof: We can simplify (5) to the following:

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$$TC(\alpha) = Q\left[\frac{u}{\alpha} + \frac{\int_{0}^{G^{-1}(\alpha)} xg(x)dx}{\alpha}\right] \tag{6}$$

Dividing (6) by Q results in the following expression for average total cost per unit:

$$UTC(\alpha) = \frac{u}{\alpha} + \frac{\int_{0}^{G^{-1}(\alpha)} xg(x)dx}{\alpha} \tag{7}$$

The result follows from the observation that (7) does not depend on Q. □

Since α fully describes the acquisition amount (Q/α) and the sorting policy ($G^{-1}(\alpha)$),

the fact that the optimal α is independent of the magnitude of Q (e.g. if the optimal α equals 0.5

for one value of Q, it equals 0.5 for all Q) means that a single acquisition and sorting policy is

optimal for *any* production amount. (This assumes that remanufacturing of the product is

economically feasible – that is, the unit sales revenue and the cost of purchasing and processing

new inputs both exceed the average total cost from (7).) In addition to this interesting insight

into the deterministic demand problem, Proposition 3 also allows us to easily extend our analysis

to cases of uncertain demand. We demonstrate this extension in Section 5.

5: Acquisition and sorting policies for stochastic demand

Ferrer and Whybark (2001) point out that some remanufacturers have limited advance

knowledge of demand, and overproduction of remanufactured items can expose them to

obsolescence risk. In these cases, the classic newsvendor problem can be used to set a

production amount which minimizes the sum of expected shortage and overage costs. We now

examine the applicability of the newsvendor problem given the variable production costs in remanufacturing.

We start by presenting a basic newsvendor formulation given uncertain demand with distribution f(.):

$$N(Q) = c_o \int_0^Q (Q-x)f(x)dx + c_s \int_Q^\infty (x-Q)f(x)dx \qquad (8)$$

where overage cost, c_o, is defined as the cost of producing one unit (a wasted expense if the unit cannot be sold); shortage cost, c_s, is defined as the lost margin per unit from producing fewer units than the actual demand (plus any penalty for disappointing customers). The newsvendor solution is the production quantity Q* which minimizes N(Q). In the context of the general remanufacturing problem (without the assumption of linear acquisition costs), c_o and c_s are as follows:

$$c_o = \frac{TC(P^*)}{Q}, c_s = A - \frac{TC(P^*)}{Q} + b$$

where TC(P*) is the optimal solution to (3), b is the unit shortage penalty, and A is unit sales revenue.

In the general case of nonlinear acquisition costs, TC(P*) depends on Q. Thus, when the newsvendor approach is applied to the general remanufacturing problem to minimize the function N(Q) as given in (8), c_o and c_s are also functions of Q, and the problem becomes more difficult to solve.

However, by Proposition 3 the optimal average total cost per unit is independent of Q given linear acquisition costs. In this case we can define c_o and c_s as follows:

$$c_o = UTC(\alpha^*), c_s = A - UTC(\alpha^*) + b$$

where UTC(α*) is the optimal solution to (7). Given that these expressions for c_o and c_s do not depend on Q, the newsvendor problem can be solved using standard techniques. The expression defining the optimal newsvendor production quantity Q*, $F\left(Q^*\right) = \dfrac{c_s}{c_o + c_s}$, can be written as follows in this context:

$$F\left(Q^*\right) = \frac{A - UTC\left(\alpha^*\right) + b}{A + b} \quad \textit{or} \quad Q^* = F^{-1}\left(\frac{A - UTC\left(\alpha^*\right) + b}{A + b}\right) \tag{9}$$

Since (7) does not depend on Q, UTC(α*) is invariant with respect to the ultimate determination of Q*. Thus, remanufacturers can set acquisition and sorting policies (as defined by α) a priori and still use the standard newsvendor model to optimize production amounts when facing uncertain demands. This implies the following 3-step procedure:

1. find α* and UTC(α*) using (7), and define the sorting policy by evaluating (1) at α*;

2. find Q* using (9);

3. acquire $\dfrac{Q^*}{\alpha^*}$ and remanufacture Q* products (using the sorting policy defined in step 1).

The above procedure will result in minimum expected costs to meet an uncertain demand in a remanufacturing setting (see the following example).

Example Problem:

Assume used products are acquired for a unit cost of $3.00. Condition follows a gamma distribution with parameters (5,2), implying an average cost to remanufacture of $10.00. Unit sales revenue is $15.00 and unit shortage penalty is $4.00. In step 1 of the above procedure, we can use (8), with u=3 and G(.)=gamma(5,2), to find α*. In this example α* equals 0.71 (so the optimal acquisition policy is (1/0.71) multiplied by the number of remanufactured products

needed). Using (1), sorting policy should be set to scrap all units with a remanufacturing cost greater than $11.93 (this is equivalent to sequencing the items by condition, processing the best α^* (or 71%) of them, and scrapping the remaining 29%). In this case, the average remanufacturing cost of the products that are not scrapped is $7.72, and $4.23 is spent on used product acquisition for every remanufactured item. Therefore, the average total cost of products that are actually remanufactured, UTC(α^*), is $11.95.

Note that the optimal acquisition and sorting policy, as defined by α^*, is known *before* demand is estimated. We now have all the information required to determine optimal policies when faced with any uncertain demand. For example, if expected demand f(.) is normally distributed with mean 1000 and standard deviation 150, then in step 2 we have

$$Q^* = F^{-1}\left(\frac{15 - 11.95 + 4}{15 + 4}\right) = 951.$$ In step 3 we calculate the optimal acquisition quantity for this production amount to be 951/0.71 =1339. By acquiring 1339 used products and processing them according to the sorting policy determined in step 1, we can expect to remanufacture the optimal quantity of 951 units at minimum cost.

6: Conclusions and Future Research

In this paper we have provided a detailed analysis of optimal acquisition and sorting policies for remanufacturers facing variable used product condition. Our work provides a foundation for continued research in this area, which has been identified as having received little previous attention from researchers (Guide et al. 2003b). The analysis in the previous two sections has shown that, given reasonable assumptions, a single optimal acquisition and sorting policy exists for a remanufacturer. When acquisition costs are linear, the optimal policy can be defined

independently of the production amount, and the optimal acquisition quantity can be calculated by applying a simple scalar, $(1/\alpha)$, to demand. Because the optimal average total cost per unit does not vary by production amount, our results are easily extended to the case of uncertain demand. For remanufacturers that follow a make from stock model for their products – such as imaging supplies and cellular phones – this paper provides algorithms to find cost-minimizing acquisition amounts and sorting policies.

As noted earlier, the model presented in this paper assumes that yield (i.e. α) is deterministic. We believe that this assumption preserves the essence of the problem and provides interesting insights into the stochastic demand model. In reality, however, the number of items that meet a specified condition cutoff would itself be stochastic, and so optimal sorting policies under random yield would be a useful extension of this research. In our preliminary work investigating random yields, we find that optimal policies for the deterministic yield case often provide very good approximations to the stochastic yield solution. Specifically, as production quantity (Q) increases, we find that the solution to the stochastic model converges asymptotically to the deterministic solution (for very large production amounts, the actual yield will converge to the expected value assumed by the deterministic model). The deterministic model also provides good approximations when overage and shortage costs due to discrepancies between expected and actual production amounts are nearly equal. Thus, for products with high demand and/or nearly equivalent shortage and overage costs, the deterministic model presented in this paper provides comparable results to the more complex stochastic yield situation.

While our results provide useful insights into this emerging area of remanufacturing research, some of our assumptions could be relaxed for wider applicability. For example, although we impose no restrictions on scrapping, a scrapping penalty can be added to our total

cost function to reflect situations in which scrapping is constrained. If penalties are assessed based on the percentage of acquired products which are scrapped, then the scrapping penalty is independent of Q. Since these penalties are also likely to be convex (increasing penalties as scrapping increases), we expect that, for most cases, the total cost will remain a convex function of α, and the results in sections 4 and 5 would remain valid. The single-period problem presented here could also be extended to the case of a remanufacturer facing multiple periods of uncertain demand. The multi-period extension of this work is an intriguing future research area, and a multi-period model could more accurately represent remanufactured products with longer life cycles.

Appendix A

To prove monotonically decreasing convexity, we show that the first derivative of the remanufacturing cost function is always negative and the second derivative is always positive: Substituting (1) into (2) gives us the following expression for remanufacturing cost as a function of P:

$$R(P) = P \int_0^{G^{-1}\left(Q/P\right)} x g(x) dx \tag{A1}$$

Recall that the derivative of an inverse function is defined as follows:

$$\frac{df^{-1}(x)}{dx} = \frac{1}{\dfrac{df\left(f^{-1}(x)\right)}{dx}}$$

Applying the above definition, as well as Leibnitz's Rule and the product rule, we have:

$$\frac{dR}{dP} = PG^{-1}\left(Q/P\right) g\left(G^{-1}\left(Q/P\right)\right) \frac{1}{g\left(G^{-1}\left(Q/P\right)\right)} \left(-\frac{Q}{P^2}\right) + \int_0^{G^{-1}\left(Q/P\right)} x g(x) dx$$

which simplifies to:

$$\frac{dR}{dP} = \int_0^{G^{-1}\left(Q/P\right)} x g(x) dx - \frac{Q}{P} * G^{-1}\left(Q/P\right) \tag{A2}$$

Factoring out $\dfrac{Q}{P}$ from (A2) gives:

$$\frac{dR}{dP} = \frac{Q}{P}\left[\frac{P}{Q} \int_0^{G^{-1}\left(Q/P\right)} x g(x) dx - G^{-1}\left(Q/P\right) \right] \tag{A3}$$

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Replacing $\dfrac{P}{Q}$ in the first term of (A3) with the equivalent expression $\dfrac{1}{G^{-1}\left(\frac{Q}{P}\right)\displaystyle\int_0^{} g(x)dx}$:

$$\frac{dR}{dP} = \frac{Q}{P}\left[\frac{\displaystyle\int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx}{\displaystyle\int_0^{G^{-1}\left(\frac{Q}{P}\right)} g(x)dx} - G^{-1}\left(\frac{Q}{P}\right)\right] \tag{A4}$$

Observe that (A4) is always negative. The term in the brackets is the difference of two terms.

The first term is equivalent to the expected value of g on $\left[0, G^{-1}\left(\frac{Q}{P}\right)\right]$, and the expected value

of g is less than its upper limit, $G^{-1}\left(\frac{Q}{P}\right)$. Therefore, the bracket term is always negative, and

thus $\dfrac{dR}{dP}$ is always negative.

We now derive the second derivative, $\dfrac{d^2 R}{d\,P^2}$. Rewrite (A4) as follows:

$$\frac{dR}{dP} = \frac{1}{P}*\frac{Q\displaystyle\int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx}{\displaystyle\int_0^{G^{-1}\left(\frac{Q}{P}\right)} g(x)dx} - \frac{1}{P}*QG^{-1}\left(\frac{Q}{P}\right) \tag{A5}$$

Note that the first term of (A5) is (2) multiplied by $\dfrac{1}{P}$, and, applying the product rule, the

derivative of the first term of (A5) is

$$-\frac{R}{P^2} + \frac{\frac{dR}{dP}}{P}$$

or

$$-\frac{1}{P^2} * P \int_0^{G^{-1}(Q/P)} xg(x)dx + \frac{1}{P^2}\left(P \int_0^{G^{-1}(Q/P)} xg(x)dx - QG^{-1}\left(Q/P\right) \right)$$

which simplifies to

$$-\frac{QG^{-1}\left(Q/P\right)}{P^2} \tag{A6}$$

Differentiating the second term of (A5) gives:

$$-\frac{Q^2}{P^3 g\left(G^{-1}\left(Q/P\right)\right)} - \frac{QG^{-1}\left(Q/P\right)}{P^2} \tag{A7}$$

Combining (A6) and (A7) we have

$$\frac{d^2 R}{d P^2} = -\frac{QG^{-1}\left(Q/P\right)}{P^2} - \left[-\frac{Q^2}{P^3 g\left(G^{-1}\left(Q/P\right)\right)} - \frac{QG^{-1}\left(Q/P\right)}{P^2} \right] \tag{A8}$$

which simplifies to:

$$\frac{d^2 R}{d P^2} = \frac{Q^2}{P^3 g\left(G^{-1}\left(Q/P\right)\right)} \tag{A9}$$

which, by inspection, is always positive. □

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CHAPTER II

THE IMPACT OF YIELD UNCERTAINTY IN REMANUFACTURING

1: Introduction and Literature Review

The inherent variation in the quality of used products presents a significant management

challenge for remanufacturing firms. Sorting policies help manage this variation by assigning a

disposition to used products based on condition. For a given sorting policy, there is some

uncertainty regarding the percentage of used products that will actually be sorted for

remanufacture (i.e. the remanufacturing yield). In this research we examine the impact of this

uncertainty on optimal acquisition and sorting policies for remanufacturers. We present a

newsvendor-type heuristic that allows us to incorporate this uncertainty into the determination of

optimal acquisition and sorting policies very easily. We also provide exact algorithms for

uncertain yield problems, which we use to verify the accuracy of the heuristic approaches. The

results of our analysis indicate that approximating the uncertain remanufacturing yield parameter

with its deterministic equivalent provides near-optimal results in many cases while avoiding the

complexity of a stochastic yield formulation.

Remanufacturing operations management has received considerable attention from

researchers in recent years. For a thorough review of the academic work in this area, we refer

the reader to Guide et al. (2003) and Galbreth and Blackburn (2006). While the variability of

used product condition has been well established (Bloemhof-Ruwaard et al. 1999, Guide and

Jayaraman 2000, Fleischmann et al. 2000, Toktay 2000, Stanfield et al. 2004), the impact of

condition variability on used product acquisition, sorting, and disposition decisions – areas

identified by Guide et al. (2003) as under-treated from an academic perspective – has received

limited attention. The problem of variable used product condition is addressed in practice using

sorting systems ranging from a simple category-based approach (e.g. excellent, good, poor) to a

more robust methodology based on the unique remanufacturing requirements of individual

products. Researchers have also used various approaches to modeling used product condition.

For example, Aras et al. (2004) present a continuous-time Markov chain model that differentiates

between used product condition using two quality categories, while Galbreth and Blackburn

(2006) present a more general model by defining used product condition along a continuum.

In their 2006 paper, Galbreth and Blackburn show the existence of optimal acquisition

and sorting policies in a wide range of situations. In order to derive their results, they assume

that remanufacturing yield is a deterministic function of sorting policy. They suggest that future

research examine the extent to which the assumption of deterministic yield limits their results. In

this paper, we explicitly consider uncertainty in the remanufacturing yield associated with a

given sorting policy, analyzing both the dichotomous condition case and the case where used

product condition is defined along a continuum. We analyze the acquisition and sorting decision

for a remanufacturer using both heuristic approaches based on deterministic estimates of yield

and stochastic yield models. Through extensive numerical analysis, we identify a wide range of

problems for which the stochastic yield model provides minimal cost benefits over the simpler

deterministic yield model, lending credibility to the deterministic yield assumption in these

environments.

The rest of this paper is organized as follows. Section 2 presents cost minimization

models given dichotomous used product condition, including formulations with and without

yield uncertainty. Section 3 extends the analysis to the case in which condition is defined along a continuum, again providing both deterministic and stochastic yield formulations. Section 4 presents the results of experiments comparing the deterministic and stochastic yield models. Section 5 concludes the analysis with a discussion of the contribution of this paper to the remanufacturing research area.

2. Optimal Policies for Dichotomous Used Product Condition

We begin by examining the case of a remanufacturer that acquires a mixed lot of used products, the remanufacturing cost of each product being either <u>low</u> or <u>high</u>, based on its condition. Some fraction α of the lot of acquired products fall into the first category and have a known, fixed cost to remanufacture, C_1. The remaining $(1-\alpha)$ products have a higher remanufacturing cost, C_2. This is representative of many simple remanufacturable items. For example, certain toner cartridges, particularly smaller ones, are simple enough that the sorting is merely driven by whether or not the cartridge has been remanufactured previously. The ones that have never been remanufactured, called "virgin" cartridges in the industry, typically have a lower remanufacturing cost than the others, called "non-virgins." In this section, we model the decision of how many used items to acquire to meet a given demand in such a situation, where used product condition is dichotomous, and how to sort those used items.

2.1 Deterministic Yield

Assuming deterministic yield, the cost to meet a given demand Q can be defined as follows:

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$$f(Q) = MIN \begin{cases} uQ + C_1\alpha Q + C_2(1-\alpha)Q \\ u\left(\dfrac{Q}{\alpha}\right) + C_1Q \end{cases} \tag{1}$$

where u is the unit acquisition cost. In this analysis, we assume that the remanufacturer always prefers to meet the demand rather than be short – in other words, given a unit shortage penalty s, we assume that the following condition holds: $s > u + C_2$. Note that (1) consists of two possible scenarios, of which the one with the lowest total cost is optimal. The top expression in (1) represents acquiring exactly Q used products and remanufacturing all of them. In the second expression, enough used products are acquired such that only the low cost items need to be remanufactured to meet the demand.

Defining the acquisition quantity as P, we can specify the optimal acquisition quantity, P*, by simplifying (1) to the following:

$$P^* = \begin{cases} Q & when \quad \alpha\tilde{s} < u \\ \dfrac{Q}{\alpha} & otherwise \end{cases} \tag{2}$$

where $\tilde{s} = C_2 - C_1$. As shown in (2), this problem reduces to a decision based on a very simple expression. Either a remanufacturer will obtain the exact quantity it needs to meet the demand and remanufacture everything, or it will obtain enough that it can sort only the low cost items for remanufacture.

2.2 Stochastic Yield

In practice, of course, there will be uncertainty regarding the actual value of α for a given lot of acquired products. We now incorporate this uncertainty into the analysis by modeling the actual

quantity of low cost items for a given acquisition amount as a binomially distributed random variable. The binomial distribution is appropriate in situations where the process is in statistical control and the production of any individual item is independent of all other items (Yano and Lee 1995, Barad and Braha 1996, Grosfeld-Nir and Gerchak 2004) – an accurate description of most remanufacturing environments. In this scenario, the remanufacturer will process all low cost items initially, setting aside the high cost items during sorting. If demand is fulfilled using only low cost items, then the high cost items are not needed and are scrapped. If demand is not met using only low cost items, then high cost items are processed until demand has been fulfilled. We incorporate stochastic yield into our analysis in two ways – first with a newsvendor-type approximation, and then with an exact algorithm.

We can adjust the deterministic yield model to account for binomially distributed yield using an approach similar to a newsvendor model with normally distributed demand. Define the likelihood of a shortage given an acquisition quantity P as follows:

$p(N < Q \mid P)$ where N is the number of low cost items

Therefore, the marginal unit of acquisition P satisfies the following:

$$p(N < Q \mid P)\alpha\tilde{s} = u \tag{3}$$

The left hand side of (3) is the expected value of acquiring one more used item: specifically, the probability that we have less than Q low cost items, multiplied by the probability that the next item acquired will be low cost, multiplied by the savings that will result from replacing a high cost item with a low cost one. When this equals the cost of acquiring one more item, u, we are at the margin.

Since the normal distribution approximates the binomial very well for all $np(1 - p) \geq 10$ (Ross 2002), we can take the quantity of low cost items acquired to be the analog of normally

distributed demand in the standard newsvendor problem. Define $\Phi(x)$ as the probability of acquiring less than Q low cost items, given that the quantity of low cost items is normally distributed with mean αP and standard deviation $\sqrt{P\alpha(1-\alpha)}$. Now (3) can be written as;

$$\Phi(P) = \frac{u}{\alpha\widetilde{s}} \qquad (4)$$

The value of P for which (4) holds can be easily obtained. Using this simple technique, we can incorporate yield uncertainty into our determination of a cost-minimizing policy (see Example 1).

In order to verify that our newsvendor approach is accurate, we present an exact formulation of the stochastic yield model. Modeling yield as binomially distributed, we have the following cost expression:

$$f(P) = uP + C_1 Q + \widetilde{s} \sum_{N=0}^{Q-1} \binom{P}{N} \alpha^N (1-\alpha)^{P-N} [Q-N] \qquad (5)$$

We now show that (5) is "discrete convex," i.e. first differences of the function are monotonically increasing in the decision variable (Barad and Braha 1996), and therefore a unique global minimizer exists for (5).

PROPOSITION 1: The cost function (5) is discrete convex with a unique minimum value.

Proof: See Appendix A.

Given Proposition 1, the numerical search for the minimizer of (5) is straightforward.

We can use the deterministic solution from the previous section to gain insights into the minimizer of (5). For example, consider the case in which the deterministic yield solution from (2) is P*=Q (i.e. $\alpha\widetilde{s} < u$). When yield is stochastic, we can calculate the benefits from obtaining one extra unit – if the extra unit is low cost, then it can be processed instead of a high cost item. This leads us to the following proposition:

PROPOSITION 2: If P*=Q assuming deterministic yield, then P*=Q when the deterministic yield assumption is relaxed.

Proof: The acquisition cost of unit Q+1 is u. The expected benefit from acquiring unit Q+1 is the probability that it can be used to replace a high cost item with a low cost one multiplied by the savings enabled by that substitution: $\alpha\widetilde{s}$. Therefore, unit Q+1 should be acquired when $u < \alpha\widetilde{s}$. From (2), we know that this is never the case when P*=Q in the deterministic yield case. □

Next, consider the case where the deterministic yield solution is P*=Q/α. Recall that the decision to acquire extra items is driven by the relative values of u and $\alpha\widetilde{s}$, i.e. what the extra item costs versus its expected reduction in remanufacturing costs. Evaluating this tradeoff when yield is stochastic, we have the following proposition.

PROPOSITION 3: If P*=Q/α assuming deterministic yield, then for sufficiently large Q, Q/α is the critical point in the stochastic yield solution:

whenever $u \le \dfrac{\alpha\widetilde{s}}{2}$, Q/α is a lower bound on P*

whenever $u > \dfrac{\alpha\widetilde{s}}{2}$, Q/α is an upper bound on P*.

Proof: See Appendix B.

Note that the bounds defined by Proposition 3 can be confirmed in the experimental results in Appendix C.

Example 1:

We now present an example of the determination of an optimal acquisition quantity to meet a given demand when condition is dichotomous. We calculate the optimal policy three ways –

first, using the simple deterministic yield approximation; next, using the newsvendor-type

approach; and finally using the exact stochastic yield model. In this example, we use parameter

values derived from data for an actual laser toner cartridge, obtained from a large

remanufacturing firm in that industry (values have been disguised):

Table 1: Parameter Values for Example 1

Parameter	Example Value
u (acquisition cost)	$1.33
C_1 (low cost)	$20.41
C_2 (high cost)	$23.19
α (proportion that are low cost)	0.5
Q (demand)	2000

Note that $\alpha\tilde{s} = 1.39$ ($\tilde{s} = C_2 - C_1$). Therefore, from (2) we know that the optimal

acquisition quantity assuming deterministic yield is: P* = Q/α = 2Q. Thus, the heuristic solution

to this example can be summarized as follows: to minimize unit costs, acquire 4000 used

cartridges and remanufacture the virgin cartridges. If demand of 2000 is not met after processing

all virgin cartridges, use non-virgins to make up the shortfall. Next, we can determine that (4)

holds when P=3893 in this case. Therefore, the newsvendor solution is to acquire 3893 used

cartridges and process them in the same way as above. Finally, we verify that our newsvendor

approach resulted in the true optimal solution by solving the stochastic yield model exactly. We

can see from Proposition 3 that 2Q is an upper bound on the optimal solution. Using a simple

numerical search beginning at the upper bound of P=4000, we can find that (5) is minimized

when P=3893. Therefore, the true optimal solution was exactly specified by the newsvendor

approach – acquire 3893 used cartridges, and if the demand of 2000 is not met after processing

all the virgin cartridges, use non-virgins to make up the shortfall. This exact match of the

newsvendor and exact solutions is not surprising given the fact that $np(1-p) = 973.25 \gg 10$.

We can calculate the expected cost of each of the solutions by evaluating (5) for each

value of P. The expected cost to meet the demand using the exact solution (P=3893) is

$46,147.90, while the expected cost when using the deterministic yield approach (P=4000)

would be $46,175.10. Note that the cost increase resulting from the use of the deterministic

yield heuristic in this example is 0.06%.

Appendix C contains the solutions to (5) for several classes of problems. In that

appendix, we report the exact optimal policy as defined by the number of used products to

acquire, P. For all examples, since np(1-p) is far greater than 10, the exact solution can be

obtained either by using the newsvendor approach (solving (4)) or by minimizing (5). We also

report the policy suggested by the deterministic yield heuristic and the cost savings that can be

achieved by using the true optimal policy.

3. Optimal Policies for Continuous Used Product Condition

3.1 Deterministic Yield

For more complex used products, the cost to remanufacture cannot be accurately reflected by a

simple dichotomous condition model. In these cases, the condition of used items could fall

anywhere along a continuum – from very low cost to very high cost. For example, a used cell

phone could have any combination of a wide array of potential remanufacturing needs (antenna, screen, microphone, speaker, faceplate, etc.). When product condition is defined along a continuum, the expected yield α is no longer exogenous. Rather, it is a decision variable, since the remanufacturer can control the expected fraction of products sorted for remanufacture by adjusting the "cutoff" condition for remanufacturing – for example, a phone for which the screen and speaker are both damaged may or may not be remanufactured, depending on the cutoff determined by the remanufacturer. Galbreth and Blackburn (2006) describe how the sorting policy can be used to control yield in this way. Given any continuous distribution describing used product condition, a manager can effectively set expected yield anywhere between zero and one by adjusting the criteria required for a used product to be sorted for remanufacture versus scrap. Figure 1, adapted from that paper, contains a graphical representation of this concept.
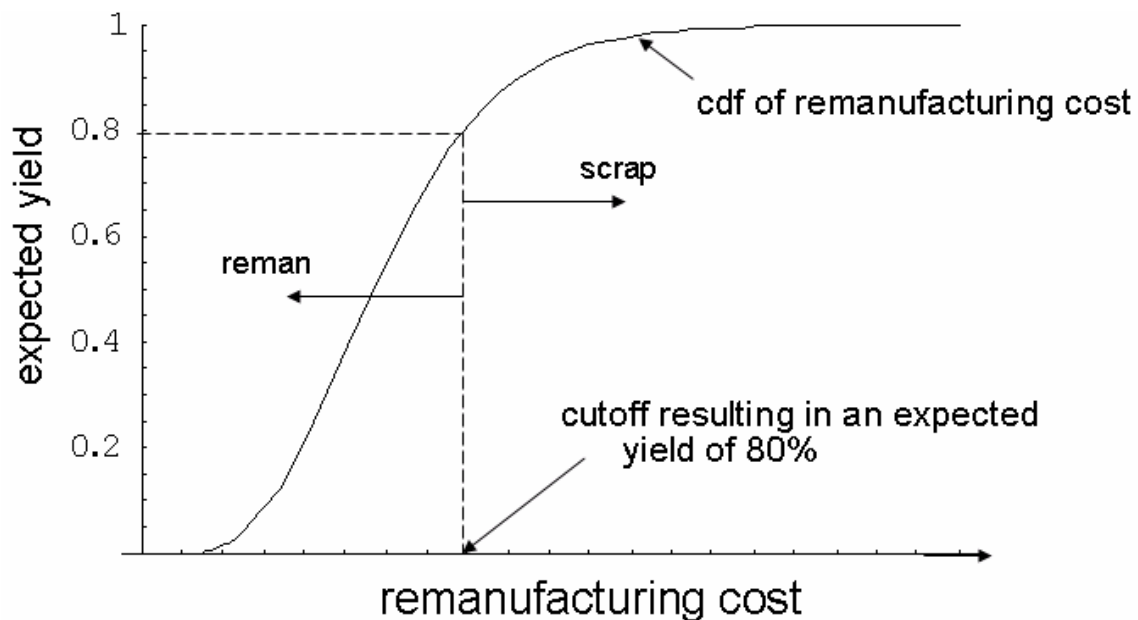
Figure 1: How expected yield can be controlled using the sorting cutoff

We now provide a summary of the formulation of Galbreth and Blackburn (2006) for the case of a known demand, continuous condition and deterministic yield. Note that, assuming deterministic yield, there is a single decision variable, the acquisition quantity P, and α is simply defined as $\alpha = Q/P$. In other words, for a given acquisition amount, yield will always be selected such that the quantity remanufactured equals the demand. The cumulative distribution of used product condition is denoted as G(.), with the corresponding density function g(.). Given G(.), the cutoff cost, i.e. the maximum cost acceptable for remanufacturing to meet a target yield, is some remanufacturing cost t such that:

$$G(t) = \alpha \text{, or } G(t) = Q/P$$

The sorting policy is defined by the value of t – products with remanufacturing cost greater than t are scrapped, while those with cost less than t are remanufactured (note that both α and t are functions of the single decision variable P in this case). We can express this cutoff cost as a function of P:

$$t = G^{-1}\left(Q/P\right)$$

and the average cost to remanufacture one unit, $\overline{C}$, as:

$$\overline{C} = \frac{\int_0^t xg(x)dx}{\alpha}$$

Adding an acquisition cost term, we have the following expression for total acquisition and remanufacturing cost:

$$TC(P) = uP + \overline{C}Q \tag{6}$$

Note that, given the deterministic yield assumption, the calculation of the optimal α once P^* is

determined is trivial, since $\alpha^* = Q/P^*$, and the optimal condition cutoff to be used in sorting the

P used products is simply $t = G^{-1}(\alpha^*)$. Similar to our dichotomous condition analysis, we

assume that the remanufacturer always prefers to meet the demand rather than be short – in other

words, given a unit shortage penalty s, we assume that the following condition holds: s>u+B,

where B is the maximum value that g(.) can attain.

Given any uniformly distributed g(.), we can extend the work of Galbreth and Blackburn

(2006) by deriving a closed form expression for the optimal P. First, note that that paper showed

that $\overline{C}Q$ is convex decreasing and has the following first derivative:

$$D = \frac{Q}{P}\left[\frac{\int_0^{G^{-1}(Q/P)} xg(x)dx}{\int_0^{G^{-1}(Q/P)} g(x)dx} - G^{-1}\left(Q/P\right) \right] \tag{7}$$

The other component of (6), uP, is obviously linearly increasing. Since $\overline{C}Q$ is convex

decreasing, its derivative D is increasing (becoming less negative) in P, so $(-D)$ is decreasing in

P. Therefore, (6) is minimized when $(-D) = u$, since acquiring the next item after this amount

would increase acquisition costs by u, but decrease remanufacturing costs by less than u.

PROPOSITION 4: Given that g(.) is any uniform distribution [A,B], the cost-minimizing

acquisition quantity P* is:

$$P^* = MAX\left(Q, \frac{Q}{\sqrt{\frac{2u}{B-A}}} \right) \tag{8}$$

Proof: substituting $\alpha = Q/P$ into (7), we have:

$$D = \alpha \left[\frac{\int_0^{G^{-1}(\alpha)} xg(x)dx}{\int_0^{G^{-1}(\alpha)} g(x)dx} - G^{-1}(\alpha) \right] \qquad (9)$$

which simplifies to:

$$D = \alpha \left[\frac{2A + \alpha(B - A)}{2} - [A + \alpha(B - A)] \right] \qquad (10)$$

further simplified:

$$D = \frac{\alpha^2 (A - B)}{2} \qquad (11)$$

To find the minimizer, we set (11) equal (-u) and solve for $\alpha*$:

$$\alpha^* = \sqrt{\frac{2u}{B - A}}$$

Since P*=Q/α*, we now have $P^* = \dfrac{Q}{\sqrt{\dfrac{2u}{B - A}}}$.

Given our assumption that shortages are never preferred, we add the condition that P* must be at least equal to Q. □

In (8) we provide a closed form solution to the deterministic yield, uniformly distributed condition problem. Given a demand (Q), an acquisition cost (u), and the upper and lower bounds on used product condition (A and B), we can determine the acquisition quantity and corresponding target yield that will minimize costs when yield is deterministic. Note that P* is decreasing in u, as expected. It is also increasing in condition variability, i.e. the spread between the highest and lowest cost items (B-A). This makes intuitive sense given that the potential

savings from acquiring more used items increases as the variability of those items increases (e.g., if all used products are in approximately the same condition, the benefit of acquiring additional ones is minimal).

3.2 Stochastic Yield

As in the dichotomous condition case, when condition is defined on a continuum the yield actually realized for a given batch might not be exactly what was expected for the sorting policy. Recall that, unlike with dichotomous condition, α is now a decision variable. Since overage and shortage costs are relevant to the analysis when yield is stochastic, a given value of P does not necessarily correspond to a specific value of α. For example, when shortage costs are high, the remanufacturer might react by increasing P *or* increasing α *or* increasing both. Thus, the remanufacturing problem with a continuous condition distribution becomes multidimensional when yield uncertainty is considered. As in the dichotomous condition case, we include the possibility of recourse if demand is not met after initial processing: the remanufacturer sets aside items that do not meet the sorting criteria, and if demand is not met using items initially sorted for remanufacture, then items that were set aside are processed until demand is met.

Before addressing the multidimensional problem, we note that if we hold α constant, a newsvendor-type approach identical to the one in Section 2 can be used to improve the deterministic solution. The only change to the approach would be to the shortage cost parameter – in the continuous condition case, shortage costs are defined as the difference between the expected cost of the items initially sorted for scrap and the items initially sorted for remanufacture. Therefore, when g(.) is uniform on [A,B], shortage cost is simply the following constant:

$$\hat{s} = \frac{B-t}{2} - \frac{t-A}{2} = \frac{B-A}{2}$$

We can adjust (4) to reflect the new shortage cost:

$$\Phi(P) = \frac{u}{\alpha\hat{s}} \qquad (12)$$

The value of P for which (12) holds can be easily determined. Using this simple technique, we can obtain an improved heuristic solution without resorting to the full multidimensional stochastic yield model (see Example 2).

Next, we present an exact formulation of the stochastic yield, continuous condition problem. Given that α and P can be adjusted independently, this problem has the following form:

$$f(P,\alpha) = uP + \overline{C}Q + \hat{s}\sum_{N=0}^{Q-1}\binom{P}{N}\alpha^{N}(1-\alpha)^{P-N}[Q-N] \qquad (13)$$

The problem of maximizing (13) is a multidimensional discrete optimization problem with decision variables P and α. Note that, unlike in the dichotomous condition case, remanufacturing costs are not constant but are functions of the decision variable α. As in the previous section, the actual condition cutoff used in sorting is defined as $t = G^{-1}(\alpha)$. We analyze the case where used product condition g(.) is any uniform distribution and obtain the following result.

PROPOSITION 5: The cost function (13) is discrete convex and attains a unique minimizer.

Proof: see Appendix D.

Given the convexity of the function, we can find the solution to (13) using a neighborhood search algorithm. The efficiency of a neighborhood search approach is significantly impacted by the quality of the starting point selected. For the starting point in our

neighborhood search we use the solution to the deterministic yield formulation as defined by (8).

Using this easily obtained starting point, we are able to locate the minimizer to (13) very

efficiently, even for large problems.

Example 2

We now present an example of the determination of an optimal acquisition quantity to meet a

given demand when condition is described by a continuous distribution. For example, consider a

cell phone whose remanufacturing cost, depending on the combination of remanufacturing

needs, could be anywhere from $0 (brand new) to $24.00 (all key components are damaged).

Parameter values representative of cell phone remanufacturing are presented in Table 2:

Table 2: Parameter Values for Example 2

Parameter	Example Value
g(x)	Uniform(0,24)
u	1
Q	100

Using (8), the optimal acquisition quantity assuming deterministic yield can be calculated as

P*=346. The corresponding target yield, α*, is therefore 100/346=0.29, and the optimal cutoff t

is G^{-1}(0.29)=$6.96. Thus, the heuristic solution can be summarized as follows: to minimize

costs, acquire 346 used phones; as each phone is processed, it will be sorted for scrap if its cost

to remanufacture is more than $6.96. If the demand of 100 is not met after initial processing, remanufacture items initially sorted for scrap to make up the shortfall.

Next, we can use the newsvendor-type approach in (12) to obtain a new approximation. Note that this limits us to solutions with the same α as the deterministic yield model solution, $\alpha=0.29$. Since (12) holds when P=362, we have a new optimal acquisition quantity of 362 using this approach.

Finally, we can solve the problem using the stochastic yield model presented above[1]. Using a neighborhood search, we can find that (13) is minimized when P=349 and $\alpha=0.30$ (corresponding to a cutoff of $7.20). Therefore, the true optimal solution would be to acquire 349 used phones as process them using a $7.20 sorting cutoff.

We can calculate the expected cost of the solutions by evaluating (13) for each one. The expected cost when using the deterministic yield heuristic solution (P=346, $\alpha=0.29$) would be $732.35. Using the newsvendor approach to improve this approximation (P=362, $\alpha=0.29$) gives an expected cost of $727.94, for a savings of 0.60%. The expected cost to meet the demand using the exact solution (P=349, $\alpha=0.30$) is $727.62, a savings of 0.65%.

Appendix E contains solutions to the continuous condition model, both as it is formulated in (13) and using each heuristic, for several classes of problems.

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[1] Note that, while we could use the same logic as in Proposition 3 to determine whether P* from the deterministic model provides an upper or lower bound on the stochastic yield solution, this conclusion would only be valid when the true α* *equals the α* defined by the deterministic model*. Since the true α* might be higher or lower than what was found in the deterministic model, a general representation of the relationship between the deterministic solution and the true P* is not available in this case (although the deterministic solution does provide a starting point for the efficient determination of the true P*, as described above).

4. Results

Appendices C and E allow us to compare remanufacturing policies obtained using the more parsimonious deterministic yield model with those obtained from the more accurate newsvendor and exact approaches. In the dichotomous condition case in Appendix C, optimal policies are fully defined by the acquisition amount, P, since condition categories for sorting are given. Appendix E presents the results for continuous condition, where sorting policy is controlled by management via the target yield. For these problem classes, the optimal policy is defined in terms of the acquisition amount, P, and the target yield, α.

A very clear pattern can be seen in the experimental results: for either type of condition distribution, the approximation provided by the deterministic yield model improves as demand increases. This is as expected given that the actual yield will converge to the expected value assumed by the deterministic model as demand increases.

Another observation regarding the results in the appendices is the role of shortage costs in the performance of the deterministic yield model. Specifically, in Appendix C, we can examine the deterministic yield results when the critical fraction, $\alpha\tilde{s}/2$, is very close to the overage cost, u. This fraction is closest to u for high α/low $\tilde{s}$, low α/medium $\tilde{s}$, and medium α/medium $\tilde{s}$. In these cases, the deterministic yield model approximates the exact solution very closely. When the critical fraction is not near u, e.g. high α/high $\tilde{s}$, the performance of the deterministic model is worse. This is not surprising given the fact that the deterministic model ignores the possibility of a shortage – when shortage and overage costs are widely disparate, the heuristic can be expected to perform worse. Although there is no simple critical fraction for the multidimensional continuous condition model, we note that when an analogous value, $\alpha\hat{s}/2$, is

very far from u, the performance of the deterministic estimate is worse, e.g. the high variability case.

It is important to note that, while the deterministic yield heuristic performed better for higher demands and some ranges of shortage costs, it always found a solution with an expected cost well within 1% of the exact solution in our experimental analysis. In most cases, the heuristic was within 0.1% of the optimal solution. Therefore, while the assumption of deterministic yield is most appropriate when demands are high and the value of the critical fraction is near u, we find that it is in fact quite reasonable in most remanufacturing settings.

5. Conclusions

The models developed and analyzed in this paper contribute to the remanufacturing research area in several ways. First, we have provided new models to address the acquisition and sorting aspects of remanufacturing operations, areas identified by Guide et al. (2003) as in need of additional research. In addition, we have performed the model extension suggested in Galbreth and Blackburn (2006) to the case of stochastic yield. We did this using both a simple newsvendor-derived algorithm and an exact solution to the discrete optimization problem. Our newsvendor approach provides optimal solutions for any reasonably-sized problem with dichotomous condition, and it performs very well in the continuous condition case also. Thus, we have provided a simple and accurate approach to incorporating yield uncertainty into acquisition and sorting decisions.

By comparing it with exact solutions, we find that the model assuming deterministic yield is often reasonable, providing near-optimal solutions in a wide range of scenarios. Specifically, that model performs very well when demand levels are high and shortage costs are more in line

with overage costs. These findings provide justification for the use of the much simpler and more tractable deterministic yield models of remanufacturing. We also note that the deterministic model for continuous condition serves an important role by providing the starting point of the solution algorithm for the stochastic yield problem.

We see several directions for future research in the area of acquisition and sorting policies for remanufacturing. One interesting extension to our model would be to include constraints or penalties for scrapping used products. An advantage of remanufacturing is its positive impact on the environment by extending the usable life of products. Given that scrapping large percentages of acquired products conflicts with this advantage, future research should explicitly consider the direct and indirect costs of scrapping. Debo et al. (2005) suggest that a unit disposal cost might be a relevant consideration in remanufacturing research, and this change can be easily incorporated into most models, including the ones presented in this paper. In addition, analysis of the usefulness of models like this one when used product condition is described by more complex distributions, ideally obtained from empirical data, would also further the knowledge in this emerging research area.

Appendix A

Note: this proof follows the approach used by Barad and Braha (1996).

For any P, we have the following function for f(P+1):

$$f(P+1)=u(P+1)+CQ+\tilde{s}\sum_{N=0}^{Q-1}\binom{P+1}{N}\alpha^N(1-\alpha)^{P+1-N}[Q-N] \tag{A1}$$

or, simplifying the notation:

$$f(P+1)=u(P+1)+CQ+\tilde{s}\sum_{N<Q}[Q-N]p[N\mid P+1] \tag{A2}$$

From elementary probability theory we have:

$$p(N\mid(P+1))=\alpha p((N-1)\mid P)+(1-\alpha)p(N\mid P) \tag{A3}$$

Subtracting (5) from (A2) and using (A3) gives the following expression for the first difference:

$$\Delta f(P)=u+\tilde{s}\alpha\sum_{N<Q}[Q-N]\{p[(N-1)\mid P]-p[N\mid P]\} \tag{A4}$$

which after some algebraic manipulation simplifies to:

$$\Delta f(P)=u-\tilde{s}\alpha p(N<Q\mid P) \tag{A5}$$

Letting $\Psi(P)=p(N\geq Q\mid P)$:

$$\Delta f(P)=u-\tilde{s}\alpha(1-\Psi(P)) \tag{A6}$$

or, equivalently:

$$\Delta f(P)=u-\tilde{s}\alpha+\tilde{s}\alpha(\Psi(P)) \tag{A7}$$

Since $\psi(P)$ is strictly increasing for all $P\geq Q$, we can say that (A7) is a monotonically

increasing function of P. We conclude the proof by noting that, since $\lim_{P\to\infty}\Delta f(P)=u>0$, (A7)

has a unique minimum. □

Appendix B

For any P, we have the following function for f(P+1):

Assume that Q/α items are acquired. Note that acquiring one item less reduces costs when the following condition holds:

$$u > \tilde{s}\left[\sum_{N=0}^{Q-1}\binom{P-1}{N}\alpha^N(1-\alpha)^{P+1-N}[Q-N] - \sum_{N=0}^{Q-1}\binom{P}{N}\alpha^N(1-\alpha)^{P+1-N}[Q-N]\right] \tag{A8}$$

That is, when the unit acquisition cost saved exceeds the expected increase in shortage costs.

From elementary probability theory we have:

$$p(N\,|\,(P)) = \alpha p((N-1)\,|\,(P-1)) + (1-\alpha)p(N\,|\,(P-1)) \tag{A9}$$

Simplifying (A8) using (A9):

$$u > \tilde{s}\,\alpha\sum_{N<Q}[Q-N]\{p[(N-1)\,|\,(P-1)] - p[N\,|\,(P-1)]\} \tag{A10}$$

which after some algebraic manipulation simplifies to:

$$u > \tilde{s}\,\alpha p(N<Q\,|\,(P-1)) \tag{A11}$$

Note that, when P=Q/α, the likelihood of shortage is 0.5 for sufficiently large Q. Thus, since P is decreased by one in (A11), we know that $p(N<Q\,|\,(P-1))>0.5$. Therefore, it is clear that, when $u \leq \dfrac{\alpha\tilde{s}}{2}$, (A11) will never hold, therefore decreasing P will never improve the solution. Similarly, when $u > \dfrac{\alpha\tilde{s}}{2}$, (A11) will always hold, therefore decreasing P will always improve the solution. □

Appendix C

demand (Q)	$\widetilde{s}$	α	deterministic yield heuristic: P, cost	exact solution: P, cost	cost savings
low (100)	low	low	200, $2314.83	177, $2308.69	0.27%
		medium	143, $2237.12	139, $2236.30	0.04%
		high	111, $2192.24	111, $2192.24	0.00%
	medium	low	200, $2320.47	198, $2320.26	0.01%
		medium	143, $2241.38	145, $2241.19	0.01%
		high	111, $2194.85	113, $2194.23	0.03%
	high	low	200, $2326.10	203, $2325.58	0.02%
		medium	143, $2245.65	147, $2243.76	0.08%
		high	111, $2197.45	114, $2195.33	0.10%
medium (500)	low	low	1000, $11552.50	947, $11538.90	0.12%
		medium	714, $11168.50	706, $11167.00	0.01%
		high	556, 10951.80	555, $10951.70	0.00%
	medium	low	1000, $11565.10	995, $11564.80	0.00%
		medium	714, $11178.40	719, $11177.70	0.01%
		high	556, $10957.00	559, $10955.90	0.01%
	high	low	1000, $11577.80	1008, $11576.40	0.01%
		medium	714, $11188.40	724, $11183.20	0.05%
		high	556, $10962.30	562, $10958.30	0.04%
high (2000)	low	low	4000, $46175.10	3893, $46147.90	0.06%
		medium	2857, $44647.10	2840, $44644.10	0.01%
		high	2222, $43791.20	2221, $43791.20	0.00%
	medium	low	4000, $46200.30	3991, $46199.60	0.00%
		medium	2857, $44666.80	2866, $44665.30	0.00%
		high	2222, $43802.70	2320, $42799.50	0.01%
	high	low	4000, $46225.50	4017, $46222.60	0.01%
		medium	2857, $44686.40	2877, $44676.20	0.02%
		high	2222, $43814.20	2234, $43804.10	0.02%

Parameter values: u=1.33; C_1=20.41; $\widetilde{s}$ =2.78, 4.78, 6.78; α=0.5, 0.7, 0.9

Appendix D

In order to prove that the multidimensional function (13) is discrete convex in both decision variables, we analyze second differences of the function when both variables are increased simultaneously. Let P increase by 1 and α increase by ε. The change in α results in changes to $\overline{C}$, since the average cost to remanufacture increases as expected yield is increased, giving the following expression (recall that $\hat{s}$ is a constant):

$$f(P+1,\alpha+\varepsilon)=u(P+1)+\overline{C_2}Q+\hat{s}\sum_{N=0}^{Q-1}\binom{P+1}{N}(\alpha+\varepsilon)^N(1-(\alpha+\varepsilon))^{P+1-N}[Q-N]\qquad(A12)$$

where $\overline{C_2}=A+\dfrac{(\alpha+\varepsilon)(B-A)}{2}$.

or, simplifying the notation:

$$f(P+1,\alpha+\varepsilon)=u(P+1)+\overline{C_2}Q+\hat{s}\sum_{N<Q}[(Q-N)p_2(N\,|\,(P+1))]\qquad(A13)$$

where $p_2>p$.

Define δ as the difference in expected underage when α is increased by ε:

$$\delta=\sum_{N<Q}[(Q-N)p(N\,|\,(P+1))]-\sum_{N<Q}[(Q-N)p_2(N\,|\,(P+1))]$$

We can rewrite (A13) using δ as follows:

$$f(P+1,\alpha+\varepsilon)=u(P+1)+\overline{C_2}Q+\hat{s}\left(\sum_{N<Q}[(Q-N)p\,(N\,|\,(P+1))]-\delta\right)\qquad(A14)$$

Using the approach from Appendix A, we can subtract (13) from (A14) to obtain the following first difference expression:

$$\Delta f(P,\alpha)=u+\left(\overline{C_2}-\overline{C}\right)Q-\hat{s}\alpha p(N<Q\,|\,P)-\hat{s}\delta\qquad(A15)$$

To further simplify the expression, let $\Psi(P,\alpha) = p(N \geq Q \mid P)$:

$$\Delta f(P,\alpha) = u + \left(\overline{C_2} - \overline{C}\right)Q + \hat{s}\alpha\Psi(P,\alpha) - \hat{s}\delta \tag{A16}$$

Note that $\left(\overline{C_2} - \overline{C}\right)$ does not depend on either decision variable and is a constant for any ε:

$$\overline{C} = A + \frac{\alpha(B-A)}{2}; \overline{C_2} = A + \frac{(\alpha+\varepsilon)(B-A)}{2}; \text{ Therefore, } \overline{C_2} - \overline{C} = \frac{\varepsilon(B-A)}{2}.$$

We now examine second differences of the function. Incrementing the decision variables in (A16) gives us the following:

$$\Delta f(P+1,\alpha+\varepsilon) = u + \left(\overline{C_2} - \overline{C}\right)Q + \hat{s}(\alpha+\varepsilon)\Psi(P+1,\alpha+\varepsilon) - \hat{s}\delta \tag{A17}$$

Subtracting (A16) from (A17) and simplifying, we have:

$$\Delta^2 f(P,\alpha) = \hat{s}\alpha\left[\Psi(P+1,\alpha+\varepsilon) - \Psi(P,\alpha)\right] + \hat{s}\varepsilon\Psi(P+1,\alpha+\varepsilon) \tag{A18}$$

An inspection of (A18) confirms that it is always positive. We conclude the proof by noting that

$\underset{\alpha \to 1, P \to \infty}{Lim}(A11) = u + \hat{s} > 0$, so the function attains a unique minimum. □

Appendix E

demand	condition variability	deterministic solution: P, α, cost	newsvendor solution: P, α, cost (savings)	exact solution: P, α, cost (savings)
low (100)	low	141, .71, $287.18	**137, .71, $286.48 (0.24%)**	**137, .71, $286.48 (0.24%)**
	medium	265, .38, $550.67	271, .38, $550.36 (0.06%)	263, .39, $550.20 (0.09%)
	high	346, .29, $732.35	362, .29, $727.94 (0.60%)	349, .30, $727.62 (0.62%)
medium (500)	low	707, .71, $1424.79	698, .71, $1422.60 (0.15%)	704, .70, $1422.52 (0.16%)
	medium	1322, .38, $2693.44	**1330, .38, $2692.77 (0.02%)**	**1330, .38, $2692.77 (0.02%)**
	high	1732, .29, $3549.33	**1760, .29, $3541.83 (0.21%)**	**1760, .29, $3541.83 (0.21%)**
high (2000)	low	2828, .71, $5680.41	2810, .71, $5674.48 (0.10%)	2837, .70, $5673.71 (0.12%)
	medium	5292, .38, $10676.90	**5292, .38, $10676.90 (0.00%)**	**5292, .38, $10676.90 (0.00%)**
	high	6928, .29, $14019.20	**6969, .29, $14011.30 (0.06%)**	**6969, .29, $14011.30 (0.06%)**

Parameter values: u=1; g(x)=U(0,4), U(0,14), U(0,24)
bold indicates that newsvendor solution is exact (whenever deterministic yield model specifies the true optimal α)

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MODELS FOR OFFSHORE REMANUFACTURING WITH VARIABLE USED PRODUCT
CONDITION

1: Introduction and Literature Review

For many products, offshore production may provide a lower cost on average without a

significant difference in quality (Venkatraman 2004). However, offshoring remanufacturing also

involves higher shipping and handling costs and longer lead times relative to domestic

production. Lead times might prevent the use of the offshore facility to meet some demands.

Even when demand is known with sufficient time to send items offshore, the production cost

savings must be considered along with increased shipping and handling costs to determine the

true cost advantage of offshoring.

In remanufacturing, the amount of processing required to restore a given used item to

saleable condition is often variable. If the remanufacturing requirement is very low, then the

additional shipping and handling costs might make offshore remanufacturing an unattractive

option. Krikke et al. (1999) describe tradeoffs such as these when evaluating a proposed copier

remanufacturing site in the Czech Republic. Carter et al. (1997) provide a discussion of the

factors leading to higher logistics costs and lead times for a firm choosing China for offshore

production, and they note that these costs could potentially outweigh any production cost

savings. The consideration of such international elements in reverse logistics has been identified

as an area in need of research (Rogers and Tibben-Lembke 2001). The focus of this research is

to evaluate how the offshoring decision changes in a remanufacturing environment, where the

processing requirements of the used items are variable.

The variability in the condition of used products obtained for remanufacturing has been noted extensively in the literature. While many studies have acknowledged the problem (Bloemhof-Ruwaard et al. 1999, Blumberg 1999, Stanfield et al. 2004, Franke et al. 2005), few have addressed its implications for remanufacturing profitability. Fleischmann et al. (1997) discuss the variability of supply quality as a distinguishing characteristic of reverse vs. forward distribution. In their 2004 paper, Savaskan et al. point to the growing interest in research which considers the quality variability in return flows. They analyze three collections options and discuss the conditions under which different collections processes and supply chain structures are appropriate. However, their model assumes homogeneous quality of returned products for each collection option and that all returned products are remanufactured for resale (100% yield). Krikke et al. (1999) mention the variability of returned product quality, but their MILP formulation of a reverse logistics network includes only a fixed unit cost of remanufacturing.

In this paper, we assume that used products are obtained as needed, e.g. from third party brokers, and are inspected immediately at a central facility (Rogers and Tibben-Lembke (2001) find that a single central collection facility is used in nearly 70% of reverse supply chains). Some products are immediately scrapped (Guide and Van Wassenhove (2002) and Blackburn et al. (2004) discuss this type of early differentiation of used items as a valuable reverse logistics strategy), and the remaining items are remanufactured at one of two locations – domestic or offshore. This choice of remanufacturing facility for each individual used product has been mentioned as a critical step in reverse logistics (Rogers and Tibben-Lembke 2001).

We determine optimal strategies for two models of offshore remanufacturing. First, we examine a two-period model in which only the second period demand is known with sufficient lead time to use the offshore facility and no demand is anticipated beyond the second period.

We use this model to analyze, in a simple setting, the tradeoffs between domestic and offshore remanufacturing. Second, we extend the analysis to the multi-period case in which demand is stable and the use of the offshore facility is always feasible in steady state. We identify situations in which offshoring all remanufacturing is always optimal when lead times allow, as well as cases in which a domestic facility has a role in the optimal solution, even when there is sufficient lead time for remanufacturing offshore. We also identify conditions under which a mixed strategy of domestic and offshore remanufacturing, with the advantages of each facility being leveraged, will result in lower costs than either a purely domestic or purely offshore approach. We show that this mixed strategy remains optimal as the product matures and fewer of the used items acquired are in good condition.

2: A two-period model of remanufacturing

In this section, we examine a two-period problem in which two locations, domestic and offshore, are available for remanufacturing used items (Figure 1). In this model, Period 1 demand is not known far in advance – when the order is received, it must be met quickly. The model is constructed in this way to reflect the fact that, as pointed out by Ferrer and Whybark (2001), some remanufacturers have limited advance knowledge of demand. The domestic facility with its shorter lead times must be used to meet Period 1 demand. Beyond this initial period, a single additional demand, in Period 2, is known. Period 2 represents the end of the product life cycle, and after that period no more demand can be anticipated. This reflects the situation in which product life cycles are short – for certain remanufactured items such as cell phones and other electronics, a stable, continuing demand is unlikely.
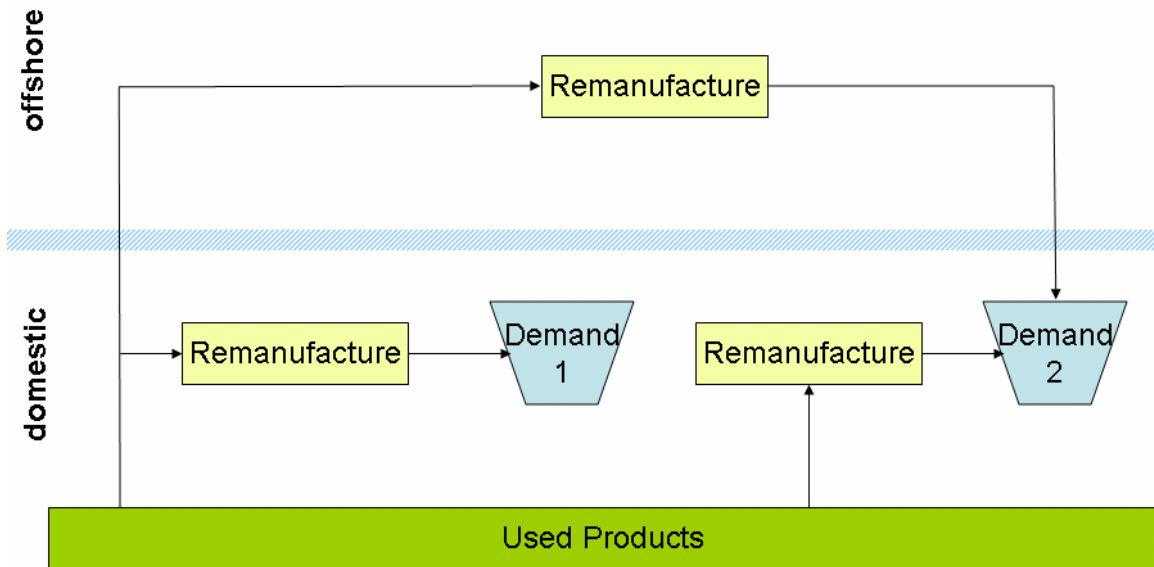
Figure 1: A Two-Period Remanufacturing Problem with Offshoring

We make the following assumptions regarding the remanufacturing environment. Used items are acquired as needed and fall into one of two categories – low-touch, indicating that the item can be made available for sale after a relatively small amount of remanufacturing, and high-touch, indicating that a higher amount of remanufacturing is required before the product can be resold (this type of dichotomous sorting is common in practice and has been used in previous remanufacturing models, e.g. Aras et al. (2004)). Items can be scrapped at the remanufacturer's discretion (scrap value is assumed to be negligible, although any positive or negative scrap value can be captured in the model by an adjustment to unit acquisition costs). Thus, the remanufacturer has the option of acquiring extra used products and meeting demand by remanufacturing only the low-touch items. The proportion of used products in the low-touch category is denoted by α, with the remaining $(1-\alpha)$ being high-touch. In this analysis, we assume no variability in α. For example, if $\alpha = .18$, then a batch of 100 used items will always

contain 18 low-touch products and 82 high-touch products. See Chapter II of this manuscript for

a detailed discussion of this assumption and situations for which it is reasonable. Table 1

contains a complete listing of the notation used in our analysis.

Table 1: Notation

Notation	Description
D	Demand
c	cost to remanufacture a low-touch item at the domestic facility
u	unit acquisition cost of used items
s	round trip shipping cost to/from domestic plant
ρ	Cost reduction factor (offshore low-touch cost = c/ρ) ($\rho > 1$)
θ	Shipping cost multiplier (offshore shipping = θs) ($\theta > 1$)
λ	Remanufacturing cost relationship (high-touch cost = λc) ($\lambda > 1$)
α	Proportion of low-touch items

Two key parameters in our model capture the tradeoffs inherent in an offshoring decision.

The parameter ρ represents the ratio of domestic remanufacturing costs to offshore, and it can be

quite high (comparing some locations in China to the U.S., this parameter could exceed a value

of 20). This cost savings is offset to some extent by the higher shipping costs to and from the

offshore facility, as indicated by θ.

We acknowledge that there are many differences in the costs of using facilities in

different parts of the world (Vidal and Goetschalckx 1997). While it is impossible to explicitly

include all of these in our model, our parameters allow us to capture many of them. Specifically,

the shipping cost parameters, s and θ, are location-specific costs that can include a variety of

components. Any additional location-specific cost – for example, taxes, local tariffs, time-value effects, inventory costs, and capacity reservation costs – could be included in the values of these parameters.

We now examine the optimal role of the domestic facility in the two-period problem. Recall that Period 1 demand must be met using the domestic facility. The interesting question, therefore, is whether or not the domestic facility would be used again to meet some or all of Period 2 demand – equivalently, we can ask: **is the optimal quantity of remanufactured items at the beginning of Period 2 ever less than D?** If so, then the optimal solution must use the domestic facility to meet at least some of Period 2 demand. This question can be answered by identifying the lowest-cost approach to meet a <u>single</u> demand when lead times allow the use of either facility (as is the case for any Period 2 demand). If an offshore approach has lower costs than any domestic approach, then no Period 2 demand will be unmet at the beginning of that period (there is time to use the cost-minimizing offshore facility, so it will be used). By answering the question above, we can specify the conditions under which the domestic facility will *never* be used in the two-period problem except in Period 1, when it is the only feasible choice.

Since, as described above, the role of the domestic facility in Period 2 can be determined by examining the case of a single demand, we now identify optimal solutions in that case. Assuming that shortages are prohibited, we know that at least D used items will be acquired. Furthermore, we know that at most D/α used items will be acquired, since this acquisition amount ensures that only low-touch items are remanufactured, and therefore there are no incremental savings from acquiring more. Between these extreme points, the total domestic cost to acquire some amount of used items, Q, and remanufacture D of them is given by:

$$uQ + sD + c\alpha Q + \lambda c(D - \alpha Q) \qquad (1)$$

Since (1) is linear in Q, we know that one of its extreme points, D or D/α, is the optimal

acquisition amount (the expression for offshore cost is similar and also linear in Q). Therefore,

the problem of meeting a single demand of D reduces to a choice between two alternatives for

each location:

      a. Obtain D used items and remanufacture all of them

      b. Obtain D/α used items and remanufacture only the D low-touch items

We begin by examining each alternative in the domestic case (denoted by subscript d). The unit

costs of each alternative are expressed below as a function of acquisition quantity:

$$C_d(D) = u + \alpha c + (1 - \alpha)\lambda c + s \qquad (2)$$

$$C_d\left(\frac{D}{\alpha}\right) = \frac{u}{\alpha} + c + s \qquad (3)$$

Therefore the optimal pure domestic strategy is to obtain D used items whenever (2) < (3). After

some algebraic manipulation, we have the following:

$$C_d(D) < C_d\left(\frac{D}{\alpha}\right) \text{ when:}$$

$$\frac{u}{\alpha} > c\lambda - c \qquad (4)$$

The above expression can be explained intuitively as follows. When the cost to acquire one

more low-touch item exceeds the savings from remanufacturing a low-touch item instead of a

high-touch one, then no additional items will be acquired. (4) can be simplified to the following

restriction on λ:

$$\lambda < \frac{u}{\alpha c} + 1 \qquad (5)$$

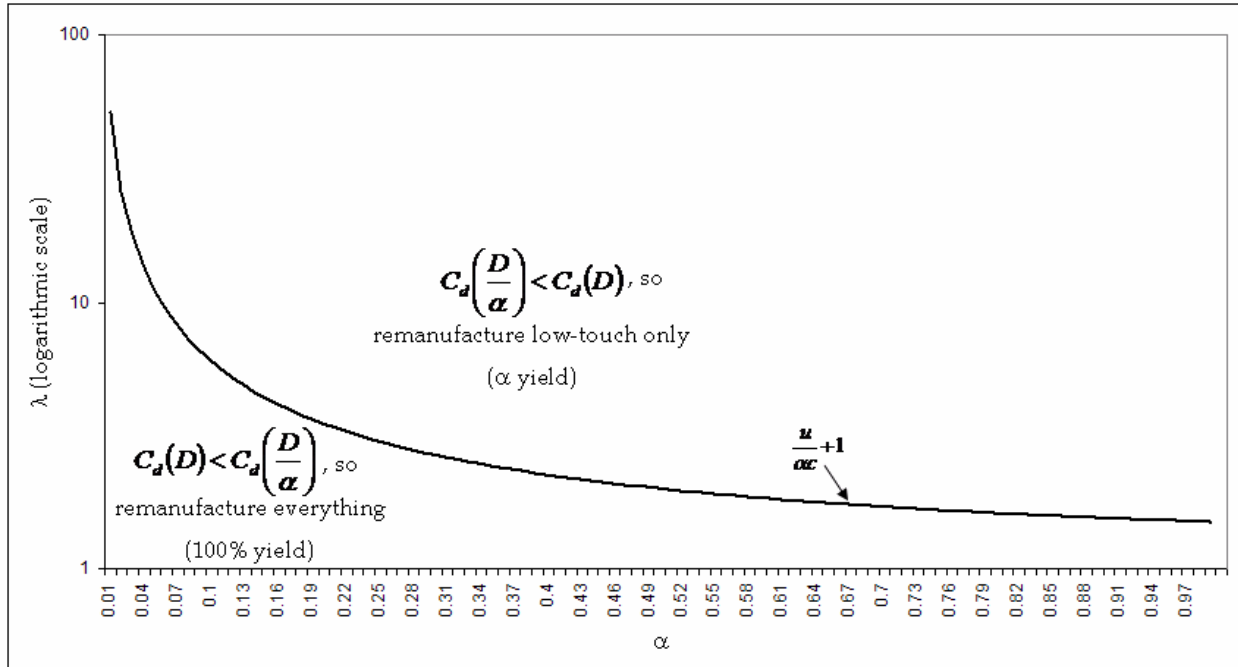(5) is depicted graphically in Figure 2 for various α:

Figure 2: Optimal approaches to domestic remanufacturing

From (5), we see that for high-touch remanufacturing requirements over a certain threshold, it is optimal for the domestic facility to incur extra acquisition costs and meet the demand using only low-touch items.

We now examine the costs of using the offshore facility (subscript o). Note that the same two alternatives exist as in the domestic case – acquire exactly the quantity demanded or acquire extra and only process the low-touch items. Unit cost, as a function of acquisition quantity, to meet a demand D is as follows:

$$C_o(D) = u + \frac{\alpha c}{\rho} + \frac{(1-\alpha)\lambda c}{\rho} + \theta s \qquad (6)$$

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$$C_o\left(\frac{D}{\alpha}\right) = \frac{u}{\alpha} + \frac{c}{\rho} + \theta s \tag{7}$$

We then obtain:

$$C_o(D) < C_o\left(\frac{D}{\alpha}\right) \text{ whenever } \frac{u}{\alpha} > \frac{\lambda c - c}{\rho}$$

As with the domestic facility, when the cost to acquire one more low-touch item exceeds the savings from remanufacturing a low-touch item instead of a high-touch one, then no additional items will be acquired. The above expression can be simplified to the following restriction on λ:

$$\lambda < \frac{\rho u}{\alpha c} + 1 \tag{8}$$

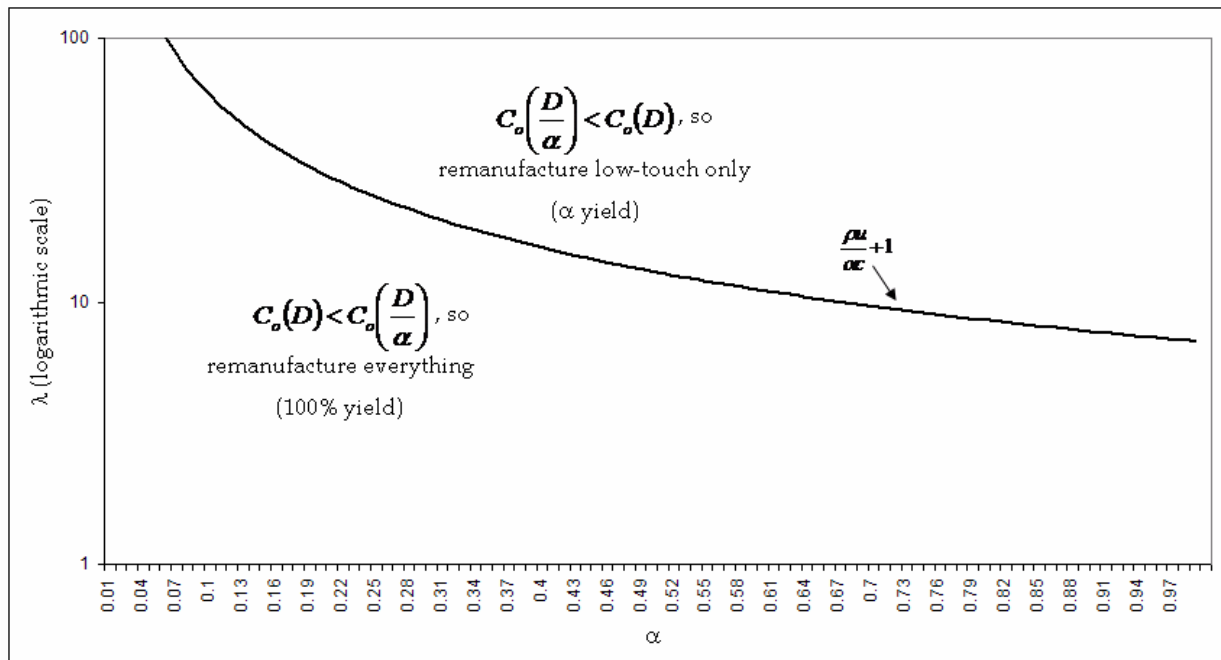which is depicted graphically in Figure 3 for various α:

Figure 3: Optimal approaches to offshore remanufacturing

We can combine Figures 2 and 3 to identify the conditions under which offshore is always lower cost than domestic for each region of λ (Figure 4).
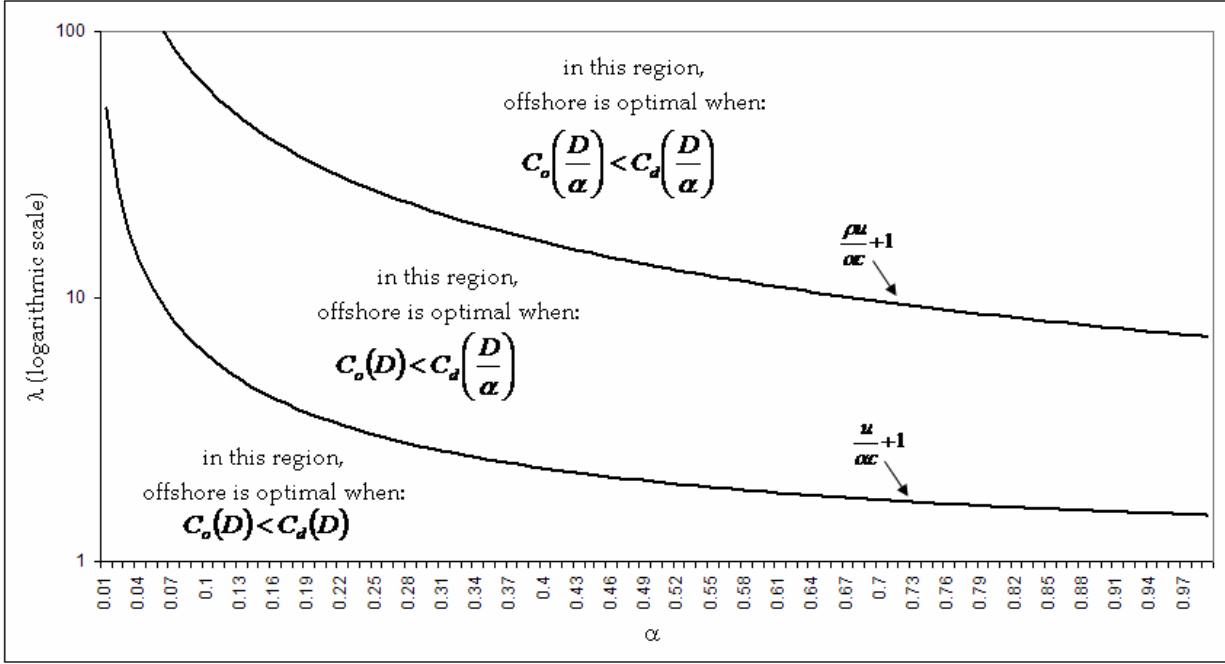

Figure 4: Conditions under which offshore remanufacturing is lower cost than domestic, by region of λ

Figure 4 defines three types of optimal policies based on λ values. In the bottom region, the optimal policy at either facility is to obtain D used items and remanufacture all of them. Given values of α and λ that define a point in this region, we can simplify the bottom inequality in Figure 4 to derive a bound on offshore shipping costs (θ). When this bound holds, offshore remanufacturing provides a lower total cost than domestic in this region of the figure:

$$\theta < \frac{c\left(1-\dfrac{1}{\rho}\right)\left(\lambda(1-\alpha)+\alpha\right)}{s}+1 \tag{9}$$

Note that (9) is an upper bound on the shipping cost penalty offshore – any value of θ above this level would result in domestic having lower cost. This upper bound on θ is strictly increasing in λ, i.e. for larger high-touch remanufacturing requirements, this upper bound is less stringent and offshoring is more likely to be optimal.

In the middle region of Figure 4, the optimal offshore policy is still to obtain D used items and remanufacture everything. However, at the domestic facility, with its higher remanufacturing costs, it is optimal to obtain D/α used items and only remanufacture the low-touch ones. We can simplify the inequality for this region to the following bound on θ:

$$\theta < \frac{u\left(\frac{1}{\alpha}-1\right)-\frac{\lambda c}{\rho}(1-\alpha)+c\left(1-\frac{\alpha}{\rho}\right)}{s}+1 \tag{10}$$

The upper bound on θ in (10) is decreasing in λ. Recall that, in this middle region, the offshore facility is processing all used items, while the domestic facility is only remanufacturing low-touch items. Therefore, as the remanufacturing cost of the high-touch items increases, offshore shipping costs must be lower for the offshore option to have lower cost.

In the top region of Figure 4, high-touch items are so expensive to remanufacture that the optimal policy at either location is to obtain D/α used items and only remanufacture the low-touch ones. The inequality for this region can be simplified to:

$$\theta < \frac{c\left(1-\frac{1}{\rho}\right)}{s}+1 \tag{11}$$

Note that (11) does not depend on λ – since no high-touch items are being remanufactured at either facility in these cases, their remanufacturing costs are irrelevant in this region.

We can use (9)-(11) to define the region in which offshore remanufacturing results in lower costs than domestic (Figure 5).
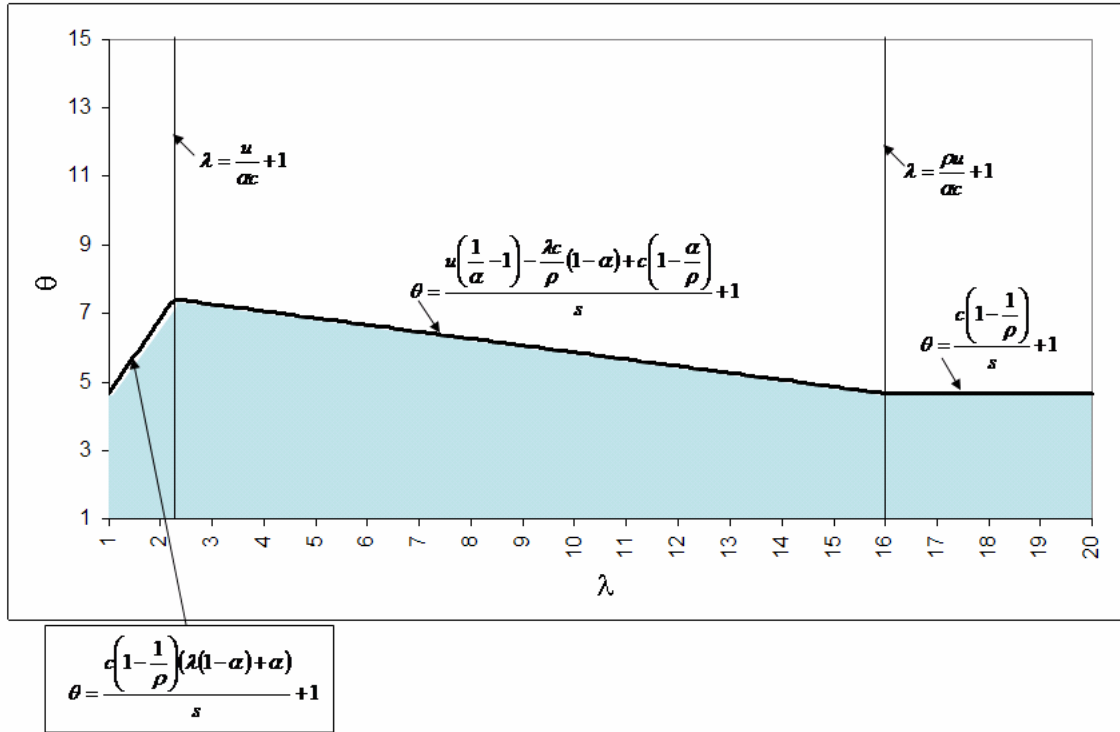

Figure 5[2]: The shaded region is where the domestic facility will **never** be used after Period 1 in the two-period problem

For parameter values in the shaded region of Figure 5, we know that the domestic facility will only be used in the first period of the two-period problem. If the values of λ and θ for a particular remanufacturing problem fall within this region (i.e. offshore shipping costs (θ) are sufficiently low given the remanufacturing requirement of high-touch items (λ)), we know that

the *only* role of the domestic facility is to meet unanticipated demand quickly. For any demand that is known with sufficient lead time, the offshore facility will always be used.

3: A multi-period model of remanufacturing with stable demands

In this section, we extend our analysis to the multi-period case, in which demand for a remanufactured item is stable and known. In this case, there is always sufficient lead time (after an initial ramp-up period) to use the offshore remanufacturing facility. In the multi-period case, we consider the fact that a mixed strategy, in which some demand is met using domestic production in the current period and some is met by offshore production in a previous period, might be preferable to both the pure domestic and pure offshore strategies. This type of strategy is depicted in Figure 6.
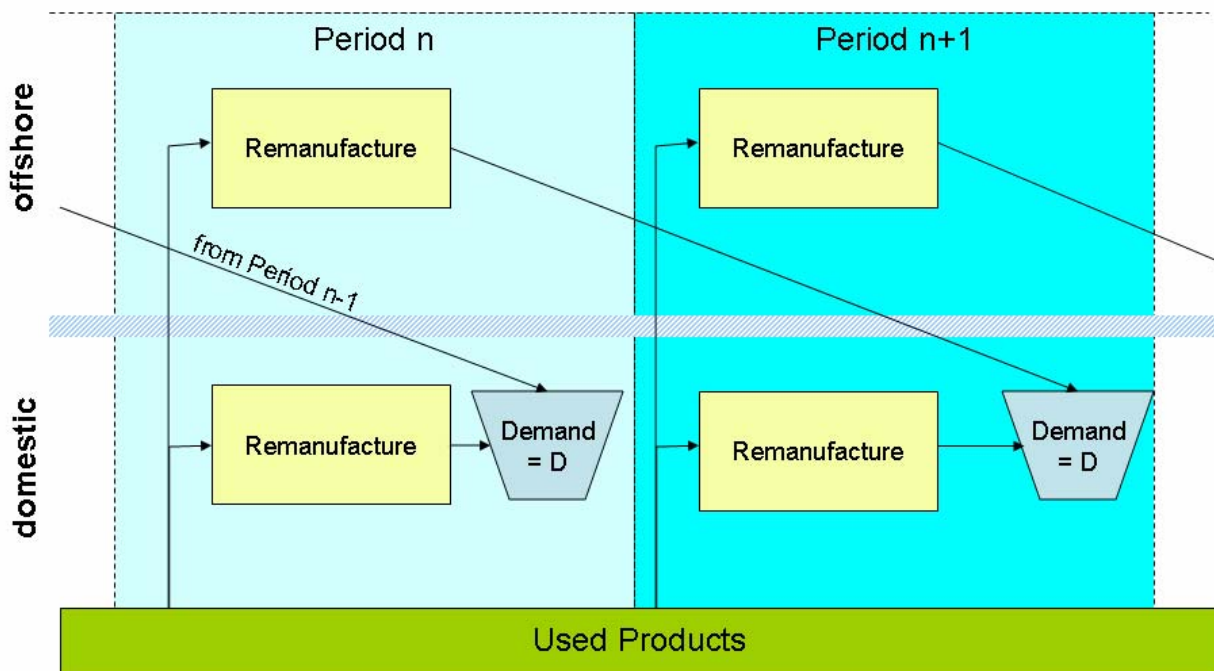
Figure 6: A mixed approach to meeting stable, multi-period demands for remanufactured items

To motivate this discussion, we introduce an example based on our experience with a large U.S.-based remanufacturing firm. We use this example to demonstrate how parametric changes can impact the optimal strategy for a remanufacturer – a detailed explanation of the dynamics of this decision is provided following the example. Consider a toner cartridge remanufacturer that must meet a set of stable, known demands. The firm has both domestic and offshore facilities available, and in steady state either facility can be used to meet any demand. In toner cartridge remanufacturing, the used items fall into one of two categories – virgins, which have not been previously remanufactured (low-touch), and non-virgins, which have already been remanufactured at least once and are in worse condition (high-touch). In this example, although the actual values are not used, the values we assume are chosen to approximate the tradeoffs faced by a toner remanufacturer. 40% of the used items acquired are virgin cartridges (α=0.4). The used items cost \$0.40 each, and it costs \$0.80 per low-touch item and \$4.00 per high-touch item to remanufacture domestically (u=0.4, c=0.8, λ=5). Shipping costs are \$0.20 per cartridge when the domestic facility is used and \$1.60 per cartridge when the offshore facility is used (s=0.2, θ=8). The offshore facility, with its much lower processing costs, can remanufacture a low-touch item for just \$0.07 and a high-touch item for \$0.33 (ρ=12). See Table 2 for a summary of the parameter values used in this example.

Table 2: Example Parameters

Notation	Example Value
c	0.8
u	0.4
s	0.2
ρ	12
θ	8
λ	5
α	0.4

Since (5) does not hold for the parameters in Table 2, that is, ($5 > \dfrac{0.4}{(0.4)(0.8)} + 1$), the

optimal domestic approach is to acquire extra items and remanufacture only low-touch

(scrapping the high-touch), for a total unit cost defined by $C_d\left(\dfrac{D}{\alpha}\right)$. The optimal offshore

approach, since (8) does hold ($5 < \dfrac{(12)(0.4)}{(0.4)(0.8)} + 1$), is to acquire D items and remanufacture all of

them, for a total unit cost of $C_o(D)$. If the remanufacturer only considers these two pure

strategies, he will have the following unit costs: $C_d\left(\dfrac{D}{\alpha}\right) = \$2.00$; $C_o(D) = \$2.23$ and use the

domestic facility, with its lower yield, to meet all demand.

However, the remanufacturer could also consider an approach that processes the low-

touch items domestically but does not scrap the high-touch ones. Since future demand is assured

in the multi-period model, each of these high-touch items has potential value. Incremental

analysis can be used to determine if remanufacturing one of these high-touch items offshore has

a lower cost than remanufacturing it domestically ($\dfrac{\lambda c}{\rho} + \theta s < \lambda c + s$) and a lower cost than

scrapping it and obtaining more low-touch items for domestic remanufacture

$(\frac{\lambda c}{\rho} + \theta s < \frac{u}{\alpha} + c + s$). If these inequalities hold, offshore remanufacturing is the lowest cost

alternative for one high-touch item. Since all costs are linear in this model, we can further state

that offshore remanufacturing is lowest-cost for **all** high-touch items when the inequalities hold,

and it makes sense to send all high-touch items offshore (since all can eventually be sold).

Therefore, we can say that a mixed strategy for the multi-period problem that remanufactures

low-touch items domestically and high-touch offshore will always remanufacture all used items,

so it **will always have a yield of 100%**. This is a key characteristic of the mixed strategy – in

the pure offshore and pure domestic cases, there are conditions under which the yield of the

optimal policy is less than 100%, but the mixed approach will always result in 100% yield. The

unit cost of the mixed strategy is as follows:

$$C_m(D) = u + \alpha(c+s) + (1-\alpha)\left(\frac{\lambda c}{\rho} + \theta s\right) \tag{12}$$

We can see that, in our example, $C_m(D) = \$1.96$. Since $C_m(D) < C_d\left(\frac{D}{\alpha}\right)$, the mixed strategy is

optimal. Note that if offshore processing became less attractive, the pure domestic approach

with its lower yield could become optimal, even over the mixed strategy. Consider the case in

which the proportion of low-touch items increases to 50% (through, for example, the

identification of a higher quality supplier). With fewer high-touch items, the high-touch

processing advantage offshore is less valuable, and an investigation of the costs indicates that

$C_m(D) = \$1.87$; $C_d\left(\frac{D}{\alpha}\right) = \$1.80$. Since we now have $C_d\left(\frac{D}{\alpha}\right) < C_m(D)$, a pure domestic

strategy that acquires extra used items and scraps the high-touch ones has a lower total cost than

the mixed strategy that sends those high-touch items offshore.

Parametric changes could also cause a shift from a mixed to a pure offshore strategy. For example, if θ were to drop to a value of 4 in our example (offshore shipping and handling drops to \$0.80 per unit), then we would have $C_m(D) = \$1.48$; $C_d\left(\dfrac{D}{\alpha}\right) = \$2.00$; $C_o(D) = \$1.43$, and the offshore costs would be low enough to justify shifting the remanufacturing of all items offshore.

We now formalize the dynamics of the choice between the pure domestic, pure offshore, and mixed strategies as described in the example above. First, we compare the mixed strategy to the pure offshore approach. Recall that, for low λ (when (8) holds), the optimal pure offshore strategy is to obtain D used items and remanufacture all of them at a cost of $C_o(D)$. It can be shown that $C_m(D) < C_o(D)$ when:

$$\theta > \frac{c\left(1 - \dfrac{1}{\rho}\right)}{s} + 1 \tag{13}$$

Note that (13) doesn't depend on λ, since the high-touch items are remanufactured offshore in either of the scenarios being compared. Importantly, it also does not depend on α (see Proposition 1 below).

In the other offshoring case, when λ is sufficiently high that (8) does not hold, acquiring D/α and remanufacturing only the low-touch items is the optimal pure offshore strategy. In these cases, the mixed strategy results in lower costs when:

$$\theta > \frac{\lambda c(1 - \alpha) - c(1 - \alpha\rho) + \rho u\left(1 - \dfrac{1}{\alpha}\right)}{\alpha s} + 1 \tag{14}$$

Condition (14) is a lower bound that strictly increases in λ – since the mixed strategy is the only one of the two being compared in this case that remanufactures high-touch items, it is less likely to be optimal as the remanufacturing requirement of those items increases.

In summary, the mixed strategy results in lower costs that the optimal pure offshore strategy in the shaded region of Figure 7:
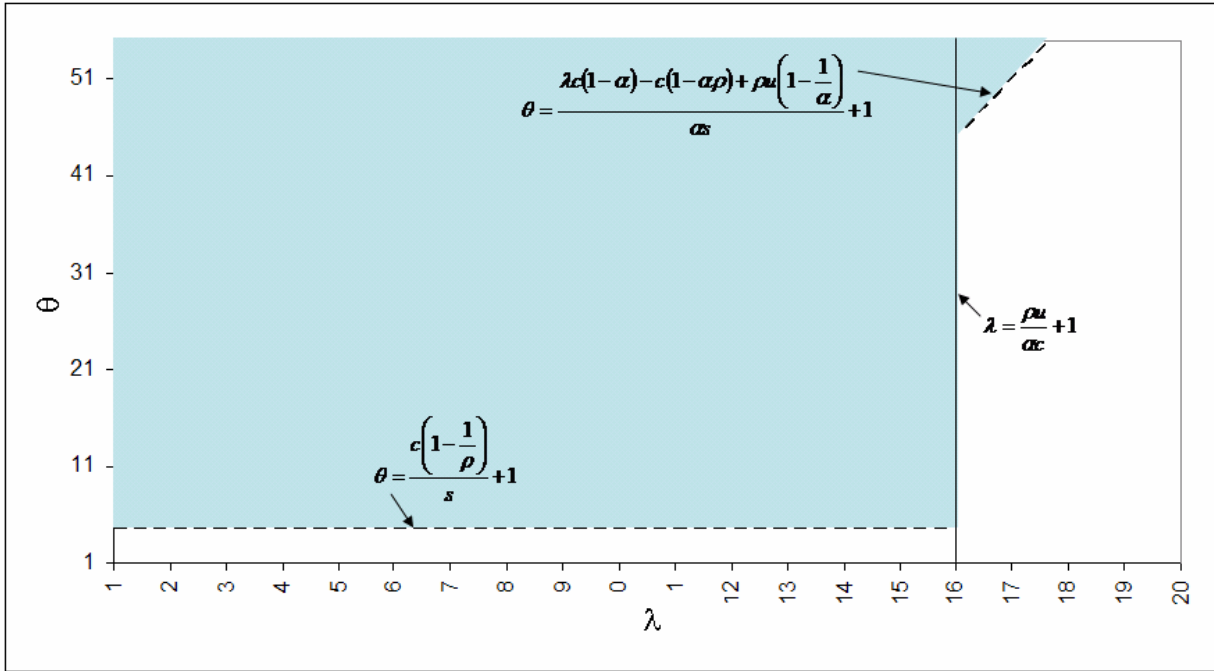

Figure 7: the shaded region is where a mixed strategy results in lower costs than the optimal pure offshore strategy in the multi-period problem

In Figure 7, we see that a mixed strategy is much more likely to be preferable to a pure offshore approach for values of λ below the threshold defined by (8) (i.e. $\lambda < \frac{\rho u}{\alpha c} + 1$).

We can perform the same type of analysis as above to compare the mixed strategy to the pure domestic approach. Recall that, for very low λ (when (5) holds), the optimal pure domestic strategy is to obtain D used items and remanufacture all of them at a cost of $C_d(D)$. It can be shown that $C_m(D) < C_d(D)$ when:

$$\theta < \frac{\lambda c \left(1 - \frac{1}{\rho}\right)}{s} + 1 \qquad (15)$$

Condition (15) is an upper bound that strictly increases in λ, as expected – since the mixed strategy sends high-touch items to the lower-cost offshore location, it is increasingly likely to have lower total costs as the remanufacturing requirement for high-touch items increases. Therefore, the restriction on θ for a mixed strategy to be preferable to a pure domestic approach becomes more slack as λ increases – even as offshore shipping costs become quite high (high θ), the mixed approach might still make sense when high-touch remanufacturing requirements are high (high λ). Note also that (15), like its analogous condition in the offshore case, (13), does not depend on α.

In the other domestic case, when λ is sufficiently high that (5) does not hold, acquiring D/α and remanufacturing only the low-touch items is the optimal pure domestic strategy. In these cases, the mixed strategy simply sends the high-touch items that would be scrapped in the pure domestic approach offshore, avoiding the necessity of acquiring extra units to meet demand. This results in lower costs when:

$$\theta < \frac{u}{\alpha s} + \frac{c\left(1-\dfrac{\lambda}{\rho}\right)}{s} + 1 \tag{16}$$

Condition (16) is an upper bound that strictly decreases in λ, as expected – since the domestic strategy in this case does not remanufacture high-touch items, an increase in the processing cost of those items (λ) only hurts the mixed strategy, resulting in a more stringent bound on the offshore shipping costs (θ) for the mixed approach to be preferable.

In summary, the mixed strategy results in lower costs than the optimal pure domestic strategy in the shaded region of Figure 8:
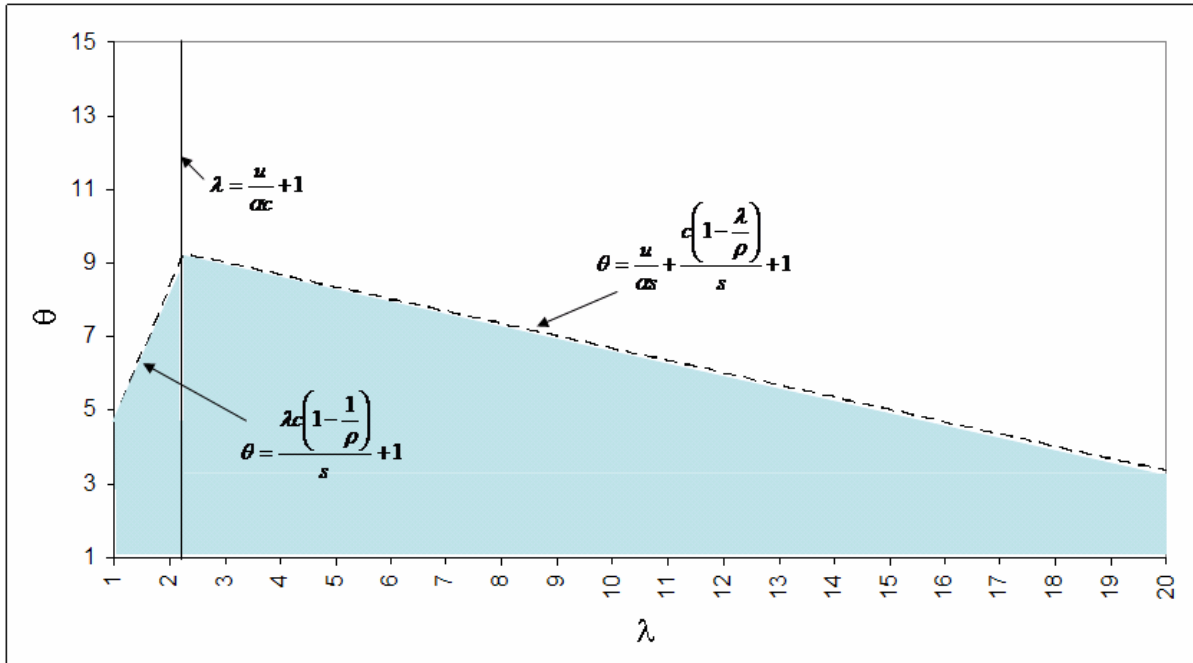
Figure 8: the shaded region is where a mixed strategy results in lower costs than the optimal pure domestic strategy in the multi-period problem

In Figure 8, we see that the mixed strategy is preferable to the optimal pure domestic strategy when the offshore shipping costs (θ) are sufficiently low. In other words, if the shipping cost of offshoring is not too high, then it lowers total costs to use the mixed strategy of remanufacturing high-touch items offshore and low-touch domestically as opposed to a pure domestic strategy.

We can combine Figures 5, 7, and 8 to define three regions in the multi-period problem, (as shown in Figure 9):

- Pure domestic is optimal (never remanufacture offshore)

- Pure offshore is optimal (never remanufacture domestically)

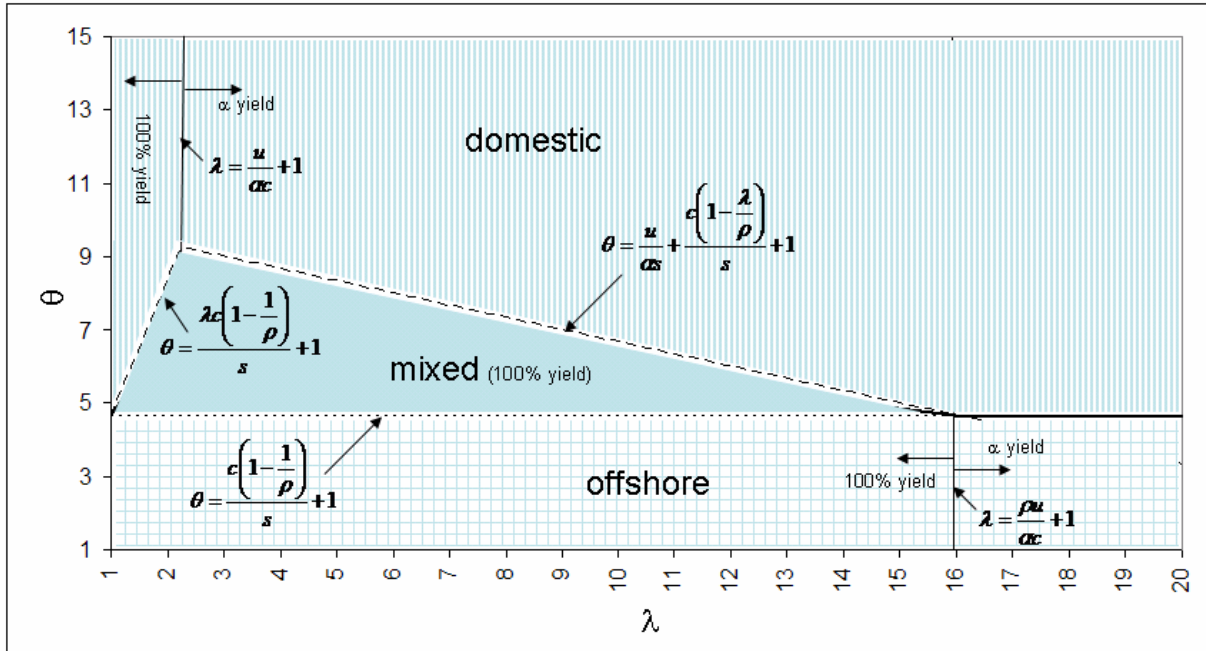- A mixed domestic/offshore strategy is optimal

Figure 9: Optimal strategies for the multi-period problem

Using Figure 9, we can make several observations about the multi-period problem.

OBSERVATION 1: When $\theta < \dfrac{c\left(1-\dfrac{1}{\rho}\right)}{s}+1$ (i.e., (11) holds), a pure offshore strategy is always

optimal; otherwise, the domestic facility will always have a role in the optimal strategy.

Observation 1 presents an interesting insight – in the decision of whether or not to pursue a

purely offshore strategy, the values of α and λ, i.e. the proportion of low-touch items and the

remanufacturing cost of high-touch items, are irrelevant. If the inequality presented above holds,

then all remanufacturing will always be offshore. In these cases, λ and α only impact the yield

achieved offshore – specifically, when $\lambda < \dfrac{\rho u}{\alpha c}+1$ the offshore yield will be 100%, otherwise it

will be α. Conversely, if $\theta > \dfrac{c\left(1 - \dfrac{1}{\rho}\right)}{s} + 1$, the domestic facility will *always* have a role in the

optimal approach to the multi-period problem (either through a pure domestic or a mixed

strategy). As opposed to only being used to meet initial demand that is not known with sufficient

lead time to go offshore, we can say that the domestic facility plays an ongoing role in the cost-

minimizing approach to meet demands over multiple periods whenever $\theta > \dfrac{c\left(1 - \dfrac{1}{\rho}\right)}{s} + 1$.

OBSERVATION 2: If the use of the domestic facility in the second period of the 2-period problem

decreases costs (i.e., we are in the unshaded region of Figure 5), then it will always have an

ongoing role in the optimal approach to the multi-period problem.

We can see that the region of Figure 8 that includes at least some domestic remanufacturing,

where $\theta > \dfrac{c\left(1 - \dfrac{1}{\rho}\right)}{s} + 1$, completely encompasses the unshaded region of Figure 5. Thus, we can

say that, if conditions are such that the domestic facility might be used in the optimal approach to

meet demand in the second period of the two-period problem, then that facility will *always* have

a role in steady state in the multi-period problem.

OBSERVATION 3: A mixed strategy, using both domestic and offshore facilities, is optimal when

the offshore shipping costs, as defined by θ, are within a defined range (i.e., neither very low nor

very high).

Observation 3 indicates the existence of a middle ground, where offshore shipping costs are not

so low that a pure offshore strategy becomes optimal (as in Observation 1), but are also not so

high that a pure domestic approach is preferred. Specifically, we can define this region using

two conditions. First, we have a lower bound on θ, as given by the case where (11) does not hold:

Condition 1: $\theta > \dfrac{c\left(1-\dfrac{1}{\rho}\right)}{s}+1$

When the above condition holds, we know that offshore shipping costs are sufficiently high, given other parameter values, to eliminate a pure offshore strategy from consideration.

Secondly, we need to ensure that a pure domestic approach doesn't result in lower costs than a mixed approach. Recall that, in the pure domestic case, we have two possibilities:

Scenario A: When $\lambda < \dfrac{u}{\alpha c}+1$ (i.e., (8) holds), the optimal pure domestic approach to

remanufacture all used items (100% yield). In this case, the upper bound on θ is given by (15):

Condition 2A: $\theta < \dfrac{\lambda c\left(1-\dfrac{1}{\rho}\right)}{s}+1$

Scenario B: When $\lambda > \dfrac{u}{\alpha c}+1$, the optimal pure domestic approach to remanufacture only low-

touch items. In this case, the upper bound on θ is given by (16):

Condition 2B: $\theta < \dfrac{u}{\alpha s}+\dfrac{c\left(1-\dfrac{\lambda}{\rho}\right)}{s}+1$

Condition 1 above, along with the Condition 2A or 2B as appropriate for the value of λ, defines the region for which the mixed strategy is optimal in the multi-period problem. It is important to note that, if Condition 1 is met but the appropriate Condition 2 is not met, then a pure domestic approach is optimal and no offshoring is used.

We now consider the fact that, as a product matures, the fraction of used items in the low-touch category could decrease. In the toner cartridge remanufacturing industry, this is seen in the diminishing proportion of virgin cartridges that can be acquired as time passes from the initial introduction of the cartridge. A consideration of the impact of the product life cycle on the condition used items leads to the following propositions:

PROPOSITION 1: If a mixed strategy is optimal for a given set of parameters, then it will always remain optimal as α decreases over time.

Proof: see Appendix A.

Proposition 1 is especially important for remanufacturers of products, such as toner cartridges, for which the proportion of low-touch used items can be expected to decrease over time. It states that, assuming the other problem parameters do not change, a remanufacturer whose optimal strategy is a mixed approach will always find it optimal to use this approach as the product matures. Since α is often the parameter most likely to change over a product's lifecycle, this result has significant implications for the long-term viability of a mixed domestic/offshore approach to meet demands for remanufactured products. If a remanufacturer is in the "mixed" region of Figure 8 (realizing 100% yield), he will remain in that region (and always realize 100% yields) as α decreases over time.

PROPOSITION 2: If a strategy that remanufactures only low-touch items is optimal for a given set of parameters, then there is some sufficiently small α for which the optimal strategy will shift to one with a yield of 100%.

Proof: see Appendix B.

Proposition 2 is fairly intuitive: as the percentage of low-touch items decreases, it becomes less likely that a policy that only remanufactures low-touch items is optimal. For example, if α drops

to 0.01, then a low-touch only approach would require an acquisition quantity of 100 times

demand, and 99 out of every 100 items acquired would be scrapped. The proof of Proposition 2

contains the exact levels of α for which this approach becomes suboptimal.

Lastly, we examine the cost-minimizing policy in terms of the yield in which it will

result. As noted, the mixed strategy always results in a yield of 100%. The yield of the optimal

pure domestic approach is 100% when (5) holds, and the yield of the optimal pure offshore

approach is 100% when (8) holds. Figure 10 simplifies Figure 9 to summarize the yield of the

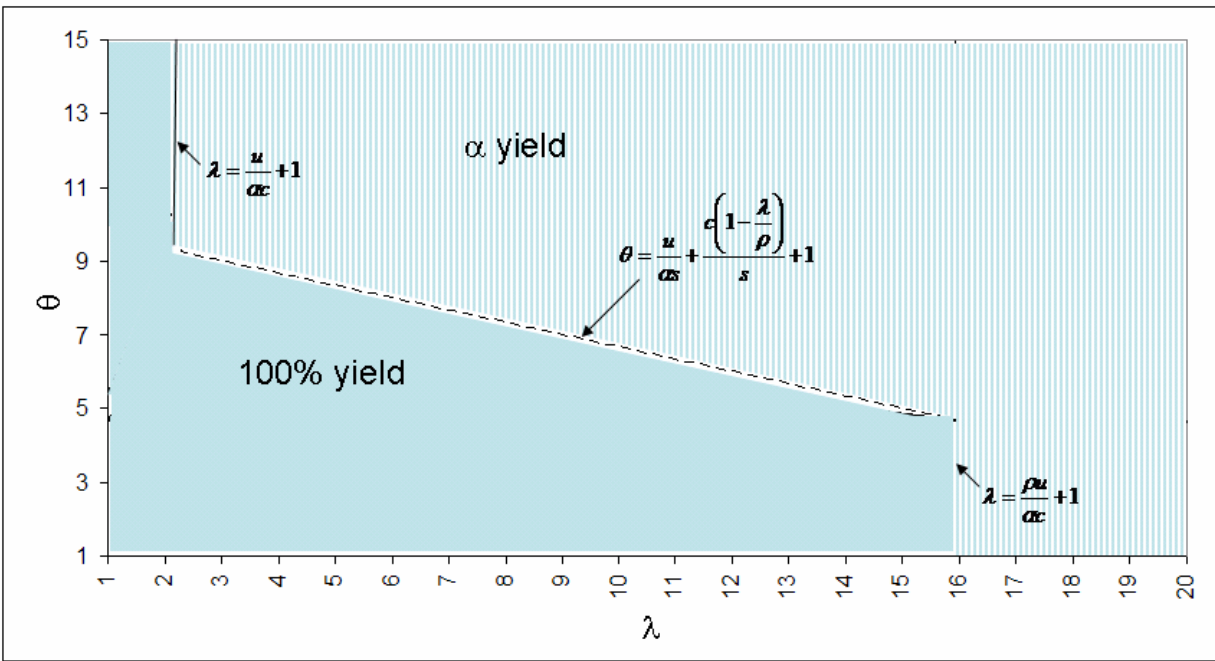optimal solution to the multi-period problem.

Figure 10: The remanufacturing yield resulting from the optimal multi-period strategy

From Figure 10, we see that for very small λ the yield of the optimal solution will always

be 100%, because high-touch items are not much more expensive to remanufacture than low-

touch in these cases. At the other extreme, for very large λ, the remanufacturing cost of high

touch items is so high that they will never be remanufactured, and yields will therefore always be α. Between these extremes, the optimal strategy still provides 100% yield when

$$\theta < \frac{u}{\alpha s} + \frac{c\left(1 - \frac{\lambda}{\rho}\right)}{s} + 1$$ (i.e. (16) holds). When θ is above this bound, offshore shipping costs are

so high that it is suboptimal to send anything offshore, and therefore a pure domestic strategy with a yield of α is optimal. Therefore, the three expressions in Figure 9, which represent conditions (5), (8), and (16), fully define the yield that can be expected from following the optimal policy for a multi-period remanufacturing problem.

4. Conclusions

This paper has extended the knowledge of the impact of used product condition variability on remanufacturing operations by considering its effect on the optimal use of offshore facilities. We have identified conditions under which offshoring will be part of the cost-minimizing approach to meet demands in both a two-period model (reflecting products with short life cycles) and a multi-period model (for items with longer life cycles). Our results indicate that, if the domestic facility has the potential to decrease costs to meet the second demand of the two-period model, then it will always have an ongoing role in the multi-period case.

We also present several insights specific to the multi-period problem. First, we note that, in the decision to pursue a pure offshore strategy in steady state (i.e. only use a domestic facility during the initial ramp-up), neither the proportion of low-touch items (α) nor the remanufacturing cost of high-touch items (λ) is relevant. Of course, these parameters impact the yield under the optimal strategy, but they do not influence the offshoring decision. We also identify conditions under which a mixed strategy of remanufacturing all low-touch items

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domestically and all high-touch items offshore is the optimal approach in the multi-period case. Importantly, we are able to show that, as the product matures and α declines, the mixed strategy will remain optimal as long as the other problem parameters are stable.

In this paper we have examined a simplified, deterministic model of the offshore remanufacturing decision in order to focus and clarify the tradeoffs involved. In so doing, we have assumed that demand for remanufactured product is known and constant over time, and that the yield is deterministic. We see several possibilities for future research in this area. Although the deterministic yield assumption used in this model has been shown to be quite reasonable in previous research, it could be relaxed. In the multi-period case, uncertain yield creates the potential for fewer low-touch items in a batch than expected. If a mixed strategy is used and such a shortfall occurs, immediate demand that was to be filled by remanufacturing low-touch items domestically could be filled in two ways – from a safety stock of finished goods or from remanufacturing some high-touch items at the domestic location. Each of these alternatives carries a cost, and an analysis of which approach is preferable to address yield uncertainty – safety stock or flexible sorting – would be an interesting study and would build on our previous work addressing the impact of yield uncertainty in remanufacturing. In addition, the case of known but *varying* demands in a multi-period setting with variable condition and two remanufacturing facilities would be an interesting lot-sizing extension. Finally, a simulation study could be used to more accurately reflect the true costs of an international supply chain, for example by including random customs delays and exchange rate fluctuations.

Appendix A

We consider the effect of a decrease in α on the advantage of a mixed strategy over each of the four pure strategies (domestic low-touch only, domestic 100% yield, offshore low-touch only, and offshore 100% yield). First, we note that the conditions for the mixed approach to have a lower cost than either 100% yield approach (as given by (13) and (15)) do not depend on α, and therefore are not impacted by a decrease in α. It remains to be shown that, if (14) and (16) hold for a given α, then they hold for all smaller α. Note that the derivative of the lower bound in (14) with respect to α is $\dfrac{2\rho u}{s\alpha^3}$, which is always positive. Therefore, as α decreases, this lower bound also decreases, and we can say that if (14) holds for a given α, it will hold for all smaller α. Finally, note that the derivative of the upper bound in (16) with respect to α is $\left(-\dfrac{u}{\alpha^2 s} \right)$, which is always negative. Therefore, as α decreases, this upper bound increases, and we can say that if (16) holds for a given α, it will hold for all smaller α. In summary, when the mixed strategy is cost-minimizing when compared to all pure strategies, it will always remain cost-minimizing as α decreases. $\square$

Appendix B

Note that a strategy that remanufactures only low-touch items will either do so domestically (the upper-right of Figure 9) of offshore (the lower-right of Figure 9). We consider each of these situations in turn below.

First, consider the case where the optimal strategy is to use the <u>domestic</u> facility to remanufacture only low-touch items (the upper-right of Figure 9). Note that, as α decreases, the upper bound on λ defining the region in which a 100% yield domestic approach is optimal (5) strictly increases. At the same time, the upper bound on θ defining the region in which the mixed strategy (with 100% yield) is optimal (16) also increases, as shown in the proof of Proposition 1. Since these upper bounds increase as α decreases, there is sufficiently small α such that one or both will hold, resulting in a 100% yield strategy being optimal. Therefore, in this case, we can say that the optimal approach will always shift to a 100% yield approach if α drops to a level such that either (5) holds ($\alpha < \dfrac{u}{\lambda c - c}$) or (16) holds ($\alpha < \dfrac{u}{s\theta - c\left(1 - \dfrac{\lambda}{\rho}\right)}$).

Next, consider the other case, where the optimal strategy is to use the <u>offshore</u> facility to remanufacture only low-touch items (the lower-right of Figure 9). Note that, as α decreases, the upper bound on θ defining the region in which a 100% yield offshore is optimal (8) strictly increases. Since this upper bound increases as α decreases, there is sufficiently small α such that it will hold, resulting in a 100% yield strategy being optimal. Therefore, in this case, the optimal approach will always shift to a 100% yield approach if α drops to a level such that (8) holds ($\alpha < \dfrac{\rho u}{\lambda c - c}$). □

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