CHAPTER III

A FRAMEWORK AND ALGORITHM FOR DYNAMIC USER CLASS EQUILIBRIUM MODELS

3.1 Overview

In this chapter, a new type of equilibrium known as the Dynamic User Class (DUC) equilibrium is proposed and compared with the traditional Multiple User Class equilibrium. In the traditional Multiple User Class equilibrium, vehicles are classified into various classes based on behavioral rules, information classes etc. For example, the class of users following the least trip-time paths from their origin to their destinations belong to user equilibrium class. Exogenously classifying the users based on pre-specified rules may not be adequate in modeling certain conditions where the user may belong to different classes at different points in time depending on the network dynamics. The 'static' Multiple User Class model in which the user class is pre-specified cannot be used to model this situation. Therefore, the main motivation in this chapter is to develop a Dynamic User Class (DUC) model in which the user classes are dynamic and dependent on the network dynamics.

This chapter presents a Dynamic User Class equilibrium framework by generalizing the 'static' Multiple User Class equilibrium framework described in Chapter 2. In particular, the generalization permits users of a given user class to change routes in accordance with the behavioral rules of the class, in response to network dynamics. More importantly it allows users to switch behavioral rules and/or classes also dynamically in response to network dynamics, thus increasing the realism in modeling network dynamics under information provision..

The Dynamic User Class equilibrium framework can also be used in modeling a richer range of behavioral rules. The non- linear interaction between user classes can be modeled providing more global/system improvement/control opportunities. The percentage of users belonging to each behavioral class and their variation with network dynamics can be studied. The 'static' Multiple User Class framework is a special case of Dynamic User Class with restrictive assumptions.

The Dynamic User Class framework can be used to address a variety of ITS related applications/questions. This framework can be used to study the compliance rate to various types of information/information sources as a function of network dynamics. The framework is useful in determining the percentage of users passing through the DMS and complying with the information provided as a function of the DMS location. The DUC framework can also be used to model coordination across multiple pre-trip/en-route information. The framework aids in developing information strategies which can exploit dynamics across classes to improve system performance. For example, through this framework, it would be possible to provide en-route System Optimal information in incident affected zones of a network to evacuate the area as fast as possible while using any other predictive strategies for other pre-trip/en-route information systems.

This chapter introduces the concept of Dynamic User Class equilibrium. The necessity of DUC equilibrium and the advantages of the DUC over the MUC are explained in this chapter. The DUC problem is formulated and an algorithm is provided to determine the DUC equilibrium. One of the applications of DUC equilibrium in terms of information strategies for Advanced Traveler Information Systems (ATIS) is also discussed. Examples of DUC model and its advantages and disadvantages to other frameworks available for modeling information systems in traffic are discussed.

3.2 Background

In a transportation network, users can be differentiated into various classes depending on the type of information available to them, behavioral rules or vehicle types. For example, certain users may not have access to pre-trip information but may receive en-route information and these users can be said to constitute the en-route information class. Other users may have access to pre-trip information at the start of the journey but may not receive en-route information and thus belong to pre-trip information class. Similarly users can be grouped into various classes based on behavioral rules. Peeta et. al., 1995 has proposed the following classes based on behavioral rules: (i) System Optimal(SO) Class consisting of equipped drivers who follow prescribed System Optimal(SO) routes, (ii) UE class consisting of equipped drivers who follow prescribed User Equilibrium(UE) routes, (iii) equipped drivers who follow boundedly rational switching rule in response to descriptive information on prevailing conditions and (iv) non equipped drivers who follow pre-specified paths which are independent on network dynamics. .

According to the Wardrop's principle (1955) – under user equilibrium conditions for a single user class, all paths connecting an origin destination pair will have equal and minimal travel times. Therefore, the travel time experienced by a user on a used path will be less than the travel time experienced by a user on an unused path. The extension of the above principle to multiple user classes was proposed by Bin-Ran (1996), Nagurney(2000) and Peeta(1995). Bin-Ran (1996) classified the users based on characteristics like: (i) route diversion willingness, (ii) income and age, (iii) driving behavior and (iv) behavioral rules. Considering users from the first three categories above, for every path Bin-Ran associated a travel disutility based on path travel time, fuel consumption and operating cost for every class. At equilibrium, the travel disutilities on all used paths are assumed to be equal and minimal for each O-D pair and departure time interval combination. When the users are classified based on behavioral rules, the equilibrium conditions associated with each of the behavioral classes also have to be satisfied simultaneously. For example, if the two user classes considered are UE and SO, then at Multiple User Class equilibrium, the convergence criteria for the user equilibrium and system optimal vehicles have to be satisfied. In other words, UE users traveling between a given O-D pair and departure time interval will select paths such that the travel times on used paths are equal and minimal. The SO users on the other hand, choose paths (for a given O-D pair and departure time interval) such that the marginal travel times on all paths used by this class are equal and minimal. Note that for the multiple user class equilibrium to hold, both conditions must be satisfied at the same time.

A common feature of the above models is the assumption that users belong to pre-specified user classes and cannot shift from one class to another. In this sense, the above models correspond to static user classes, and are referred to as ‘static’ multiple user class models hereafter, although, they are dynamic in representing route choice within a given user class. In contrast, in a dynamic user class model the assumption of static user classes is relaxed, i. e., a user can belong to different user classes at different points in time. Consequently, the equilibrium conditions for the dynamic user class context are also different. The within-class equilibrium conditions noted above for the ‘static’ multiple user class model also holds in the DUC case. In addition, under the DUC equilibrium, the following additional set of conditions must also hold between classes. As per these conditions, at equilibrium, no user has any incentive to shift between user classes unilaterally. For instance, dynamic user class model can allow the user to receive pre-trip time-dependent user equilibrium information and en-route he can receive time-dependent system optimal or prevailing information. At equilibrium when there are dynamic user classes, there is no incentive for a user to shift from one user class to other user classes.

The need for dynamic user classes in the context of Intelligent Transportation systems is illustrated by the following three examples:

Example 1: ATIS information strategy varies over time

Consider an intelligent transportation network where some users may receive pre-trip information, others may receive only en-route information, and still others receive both pre-trip and en-route information. Suppose the information provider/controller has the ability and flexibility to choose from among several alternative information strategies at any point in time depending on the traffic conditions and network dynamics (e.g. user equilibrium route guidance, system optimal route guidance etc.). In this traffic environment, for a user receiving information from both sources, it is possible that he may receive UE pre-trip information, whereas, en-route he may be provided SO information. The type of information received and the paths selected will vary dynamically in response to information strategy of the controller, type of pre-trip information, and network conditions encountered. Therefore, in this context, due to the potential switching of information strategies by the controller, it is not possible to statically pre-specify the class to which a given user belongs. For the same reason, the number of users belonging to each class cannot be known a priori. This example highlights the limitation of the static multiple user class model and illustrates the need for a more flexible dynamic user class model.

Example 2: Different information sources using the same information strategy

Consider, once again, a network with two sources of information: pre-trip and en-route information. In contrast to example 1, assume that the controllers do not change the information dynamically. For example, it may be assumed that both sources provide information based on UE route guidance at all times. The pre-trip information is provided to all users, and en-route information is provided through Dynamic Message Signs (DMS). It is generally likely that DMS are located at certain fixed locations on the network. Therefore, only those users that pass through the DMS locations will access the information en-route. Note that the drivers who access DMS information (DMS routes), in turn, depends on the routes provided by pre-trip information. For example, even for a given O-D pair, a user who departs earlier may have been provided a route that passes through a DMS location, whereas, another user traveling the same O-D pair may be provided a pre-trip path that does not pass through a DMS location, depending on the network conditions. Thus, the number of users receiving en-route information cannot be pre-specified a priori, since it depends on how the system performance changes over time. Furthermore, the DMS information can affect the congestion in the system and link travel times by diverting vehicles to alternative paths. Thus DMS, in turn, will influence the pre-trip paths through the changes it induces in network dynamics. Therefore, it is not possible to specify a priori whether a given vehicle will receive en-route information or not. As a result, the static MUC model, where user classes are pre-specified a priori, does not provide a realistic representation in this context, despite the fact that the information strategy is identical for the two sources, and does not vary over time.

Example 3: Time varying compliance with DMS information

Consider the same network as in example two, with pre-trip and en-route information. In the previous examples, it was assumed that compliance with information is perfect. Instead, consider the (more realistic) case that users do not perfectly comply with en-route information. The decision to access and comply with information itself may depend on the congestion encountered until the en-route information location. If the congestion encountered from the origin to the DMS location is unusually high, then a user may decide to access and comply with DMS information to optimize the rest of his journey. In this case also, the class to which a user belongs (especially whether they access DMS information) cannot be pre-specified fully. Therefore, the static MUC models, where compliance rates are obtained exogenously and are static over time, are not applicable.

The examples above highlight two key differences between static and dynamic multiple user class models. In the dynamic MUC models, the proportion of drivers belonging to different user classes cannot be prespecified since this proportion depends on system dynamics. Another difference relates to the nature of equilibrium conditions. The static MUC equilibrium is a path-based equilibrium i.e. at convergence users do not have any incentive to shift from one path to another within each user class. However, the Dynamic User Class equilibrium is both a path-based equilibrium and a class-based equilibrium. Dynamic User Class equilibrium is a path-based equilibrium in the sense that at convergence there is no incentive for a user to shift from one path to another that corresponds to his/her behavioral user class. It is also a class-based equilibrium in that there is no incentive for the user to shift from one user class to another. These properties are exploited in developing an algorithm to solve for the DUC equilibrium conditions in the next section.

3.3 Dynamic User Class Model

As explained in the previous section, drivers in a traffic network can be classified into various user classes based on behavioral rules and/or information available to them. In this section, an illustrative DUC model and associated DUC condition are presented for a network in which there are two information sources: pre-trip information source and an en-route information source. The en-route information source is assumed to be located at a fixed point in the network and provides information from that point to all the destinations in the network. This assumption is made for ease of explanation and is relaxed in the next chapter. The pre-trip information source is assumed to provide information from each origin to every destination in the network. In this network, the users are classified based on information accessibility into two user classes (class 1 and class 2). Users belonging to user class 1 receive information from the pre-trip information source alone, but no en-route information. Users belonging to user class 2 receive information from both sources – pre-trip and en-route information. Note that the event that user accesses en-route information depends on whether the initial route assigned to him by the pre-trip source passes through the en-route device location. It is assumed that all users are perfectly compliant with pre-trip information. This assumption is not essential, and is made to simplify the presentation. Under this assumption, if the pre-trip route passes through the en-route information location, it is assumed that the user receives information from the en-route information source. Hence the user belongs to class 2. If the pre-trip route does not pass through the en-route location, then the user only has access to pre-trip information.

3.3.1 Dynamic User Class Equilibrium

According to the Wardrop's principle, at user equilibrium – all paths connecting a given origin-destination pair have equal and minimal travel times. Hence the travel times on a used path will be smaller than the travel time on any unused path. Therefore, at equilibrium, no user has an incentive to switch paths unilaterally.

In the DUC equilibrium, there are two equilibrium conditions. A dynamic user equilibrium condition exists both at the network level (pre-trip) and at the local level (en-route) level. The dynamic user equilibrium for the pre-trip level assigns for a given O-D pair to paths such that the travel time on all used paths between the origin to the destination are equal and minimal. Similarly, the dynamic user equilibrium for the DMS assigns vehicles on paths (from the DMS location to a given destination) which are equal and minimal. For the problem under consideration there are two user classes – user class 1 and user class 2, as explained earlier. In the dynamic user class model, three important dynamic interactions (within and between classes) must be explicitly considered:

(i) Users belonging to class 1 can select from paths (between their origin to their destination) based on pre-trip information i.e class 1 users redistribute themselves along various class 1 paths.

(ii) In addition, users belonging to class 2 receiving information from both pre-trip and en-route sources) can choose from alternative paths from the DMS location to their destination in response to network conditions and information.

(iii) Users can switch between classes across iterations (i.e. users that received pre-trip only information may improve their travel time by switching to class 2 and vice-versa).

Note that there exist strong interactions between the two user classes on the network. The en-route assignment affects the travel time on the downstream links which in turn affects the pre-trip assignment. Hence, in the Dynamic User Class (DUC) model the pre-trip flow assignment is dependent on the en-route assignment. The pre-trip paths provided affect the time dependent loading of vehicles on the en-route node. Depending on the arrival rate of vehicles on the en-route information node, the path provided by the en-route information node can change. Thus the pre-trip information affects and is affected by the en-route DMS information.

Due to the dynamic interactions within and between classes noted above, the dynamic user class equilibrium conditions can be stated as follows for a given O-D pair r-s:

(i) The travel time between all used paths between r-s are equal and minimal (class one equilibrium conditions, if none of the used paths pass through the en-route location e).

(ii) Suppose a used path (for this O-D pair) passes through the en-route location e, then for class 2 users that reach the en-route location at a given time, the travel times on all used paths from e to s should be equal and minimal.

(iii) At equilibrium, there is no incentive for a user to switch classes. In other words, a user who receives only pre-trip information (whose path does not go through e) has no incentive to switch to a path passing through the en-route location e. Similarly, there is no incentive for a user in class 2 (whose path passes through e) to switch to a path that avoids the en-route location.

Suppose for a given O-D pair s-t, none of the used paths pass through the en-route information location, then only the class 1 equilibrium conditions need to be satisfied for this O-D pair. The within-class equilibrium condition for user class two requires that both conditions (i) and (ii) above need to be satisfied. In practical terms, this means that for a user that receives information from both sources, the travel times on used paths from the origin to the destination for a given departure time should be equal and minimal. In addition, the travel time on the used sub-path from the DMS location to the destination should also be equal and minimal (from e to the destination). The latter condition is not imposed in a static MUC model. The interactions between the two classes arises because of their contribution to the total link volumes, queues, and travel times on all network links, which in turn are a function of paths selected by the users from the two classes. Note that users from both classes may be present on a given link even for the same O-D pair.

Hence at dynamic user class equilibrium, there is no incentive for a user to shift from user class 1 to user class 2 and vice-versa. i.e. , at equilibrium there is no incentive for a user to shift from the class receiving pre-trip information only to a class receiving both pre-trip and en-route information and vice versa. In other words, there is no incentive for a user to shift from a route which does not pass through the DMS to a route which passes through the DMS. Similarly, there is no incentive for a user to shift from a route which does not pass through the DMS to a route which passes through the DMS.

Equilibrium conditions are of interest because they provide a benchmark for comparing performance of ITS in real world applications. In solving for the DUC, at every iteration users are shifted from one user class to another until at convergence where equilibrium conditions (i),

(ii) and (iii) mentioned earlier in the section are satisfied. Therefore, the number of users belonging to each class can be obtained only at convergence where equilibrium conditions hold and cannot be pre-specified.The DUC model is formulated and algorithms for solving the model are provided in the resulting sections.

3.3.2 Formulation

In this section the dynamic user class equilibrium conditions are formulated. The network flow constraints involving demand supply balance at nodes and on links over time, dynamic constraints consisting of the link path incidence relationship, the directional and non-negativity constraints are also presented..

3.3.2.1 Notations

R	the set of all origins in the network
S	the set of all destinations in the network.
r	an origin in the network, $r \in R$
s	a destination in the network, $s \in S$
e	node on which the en-route information source is located
P	set of all paths in the network
P_1	the set of all paths in the network which do not pass through the en-route information source.
P_2	the set of all paths in the network which pass through the en-route information source.
P_3	the set of all paths connecting node e to the destinations.
p	a path in the network, $p \in P$
T	the time until which the analysis is conducted.
L	the number of intervals to which the time period [0,T] is discretized.
l	a time interval in the period of interest l=1,2,3….L.
M	the set of all departure time intervals
m	an index for a particular departure time interval m=1,2,3,…M.
$\eta_p^{rs}(t)$	actual travel time experienced from origin r to destination s at time t.
$\pi^{rs}(t)$	minimum travel time experienced from origin r to destination s at time t.
$f_p^{rs}(t)$	flow of vehicles from origin r to destination s through route p at time t
n	a node in the network
A(n)	the set of links that terminate at node n.
B(n)	the set of links that emanate at node n.
$x^a(l)$	number of vehicles on link a at time l
$d^a(l)$	number of vehicles which entered link a at time l
$e^a(l)$	number of vehicles which exited link a at time l
$I_n(l)$	number of vehicles which was generated at node n at time l
$O_n(l)$	number of vehicles which leave the network through node n at time l.

δ_{rsp}^{lma} time-dependent link path indicator, equal to 1 if vehicles going from origin r to destination **s** on path p departing at time m are on link a in time period l.

γ_{sp}^{lma} time-dependent link path indicator, equal to 1 if vehicles going from en route information source e to destination **s** on path p departing at time m are on link a in time period l.

Δ simulation interval.

3.3.2a Dynamic User Class Equilibrium Conditions

As mentioned in the previous section, the dynamic user class equilibrium conditions involve separate within class equilibrium conditions for class 1 users and class 2 users.

Class 1 conditions:

For all vehicles which do not pass through the en-route information source, for every origin destination pair, the travel time on all used paths must be equal and minimal. The route time based DUO conditions at the network level can be defined as

$$\eta_p^{rs*}(t) - \pi_p^{rs*}(t) \geq 0 \qquad \forall p \in P_1, r, s \qquad (3.1a)$$

$$f_p^{rs*}(t)[\eta_p^{rs*}(t) - \pi^{rs*}(t)] = 0 \qquad \forall p \in P_1, r, s \qquad (3.1b)$$

$$f_p^{rs*}(t) \geq 0 \qquad \forall p \in P_1, r, s \qquad (3.1c)$$

Hence if $f_p^{rs*}(t) \geq 0$ then $\eta_p^{rs*}(t) = \pi^{rs*}(t)$

And if $f_p^{rs*}(t) = 0$ then $\eta_p^{rs*}(t) \geq \pi^{rs*}(t)$

The above two equations imply that if the flow on a path is positive then the travel time on that link is equal to the minimum travel time. The travel time on all paths were flow is zero is found to be greater than the minimum travel time. Hence the travel time on all used paths is equal to the minimal travel time and the travel time on all unused paths is greater than the minimal travel time. The above equations can be written in the form of a variational inequality as

$$\int_o^T \sum_{rs} \sum_{p \in P_1} \eta_p^{rs*}(t)[f_p^{rs}(t) - f_p^{rs*}(t)]dt \geq 0 \qquad (3.2)$$

The equivalence of the variational inequality formulation and the dynamic user equilibrium condition can be made using similar arguments as for the static MUC case which is given by Bin-Ran (1996).

Class 2 conditions:

Similarly for class 2 users who receive en-route information the variational inequality formulation of the dynamic user equilibrium condition can be written as

$$\int_0^T \sum_s \sum_{p \in P_3} \eta_p^{es*}(t)[f_p^{es}(t) - f_p^{es*}(t)]dt \; + \int_o^T \sum_{rs} \sum_{p \in P_2} \eta_p^{rs*}(t)[f_p^{rs}(t) - f_p^{rs*}(t)]dt \geq 0 \quad (3.3)$$

In addition to this condition, class 2 users who receive pre-trip and en-route information, the within class equilibrium conditions also hold at the network level between the origin destination pairs. This condition takes the same form as those shown for class 1 above, except that the set P_1 is replaced by P_2.

Interclass interactions:

The above formulation captures two sources of interactions between user class 1 and 2. Note that for users belonging to class 2, the path traversed ($P_{r\text{-}s}$) is the union of their paths from their origin to the DMS location ($P_{r\text{-}e}$) and the path from the DMS location to the destinations ($P_{e\text{-}s}$) where:

(i) Travel time on path $P_{r\text{-}s}$ is the minimal travel time between the origin and destination.

(ii) Travel time on path $P_{e\text{-}s}$ is the minimal travel time between the DMS location and destination for that particular arrival time at the DMS location.

Therefore, for a specific origin destination pair, the travel time experienced on all paths which passes through the en-route information source must be equal and minimal and equal to the travel time on all paths which do not pass through the en-route information source. This ensures that class 2 users do not have any incentive to unilaterally shift to paths which do not pass through the en-route information source and become class 1 users and vice-versa.

The second set of interactions comes from the travel time equation:

Travel time (link i) = f(volume of class1 on link j, volume of class 2 on link j)

The travel time on a link j is the function of the volume of class 1 vehicles on link j and the volume of class 2 vehicles on link j.

For dynamic user class equilibrium, the equilibrium conditions should be satisfied for both the pre-trip and en-route information source. Hence

$$\int_{o}^{T}\sum_{rs}\sum_{p\in(P_1\cup P_2)}\eta_p^{rs*}(t)[f_p^{rs}(t)-f_p^{rs*}(t)]dt+\int_{0}^{T}\sum_{s}\sum_{p\in P_3}\eta_p^{es*}(t)[f_p^{es}(t)-f_p^{es*}(t)]dt\geq 0 \qquad (3.4)$$

The above link flow based formulation is a continuous time Variational Inequality which can be converted into a discrete time variational inequality by discretizing the time period [0,T] into L small intervals such that

$$\eta_p^{rs}(l)=t \quad \text{if } t\text{-}0.5\leq\eta_p^{rs}(l)< t\text{+}0.5 \qquad (3.5)$$

where $0\leq t\leq T$.

The discretized variational inequality can be written as

$$\sum_{m=1}^{M}\sum_{rs}\sum_{p\in(P_1\cup P_2)}\eta_p^{rs}(m)[f_p^{rs}(m)-f_p^{rs*}(m)]+\sum_{m=1}^{M}\sum_{s}\sum_{p\in P_3}\eta_p^{es}(m)[f_p^{es}(m)-f_p^{es*}(m)]\geq 0 \qquad (3.6a)$$

The above discrete formulation can be converted to the following optimization problem subject to the set of constraints described in Sections 3.3.2b – 3.3.2d

$$\text{Min } Z=\sum_{m=1}^{M}\sum_{rs}\sum_{p\in(P_1\cup P_2)}\int_{0}^{f_p^{rs}(m)}\eta_p^{rs}(m)dw+\sum_{m=1}^{M}\sum_{s}\sum_{p\in P_3}\int_{0}^{f_p^{es}(m)}\eta_p^{es}(m)dw \qquad (3.6b)$$

3.3.2b) Network Flow Constraints:

The demand between origin destination pair (r,s) must be equal to the sum of the flows on the various paths connecting (r,s)

$$f^{rs}(m)=\sum_{p}f_p^{rs}(m) \quad \text{where } p\in P^{rs},\forall m\in M,r,s \qquad (3.7a)$$

Note that the above flow constraint is a general flow constraint. For the two classes in consideration the above flow constraints can be extended as

$$f_1^{rs}(m)=\sum_{p}f_p^{rs}(m) \qquad \forall m\in M,r,s \qquad (3.7b)$$

where p represents the set of paths connecting (r,s) and does not pass through the node e.

For class 2

$$f_2^{rs}(m) = \sum_p f_p^{rs}(m) \qquad \forall m \in M, r, s \tag{3.7c}$$

where p represents the set of paths connecting (r,s) and passes through the node e.

Demand Supply Balance at nodes:

The number of vehicles entering through all links incident on node n at time l $d^a(l)$ is equal to the number of vehicles exiting the node n plus the net generation at this node.

$$\sum_a d^a(l) = \sum_b e^b(l) + I_n(l) - O_n(l) \;\; \forall l, n \;\; a \in A(n) \text{ and } b \in B(n) \tag{3.7d}$$

Demand Supply Balance on links over time:

The number of vehicles on link a at the beginning of time l $x^a(l)$ is equal to the number of vehicles on the link at time l-1 plus the number of vehicles which entered the link during the time interval l-1 minus the number of vehicles which left the link during the time interval l-1.

$$x^a(l) = x^a(l-1) + d^a(l-1) - e^a(l-1) \qquad \forall l, a \tag{3.7e}$$

3.3.2b) Dynamic Constraints: ***(Link Path incidence relationship)***

$$x^a(l) = \sum_m \sum_{rs} \sum_{p \in (P_1 \cup P_2)} f_p^{rs}(m).\delta_{rsp}^{lma} + \sum_m \sum_s \sum_{p \in P_3} f_p^{es}(m).\gamma_{sp}^{lma} \;\; \forall m, l, r, s, a \tag{3.8a}$$

where

δ_{rsp}^{lma} = 1, if $f_p^{rs}(m)$ is on arc a during period l,

= 0, if arc a does not belong to path $p \in (P_1 \cup P_2)$

= 0, if $m > l$,

= 0, if $f_p^{rs}(m)$ is not on arc a during period l

γ_{sp}^{lma} = 1, if $f_p^{es}(m)$ is on arc a during period l,

= 0, if arc a does not belong to path $p \in P_3$

= 0, if $m > l$,

= 0, if $f_p^{es}(m)$ is not on arc a during period l

$$\delta_{rsp}^{lma} = F(f_p^{rs}(m), f_{p'}^{es}(m)) \text{ where } \mathrm{p} \in (\mathrm{P}_1 \cup \mathrm{P}_2) \text{ and } \mathrm{p}^{|} \in \mathrm{P}_3 \qquad (3.8b)$$

$$\gamma_{sp}^{lma} = G(f_p^{rs}(m), f_{p^{|}}^{es}(m)) \text{ where } \mathrm{p} \in (\mathrm{P}_1 \cup \mathrm{P}_2) \text{ and } \mathrm{p}^{|} \in \mathrm{P}_3 \qquad (3.8c)$$

Note that in the above equations the time dependent link path incidence variables are a function of all the assignment decisions made at the pre-trip and en-route level. The functions F and G cannot be analytically modeled as they represent the complex dynamic traffic phenomena, link interactions, satisfaction of FIFO, queuing and other dynamic constraints. Therefore, the function is evaluated and represented using a simulation-based model.

$$\eta_p^{rs}(m) = \sum_l \sum_a \delta_{rsp}^{lma}.\Delta + \sum_l \sum_a \gamma_{sp}^{lma}.\Delta \qquad \forall m, l, r, s \qquad (3.8d)$$

where Δ is the simulation interval.

3.3.2d) Definitional and Non-Negativity constraints

$\mathrm{I}_\mathrm{n}(\mathrm{l})$ refers to the number of vehicles entering the network at node n and time l. Assuming that vehicles can enter the network at the origins only, we can say that

$$\mathrm{I}_\mathrm{n}(\mathrm{l}) = 0 \;\; \forall \mathrm{l}, \mathrm{n} \notin \{\mathrm{R}\} \qquad (3.9a)$$

If the node n is an origin node then

$$\mathrm{I}_\mathrm{n}(\mathrm{l}) = \sum_\mathrm{s} \mathrm{f}^{\mathrm{ns}}(\mathrm{l}) \quad \forall \mathrm{n} \in \{\mathrm{R}\}, \mathrm{l} \qquad (3.9b)$$

Non Negativity constraints:

All the variables are assumed to be greater than equal to zero.

Also for every constraint the departure time intervals m is assumed to be less than the equal to the actual simulation interval.

$$\mathrm{m} \leq \mathrm{l} \qquad (3.10)$$

The feasible region of the flows is bounded above by the sum of the O-D demands connected by that path. As the feasible region is closed and bounded, it can be shown that a solution exists for the above minimization problem (Nagurney, 1994). However as the feasible region is not convex (Peeta et. al., 1994) the uniqueness of solution is not guaranteed. The Dynamic User Class problem has been formulated as an equivalent minimization problem and an algorithm to solve this formulation is provided in the next section.

3.4.Algorithm Overview and Description

3.4.1 Algorithm Overview

The previous section formulates the dynamic user class problem as a variational inequality. The variational inequality formulation is converted into an equivalent non linear program. The equivalent non linear formulation can be solved using the Frank-Wolfe method. The Frank- Wolfe method is an iterative procedure in which, at every iteration the objective function is linearized and evaluated at the feasible flow solution from the previous iteration. In every iteration, the auxiliary path or the shortest path is determined for every origin destination pair. If the auxiliary path passes through the location of the en-route information source then the sub-path of the original path between the en-route information source and destination is replaced by the shortest path between the en-route information source and destination. The auxiliary path constitutes the direction of descent. The descent step size is determined by the convex combinations method. The process is repeated till convergence.

3.4.2 Algorithm Description

Let d^{m-n} represent the total demand from origin m to destination n. Let e represent the location of the en-route information source. SP(orig-dest) refers to the shortest path between the various origins and destination and SP(e-dest) refers to the shortest path between the en-route information source and destination. Recall that there are two classes of users: class 1 and class 2. Users belonging to class 1 receive information from the pre-trip information source alone and do not change routes en-route. Similarly users belonging to class 2 receive both pre-trip and en-route information. Let k represent the iteration counter. Let x_{il}^{mnk} represent the flow of class i users(where i=1,2) on path l between the origin-destination pair m-n during iteration k. Let P_i^k denote the set of paths found for class i until the k^th^ iteration.

Step 0: Initialization

Set the iteration counter k=0. Set the intial flows $x_i^k = 0$ where i=1,2.

Step 1: Computing link travel times

For current network flows $\mathrm{x}^{\mathrm{k}} = \{\mathrm{x}_1^{\mathrm{k}}, \mathrm{x}_2^{\mathrm{k}}\}$, compute trip times $\widetilde{t}(x^k)$ using a dynamic network assignment model. Note that the current flows consists of two parts flows belonging to class 1 – $\mathrm{x}_1^{\mathrm{k}}$ and flows belonging to class 2 – $\mathrm{x}_2^{\mathrm{k}}$.

Step 2: Finding descent direction

Find shortest path sets SP (orig-dest) and SP (e-dest) corresponding to $\widetilde{t}(x^k)$ - $\{r_{i-j}, ...\}$, where i-j is an O-D pair, by using label correcting algorithm. These paths are called auxiliary paths.

For each Origin-Destination pair m-n:-

(i) Check if the auxiliary path between m-n r_{m-n} passes through the node e. If yes, set $\delta_{m-n}(k+1)$ =1 else set $\delta_{m-n}(k+1)$=0.

(ii) If $\delta_{m-n}(k+1)$= 0, then the auxiliary path r_{m-n} does not pass through the location of the en-route device 'e'. Hence vehicles on this path belong to class 1. The class 1 auxiliary path set is then augmented as r_{m-n} i.e. $P_1(k+1) = P_1(k) \cup \{r_{m-n}(k)\}$. The class 2 auxiliary path set is retained $P_2(k+1) = P_2(k)$.

If $\delta_{m-n}(k+1)$= 1, then the auxiliary path r_{m-n} passes through the location of the en-route device 'e'. Hence vehicles on this path belong to class 2. The class 2 auxiliary path set is augmented as $P_2(k+1) = P_2(k) \cup \{r_{m-e} \cup r_{e-n}\}$ i.e the sub path of the auxiliary path between e and the destination n is replaced by the shortest auxiliary path between e and the destination. The class 1 auxiliary path set is retained $P_1(k+1) = P_1(k)$.

Step 3: Flow Update

Calculate the updated flows of each class on each path j connecting m-n (X$_j$(1a,k)) using the Method of Successive Averages (MSA).

$$X_j(1,k+1) = \delta_{m-n}(k+1) * \frac{k}{(k+1)} * X_j(1,k) + (1-\delta_{m-n}(k+1)) * \{\frac{k}{k+1} * X_j(1,k) + \frac{d^{m-n} * Y_a}{(k+1)}\} \tag{3.11a}$$

$$X_j(2,k+1) = \delta_{m-n}(k+1) * \{\frac{k}{(k+1)} * X_j(2,k) + \frac{d^{m-n} * Y_b}{(k+1)}\} + (1-\delta_{m-n}(k+1)) * \frac{k * X_j(2,k)}{(k+1)} \quad (3.11b)$$

Where Y_a =1 if the current path and the auxiliary path are same and belong to class 1; Y_a =0 otherwise. Similarly Y_b =1 if the current path j and the auxiliary path are the same and belong to class 2; Y_b =0 otherwise.

Step 4: Convergence

There are two convergence conditions:

(i) Flow Convergence: The flow difference in path flows across successive iterations falls below the convergence threshold.

(ii) Class Convergence: The difference in the number of users belonging to various user classes across successive iterations falls below the convergence threshold.

If both conditions are satisfied, terminate the algorithm. If either of the conditions are not satisfied, set iteration counter k=k+1 and repeat steps 1-4.

3.4.3 Justification of Algorithm

In the descent direction step, the objective function is linearized with respect to the path flows. The linearized sub-problem is minimized by obtaining the least travel time path between every origin and destination and assigning the flows to this path. The optimal descent step size is obtained by the Method of Successive Averages in the flow update step. Note that in step 2, if the auxiliary path passes through the en-route information source, then the sub-path of the original path between the en-route information source and destination is replaced by the shortest path between the en-route information source and destination. Thus vehicles which pass through the en-route information source are assigned to their shortest path from the en-route information source and destination.

Convergence criteria 2 implies that at equilibrium there is no incentive for a user to shift from one class to another. This means at equilibrium, a user who receives only pre-trip information does not have any incentive to shift to a path through which he receives pre-trip and en-route information. Convergence criteria 1 implies that at equilibrium there is no incentive for a user to unilaterally shift from one path to another. This means that within each class, a user does not have any incentive to shift to other paths unilaterally. All the paths between the en-route information source and the destination have equal and minimal travel time. The proposed algorithm is based on a Frank-Wolfe linearization scheme, which has a polynomial time complexity (Frank et al., 1952).

3.5 Applications of the Dynamic User Class Framework

The Dynamic User Class framework presented above has numerous applications in generating information strategies for Advanced Traveler Information Systems. Some of the applications of the Dynamic User Class framework and its modeling applications are presented in the current section.

3.5.1 Coordination between pre-trip and DMS information

Consider a network where there is a pre-trip information source: assigning vehicles to paths between the various origins(O) and destination(D) and a DMS which assigns vehicles to paths between the DMS location(E_o) and a downstream node(E_d). The location of the downstream node (E_d) depends on the activation zone or the zone of influence of the DMS. The pre-trip information source assigns vehicles to time-dependent shortest paths between O and D, whereas the DMS assigns vehicles to time-dependent shortest paths between E_o and E_d.

Depending on the DMS assignment, the travel times on the downstream links change. This affects the pre-trip assignment strategy. Similarly, depending on the pre-trip assignment strategy, the time dependent loading of vehicles on the DMS node changes. Thus the DMS information strategy is found to be dependent on the pre-trip assignment strategy. This inter-dependency between two information sources in a network is termed as coordination.

The example presented in the previous section can be easily modified to model coordination. As explained previously in this section, the pre-trip information source is found to provide time dependent shortest paths between the various network level origins (O) and destinations (D). The DMS is found to provide time dependent shortest paths between the DMS origin (E_o) and the DMS destination (E_d). As explained in the algorithm, at a particular iteration for the current set of flows, the time-dependent shortest paths between every origin destination pair (O,D) is calculated. This time-dependent shortest paths are called auxiliary paths. If the time dependent shortest path (P_{o-d}) between an origin destination pair passes through the DMS locations i.e. (E_o and E_d) then the sub-path of P_{o-d} between E_o and E_d is replaced by the time dependent shortest path between E_o and E_d ($P_{o-d} = P_{o-e} \cup P_{e-d}$) . As P_{o-d} passes through the DMS location, auxiliary path set of class 2 is augmented with the path P_{o-d}. The auxiliary path set of class 1 is not augmented. However if the time dependent shortest path (P_{o-d}) between an origin destination pair does not pass through the DMS location, then the auxiliary path set of class 1 is augmented with (P_{o-d}). The auxiliary path set of class 2 is not augmented. The time dependent flows for all the paths are calculated using the Method of Successive Averages. At convergence, difference in the flows of vehicles across various classes and various paths from one iteration to the next is assumed to be below a certain threshold value.

The equilibrium conditions imply that: i) the time-dependent link travel times correspond to the equilibrium flows and vice-versa, ii) for users of class 1, their equilibrium paths correspond to the shortest time-dependent paths between their origin and destination given the equilibrium between the two classes, and iii) the equilibrium paths of users from class 2 are dependent on the equilibrium paths of users from class 1. Further, their equilibrium paths are composed of two sub-paths – one sub path belongs to the set of shortest time-dependent (equilibrium) paths between the DMS origin and terminal node. The second sub path is composed of two segments from the shortest time-dependent path between the vehicle's origin and destination, the first segment relates to travel between the vehicle's origin and DMS location, and the second covers the path between DMS terminal node and vehicle's destination. In solving for this equilibrium, the flows are iterated until convergence, such that the travel times of class 1 and class 2 are mutually compatible.

The equilibrium obtained through the example from above can be considered to be a predictive equilibrium. This is because both the DMS and pre-trip strategies are predictive in nature. The predictive DMS strategy accounts for the future time dependent arrival of vehicles while routing vehicles from E_o to E_d. Similarly the predictive Pre-trip strategy routes accounts for future time dependent link travel times due to the DMS routing strategy while providing information to vehicles.

A variant of the above strategy is the reactive DUC where one of the information strategy is based on prevailing or instantaneous travel times on links. For example, the pre-trip strategy may provide predictive information whereas the DMS may provide prevailing information. The

DMS can be assumed to route vehicles on shortest paths which are based on prevailing travel times. The above equilibrium can be solved for by making a simple change to the algorithm for the predictive equilibrium strategy. In the algorithm for the reactive DUC, if the time dependent shortest path between the origins and destination ($P_{o\text{-}d}$) passes through the DMS location (E_o and E_d), then the sub-path of $P_{o\text{-}d}$ between E_o and E_d is replaced by the prevailing shortest path between E_o and E_d. The convergence check for reactive Dynamic User Class involves only one criterion, the difference in the path flows across iterations must be lesser than a threshold value.

3.5.2 Switching pre-trip UE and en-route SO equilibrium

As explained in the previous section SO strategies tend to minimize the overall system travel. Hence SO strategies are extremely effective under incident condition or under emergency evacuation. If a severe incident has occurred in a network then SO information can be provided using en-route information devices to route vehicles in order to minimize the average system travel time in the incident affected area. Thus using SO strategies vehicles can be evacuated from the incident affected areas more efficiently.

This mixed information strategy can be modeled using the Dynamic User Class framework presented in the previous section. Two sources of information are assumed to be present in the network: pre-trip information which provides time dependent UE information from the various network level origins and destination and en-route information which provides time dependent SO information in the activation zone of the DMS(from DMS location E_o to DMS destination E_d). The algorithm for solving this mixed information strategy can be obtained by modifying the algorithm for solving the DUC framework as mentioned in the previous sections. At any iteration, for the current link flows, the time dependent shortest paths are solved for all the network level origin destination pairs. The time dependent least marginal cost path is obtained from the DMS origin (E_o) to the DMS destination (E_d). For any origin destination pair O-D, if the time dependent least cost path between the origin destination pair (o, d) - $P_{o\text{-}d}$ passes through the DMS location (Eo and E_d) then the sub path of $P_{o\text{-}d}$ between E_o and E_d is replaced by the least cost marginal path between E_o and E_d. As the path passes through the DMS location, the auxiliary path set of class 2 is augmented with the new path. Auxiliary path set of class 1 is retained. If the time dependent least cost path between the origin destination pair (o,d)-$P_{o\text{-}d}$ does not pass through the DMS location then the auxiliary path set of class 1 paths is augmented with the new path $P_{o\text{-}d}$. The auxiliary path set of class 2 paths is retained. Vehicles are allocated among the various paths using Method of Successive Averages. There are two criterions for convergence: (i) the difference between path flows across iterations are below a certain threshold, (ii) the difference between flows of each user class across iterations is below a certain threshold.

3.6 Summary

In this chapter the concept of dynamic user class equilibrium has been introduced and the dynamic user class framework for modeling information strategies has been developed and implemented. The dynamic user class problem has been formulated and an algorithm for solving for the dynamic user class has been proposed. The proposed DUC model relaxes the assumptions of pre-specified user classes and hence provides more scope for system/local optimization when compared to the multiple user class. Two applications of the DUC framework have been presented: (i) coordination among multiple information sources, and (ii) switching pre-trip UE and en-route SO information.

The DUC framework has numerous applications in modeling network dynamics under information. The application of the DUC in determining a predicted consistent and coordination information strategy for dynamic message signs is illustrated in the next chapter.