Cognitive Profiles of Inadequate Responders to Fractions Intervention

By

Sarah K. Krowka

Dissertation
Submitted to the Faculty of the
Graduate School of Vanderbilt University
in partial fulfilment of the requirements
for the degree of
DOCTOR OF PHILOSOPHY
in

Special Education
May 10, 2019
Nashville, Tennessee
Approved:
Lynn S. Fuchs, Ph.D.
Bethany Rittle-Johnson, Ph.D.
Christopher Lemons, Ph.D.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my adviser, Dr. Lynn Fuchs, for her continuous support of my studies. It has been an honor to have her as my adviser throughout my work at Vanderbilt. I would also like to thank Dr. Doug Fuchs, Dr. Bethany Rittle-Johnson, and Dr. Christopher Lemons for their support and invaluable feedback and advice as members of my dissertation committee.

Thank you also to my friends and colleagues without whom all of this would not have been nearly as much fun: Amber Wang, Amelia Malone, Rachel Pachmayr, Lindsay Foreman-Murray, Brooke Lee Morgan, Emily Weaver, Meagan Walsh, Katie Zimmerman, Casey Chauvin, Jodi Heidlage, Emily Quinn, and so many more.

A big "thank you" to my partner, Jason, for unreservedly supporting me in uprooting our lives in pursuit of this endeavor. I also extend my deepest gratitude to the Petruzzelli family. Without the generous love and support of Mary Beth, Tom, Tommy, and Elizabeth, completion of this project would not have been possible. Finally, and by no means least, thanks go to my family for their continued love and support through it all. Dad, Yvonne, Jenny, Adam, Hannah, and Mead…I'm glad you're in my corner.

For Leslie.

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

Figure Page

CHAPTER 1

INTRODUCTION

Competency in general mathematics is critical for academic and occupational success. Proficiency with fractions, in particular, is central to students' mathematical development because it plays a foundational role in advanced mathematics learning (National Math Advisory Panel [NMAP], 2008; Siegler et al., 2012). As Siegler and colleagues' (2012) demonstrated, nationally representative data from both the United States and United Kingdom indicated that competence with fractions in middle school (fifth or sixth grade) reliably predicts mathematics achievement five or six years later in high school. Yet, fractions frequently pose a major difficulty for students as they move from basic mathematics to more advanced topics, such as algebra (e.g., Bright, Behr, Post, & Wachsmuth, 1988; Kilpatrick, Swafford, & Findell, 2001; Hiebert, 1985).

Fractions instruction involves difficult-to-teach and difficult-to-learn concepts that present ongoing pedagogical challenges to mathematics educators. Difficulty with fractions learning begins early in the elementary years (Empson & Levi, 2011) and persists through high school and adulthood (Lipkus, Samsa, & Rimer, 2001; Reyna & Brainerd, 2007). Poor fractions comprehension can prevent individuals from pursuing advanced mathematics courses, and this becomes an obstacle for accessing certain career opportunities. Helping students develop a sound foundation in mathematics in general and on fractions in particular has long-term and high-stakes implications. The mathematics and education research communities have clear reason to explore strategies for resolving these challenges.

Difficulty in fractions learning is experienced by a broad range of learners but is especially common for students identified with or at risk of difficulty in mathematics (e.g., Hansen, Jordan, & Rodrigues, 2017; Tian & Siegler, 2017). Converging evidence suggests that this population makes up 5 to 7% of the student population in the U.S. (Berch & Mazzocco, 2007; Geary, 2004). Namkung, Fuchs, and Koziol (2018) found that, when compared to students with adequate whole-number competence, students with severe whole-number difficulty were about 32 times as likely to experience difficulty with fractions understanding. However, research also shows that for the majority of at-risk students, fractions knowledge improves with generally effective intervention (e.g., Fuchs, Malone, Schumacher, Namkung, & Wang, 2016; Fuchs, Schumacher, et al., 2013, 2016; Fuchs, Sterba, Fuchs, & Malone, 2016).

At the same time, correlational evidence (e.g., Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Mazzocco & Myers, 2003; Murphy, Mazzocco, Hanich, & Early, 2007) suggests that students with severe mathematics difficulty have differing patterns of cognitive strengths and weaknesses, or *cognitive profiles*, when compared to students with less severe mathematics difficulty. These students are most likely to respond inadequately to otherwise effective intervention and thus may require different or more intensive forms of intervention. Developing deeper understanding of the cognitive profiles of students with severe mathematics difficulty can help researchers identify strategies for increasing the efficacy of interventions and ultimately for applying diagnostic models of learning difficulties and personalizing instructional design.

Apart from one prior study (Krowka & Fuchs, 2017), little is understood about the cognitive profiles of *inadequate responders* to mathematics intervention. Krowka and Fuchs explored cognitive profiles of fourth graders at risk of developing mathematics difficulties. After grouping students according to adequacy of responsiveness to a generally effective fractions intervention on number-line estimation, calculation, and word problems, we examined differences in cognitive processing on each outcome. Findings supported correlational research demonstrating that working memory, reasoning ability, concept formation, listening comprehension, processing speed, and behavioral attention are associated with responsiveness to fractions intervention (e.g., Fuchs, Geary et al., 2013; Fuchs, Malone, et al., 2017; Seethaler, Fuchs, Star, & Bryant, et al., 2011).

Further research is, however, needed to deepen understanding of factors potentially related to responsiveness to fractions intervention. Such understanding would provide guidance for identifying and remediating mathematics difficulties and inform intervention design targeted for this population. The purpose of the present study was to extend Krowka and Fuchs (2017) by further examining the cognitive profiles of at-risk students in terms of responsiveness to fractions intervention on three similar types of fractions knowledge, but this time focused on a younger population of learners.

In this introduction, I first provide the rationale for examining the three types of fractions knowledge included in these analyses. Next, I summarize prior work on cognitive predictors of these knowledge types. Then, I discuss studies examining cognitive moderators of responsiveness to fractions intervention. Finally, I describe the present study's analytic methods.

Rationale for the Three Types of Fractions Knowledge

In examining the cognitive profiles associated with responsiveness to intervention, I focused on three types of fractions knowledge: calculations, ordering, and word problems. I chose these types of fractions knowledge because they have each been associated with mathematics achievement and have been investigated in the fractions literature.

Calculations. Calculating with fractions poses a strong challenge for many learners. Unlike whole-number calculations, which the majority of learners eventually master for single-digit numbers, many never reach proficiency with fraction calculations (Gabriel et al., 2013;

Lortie-Forgues, Tian, & Siegler, 2015). For example, during the 2006-2007 school year, a large (*N* = 1110) and nationally representative sample of Algebra 1 teachers in U.S. public schools reported skill with operations involving fractions and decimals as one of three skill areas for which their students are most poorly prepared, creating a significant obstacle to their algebra learning (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). For these reasons, it is important to explore the cognitive profiles associated with responsiveness to fractions intervention on calculations outcomes.

Prior work in this area consistently demonstrates associations between fraction calculations and mathematics achievement more generally. For example, Hecht (1998) found that calculations with fractions predicts outcomes on other measures of general mathematics. After investigating the fractions knowledge of typically performing sixth-grade students, Bailey, Hoard, Nugent, and Geary, (2012) administered a follow-up assessment at seventh grade and found that measures of fluency with fraction calculations significantly predicted seventh-grade mathematics achievement. Siegler and Pyke (2013), who examined developmental and individual differences in sixth and eighth graders' fraction calculations, found significant relations with overall math achievement test scores, which increased with age. Siegler and Pyke also demonstrated that difficulty with fraction calculations is persistent. They found that low-achieving students' calculation accuracy remained low across sixth through eighth grade while high-achieving students' accuracy improved.

Other types of data have led to similar conclusions. Siegler et al. (2012) analyzed large datasets from the U.S. and U.K. and found that performance on fraction calculations in fifth grade uniquely predicts students' knowledge of algebra and general mathematics achievement in tenth grade. These results were found even while controlling for whole-number arithmetic proficiency, verbal and nonverbal IQ, working memory, family education, race, ethnicity, and family income. Given the importance of proficiency with fractions for general mathematics achievement and subsequent academic and occupational success, poor understanding of fraction calculations is a notable problem.

Ordering. It is well documented that learners have substantial difficulty in assessing and comparing the magnitude of fractions (e.g., Siegler, Thompson, & Schneider, 2011; Stafylidou & Vosniadou, 2004). Knowledge of fraction magnitudes is assessed in multiple ways, which includes ordering fractions from least to greatest. For example, almost 50% of eighth graders in the U.S. could not correctly order ½, 2/7, and 5/9 (National Center for Education Statistics, 2007). Importantly, the NMAP (2008) posited that understanding of fraction magnitudes, by way of comparing, ordering, and placing fractions on number lines, may be more foundational for fraction understanding than part-whole understanding. Support for this comes from previous intervention research (e.g., Cramer, Post, & delMas, 2002; Fuchs, Sterba, et al., 2016), which shows that explicitly teaching at-risk fourth-grade students about fraction magnitudes improves fractions outcomes including released items from the National Assessment of Educational Progress and calculations.

Additionally, whole-number bias may play a significant role in students' difficulty with ordering fractions and fraction magnitudes in general. Common obstacles caused by whole-number bias include viewing numerators and denominators as independent numbers and comparing fraction magnitudes based on whole-number knowledge. For example, students may consider 3/12 as greater than ½ because 3 and 12 are greater than 1 and 2. It stands to reason, then, that students with poor knowledge of whole numbers may struggle with activities involving fraction magnitudes more than students with adequate whole-number competence. Namkung et al. (2018) and Malone and Fuchs (2017), who examined the error patterns of fourth graders working with fractions, found that approximately 65% of errors in ordering of fractions were due to whole-number bias. That is, students misapplied whole-number properties when attempting to determine fraction magnitudes.

Word problems. I chose word problems as the third type of fractions knowledge for three primary reasons. First, word problems are one of the best school-age predictors of adulthood employment and wages (e.g., Bynner & Parsons, 1997; Gross, Hudson, & Price, 2009; Murnane, Willett, Braatz, & Duhaldeborde, 2001). Second, by the end of elementary school, students are expected to master word problems involving fractional quantities (Kilpatrick et al., 2001). Third, solving word problems, whether with whole numbers or with fractions, is generally challenging for students at risk for difficulty in mathematics (e.g., Jitendra, Hoff, & Beck, 1999; Montague, Enders, & Dietz, 2011).

Unfortunately, little is known about the role of *fractions* in word problems. Performance on word problems in general declines substantially when additional features are included, such as extraneous information or multiple steps (e.g., Hawkins, Stancavage, & Dossey, 1998; Parmar, Cawley, & Frazita, 1996). Small changes in the context of a problem, the wording of a story, or other factors of word-problem presentation can yield dramatic changes in student success. Solving word problems with *whole numbers*, a task already difficult for at-risk learners, appears to exert strong cognitive demand. Given students' struggle with fractions, generally, fraction word problems likely increase the challenge.

Prior Work on Cognitive Predictors of Development and Responsiveness to Intervention on These Types of Fractions Knowledge

Prior literature on cognitive predictors of students' development of fractions knowledge suggest that several cognitive processes play a role in responsiveness to fractions intervention. The literature examining cognitive predictors of fractions ordering is more limited than for calculations or word problems, but studies exploring cognitive predictors of number line estimation are available. This literature on number line estimation is relevant here because ordering tasks are similar to number line estimation tasks in that they both require students to demonstrate knowledge of fraction magnitudes and to place fractions in order from least to greatest.

Ordering. I identified no prior studies that explored the cognitive predictors of understanding of fraction magnitudes when assessed using fractions ordering tasks. However, I discuss three studies that have explored underlying cognitive processes associated with students' understanding of fraction magnitudes when assessed via fractions *number line estimation* tasks (e.g., Bailey, Siegler, & Geary, 2014; Namkung & Fuchs, 2016).

First, with fourth-grade students at risk of mathematics difficulties, Namkung and Fuchs (2016) contrasted whole number and fraction number line estimation to examine shared and distinct predictors. In terms of *fractions* number line estimation, they identified language comprehension (i.e., vocabulary, listening comprehension) and reasoning as unique predictors. Second, Bailey, Siegler, and Geary (2014) analyzed longitudinal data to look at predictors of middle-school fractions knowledge; they found that working memory at first grade predicts competence with fractions number line estimation at eighth grade. Third, in Krowka and Fuchs' (2017) exploration of the cognitive strengths and weaknesses of adequate versus inadequate responders to fractions intervention, they found significant differences between groups on reasoning, processing speed, concept formation, listening comprehension, listening recall, and behavioral attention.

Calculations. Prior research has identified several cognitive processes associated with fraction calculations. First, working memory predicts skill with calculations (e.g., Hecht, Close, & Santisi, 2003; Krowka & Fuchs, 2017; Siegler & Pyke, 2013). For example, Siegler and Pyke

(2013) examined two different aspects of eighth graders' working memory (updating and inhibitory) and found correlations with accuracy in fraction calculations.

Further, when studying third-grade cognitive predictors of fifth-grade fraction skill, Seethaler et al. (2011) found different aspects of language ability (vocabulary and listening comprehension) to support fraction calculations. Findings by Namkung and Fuchs (2016) corroborated this finding in with fourth grade students. Additionally, Krowka and Fuchs (2017) used a fraction calculations outcome to examine addition and subtraction calculations with fractions. They found that inadequate responders to the fractions intervention on the calculations outcome scored significantly lower than adequate responders on reasoning, concept formation, listening comprehension, and listening recall.

Behavioral attention has also been identified as a predictor of fraction calculations skill (Fuchs, Fuchs, & Compton, 2013; Hecht et al., 2003; Hecht & Vagi, 2010; Namkung & Fuchs, 2016). In one study, Hecht et al., (2003) found that behavioral attention in the classroom was a relatively strong unique predictor of concurrent individual differences in fraction computation at fifth grade, while controlling for mathematical knowledge, working memory, and word level reading. Supporting these findings, but with fourth-grade students at risk of difficulty with mathematics, Namkung and Fuchs (2016) found that behavioral attention uniquely predicts fraction calculations. Moreover, risk for difficulty in mathematics and poor behavioral attention are often comorbid (Fletcher, Shaywitz, & Shaywitz, 1999; Gross-Tsur, Manor, & Shalev, 1996; Peng, Wang, & Namkung, 2018).

Children with mathematics disabilities have been shown to manifest less attentive behavior during math instruction (Bryan, 1974; McKinney & Speece, 1986), which may be an important contributor to how well children benefit from wholeroom mathematics instruction. A child's ability to engage in on-task behavior during learning activities, such as attentional inhibition or switching to an on-task behavior after an attentional shift (e.g., classroom distraction), is necessary to support the learning process. The observed inadequacy in classroom attentiveness among students with mathematics disabilities is unsurprising considering that performing fraction calculations accurately (a largely procedural task) requires students to engage behavioral attention to follow a sequence of procedural steps.

With regard to processing speed, students with or at risk of mathematics difficulties process information more slowly than typically performing students (e.g., Bull & Johnston, 1997). Slower cognitive processing may result in performance deficits in multiple areas of mathematics, transparently so in calculations. Processing speed, or the efficiency with which cognitive tasks are executed (Case, 1985), may influence how quickly and accurately numeric knowledge can be processed. However, mixed results have been found in the exploration of processing speed as a predictor of fraction calculations. While Fuchs, Schumacher, et al. (2013) along with Namkung and Fuchs (2016) found relations between these factors, Krowka and Fuchs (2017) and Seethaler et al. (2011) did not identify a significant association between processing speed and fraction calculations.

Word problems. I identified only two studies (Hecht et al., 2003; Krowka & Fuchs, 2017) that investigated associations between cognitive processes and performance on fraction word problems. First, Hecht and colleagues evaluated relations among types of mathematical knowledge, student characteristics, and fractions outcomes for fifth-grade students. They found evidence that working memory (on a counting span task) and classroom behavioral attention uniquely contributed to fraction word-problems performance. In terms of responsiveness to intervention, Krowka and Fuchs (2017) examined the role of several cognitive processes in the responsiveness to fractions intervention on the fraction word-problem solving skills of at-risk fourth graders. They found significant differences between responder groups on reasoning, processing speed, and behavioral attention. Although these two studies examined different student populations and used differing analyses, it is interesting to consider that Krowka and Fuchs also explored the role of working memory as a cognitive predictor via a counting recall task and did not find it to be associated with responsiveness to intervention.

To summarize, prior work suggests that performance on fraction *calculations* may be uniquely predicted by aspects of working memory, language (i.e., vocabulary, listening comprehension), concept formation, processing speed, and behavioral attention. For *word-problem solving*, research indicates performance on fraction word problems may be uniquely predicted by working memory, reasoning, processing speed, and behavioral attention. Although no prior studies have explored the potential roles of cognitive predictors of performance on fraction *ordering* tasks, studies examining performance on fraction *number line estimation* tasks, a fraction performance task that likely taps similar cognitive resources, have identified working memory and language as unique predictors of performance.

The empirical studies focusing on fractions learning along with Geary's (2004) theoretical model of general mathematics learning informed the conceptual framework for identifying potential predictors of responsiveness to fractions intervention. Within Geary's model, "an array of cognitive systems" (p. 8) support students' learning of mathematics concepts and procedures. Specifically, general cognitive processes such as working memory and aspects of attention are posited to play a substantial supporting role in mathematics learning. Alongside Geary's theoretical model, prior research guides hypotheses about potentially salient predictors of the fractions outcomes.

Across all three fractions outcomes, I expected inadequate responders to exhibit weakness in behavioral attention. I anticipated this pattern due to the potential for deficits in attentional control to disrupt mathematics learning. I also expected a pattern of weakness in working memory capacity to accompany behavioral attention deficits for all three outcomes. With respect to working memory on the calculations and ordering outcomes, I anticipated weakness on the counting recall task, but not on the listening recall task because numerical measures of working memory (e.g., counting recall) have more frequently been associated with mathematics difficulty than non-numerical measures (e.g., listening recall) (Passolunghi & Cornoldi, 2008; Passolunghi & Siegel, 2001, 2004) except in the case of word problems (e.g., Fuchs et al., 2010).

On the word-problem outcome, I expected listening recall to emerge as a weakness in working memory capacity. In this study, during administration of the word problems measure, examiners read each word problem aloud while students followed along. This is of interest because students identified as at-risk for math difficulties often also have comorbid reading difficulties (e.g., Badian, 1999; Landerl & Moll, 2010), thus many of the students in the present study may also struggle with reading. Presumably, students with weaker reading skills at third grade (e.g., poor decoding or fluency) would need to rely on stronger listening recall ability to demonstrate adequate responsiveness to fractions intervention on the word-problem outcome.

On the calculations outcome, I also anticipated vocabulary to emerge as a weakness associated with responsiveness to fractions intervention. Prior work suggests a role for vocabulary in initial acquisition of fractions knowledge (Jordan et al., 2013) which, in turn, may affect students' later development of procedural knowledge with fractions. Procedural knowledge is foundational to performing fraction calculations. This notion is supported by evidence of a role for vocabulary in the learning of fraction calculations (Seethaler et al., 2011; Namkung & Fuchs, 2016). Finally, in terms of the word problem outcome, I expected reasoning to emerge as a unique predictor of responsiveness in addition to behavior attention. Adequate reasoning ability is transparently required for successful employment of the schema-based problem-solving strategies used in fraction intervention (Fuchs, Malone et al., 2016).

Cognitive Moderators of Responsiveness to Fractions Intervention

Recent initiatives in the intervention field have encouraged researchers working with at-risk learners to use moderation analysis to consider broader and deeper exploration of student characteristics associated with responsiveness to generally efficacious intervention (Fuchs & Fuchs, 2019). The studies discussed in this section explored responsiveness to intervention by looking at interactions between cognitive process variables and intervention effects using this method. With moderation analysis, investigators identify cognitive variables that interact with intervention effects, and these analyses can determine where along the distribution of a cognitive variable the effects of intervention transition from significant to nonsignificant. An advantage of such moderation analysis is that it avoids setting an arbitrary cut-point on responsiveness and instead identifies the cut-point empirically.

The aim of these recent studies is to encourage researchers to explore *why* certain students respond inadequately to generally efficacious interventions. This is the same goal of cognitive profile analysis when used to determine which cognitive variables are and are not associated with adequate versus inadequate responders to intervention. Deeper understanding of moderator effects in this area may provide an empirical basis for developing different intervention components tailored to students with differing cognitive strengths or weaknesses.

Only one study was identified that explored potential cognitive-process moderators of the types of fractions knowledge (or related strands) investigated in the present study. Further exploring the role of working memory while also considering reasoning and language comprehension, Fuchs, Malone, and colleagues (2016) extended the literature on fractions learning of at-risk fourth graders by investigating the effects of teaching students to provide explanations for their mathematics work. They explored two variants of a multi-component fractions intervention: (1) using self-explanation while comparing fraction magnitudes or (2) solving fraction word problems. First, they investigated whether the effects of a self-explaining intervention occurred via a compensatory mechanism. Specifically, they tested whether individual differences in three cognitive processes (reasoning, working memory, and language comprehension) moderated responsiveness to the intervention on a measure of fraction magnitudes knowledge. While controlling for the effects of reasoning and language comprehension, they found a compensatory moderator effect for working memory as follows. Students who received the intervention condition with practice explaining why two fractions differ on magnitude generated comparably strong explanations at posttest, regardless of their pretest working memory capacity. By contrast, in the control group, a significant correlation existed between pretest working memory capacity and the quality of students' explanations.

While this finding is important to consider in the context of moderators of fraction magnitudes, it did not inform my hypotheses for the cognitive profiles resulting from the ordering outcome. This is because the measure Fuchs, Malone, et al., (2016) used to assess fraction magnitudes is dissimilar to the fraction magnitudes task of ordering fractions from least to greatest used in the present study. In this 2016 study, fraction magnitudes knowledge was measured using a task requiring students to place a greater-than or a less-than symbol between fractions and write words or draw pictures to explain why the fractions differ in magnitude. This task appears distinct from those requiring students to arrange fractions in order from least to greatest as is required by ordering tasks or number line estimation tasks.

In the same study, Fuchs, Malone, and colleagues (2016) also identified a second interaction, this time involving the contrast between word-problems intervention and control and pretest reasoning ability. The interaction indicated that the effect on fraction word-problems was moderated by reasoning ability. For students in the word-problems condition, word-problems outcomes correlated more strongly with reasoning ability than was the case in the control group. Students with larger deficits in reasoning ability thus demonstrated less inadequate response to fraction word-problems intervention.

Considering the demonstrated role of reasoning in fraction word-problems learning in the related literature, both through correlational evidence (Krowka & Fuchs, 2017) and moderator analyses (Fuchs, Malone, et al., 2016), I hypothesized that inadequate response on the word problem outcome would be positively associated with reasoning ability. I also considered the substantial role of behavioral attention found across all three fractions outcomes within the correlational literature and anticipated that it would emerge in the within-group cognitive profiles as a key limitation for inadequate responders relative to each groups' own performance on the other cognitive processes.

How Prior Work Guided the Selection of Cognitive Variables Investigated in the Present Study

When taken together, the literature thus reveals five cognitive processes as potentially salient to responsiveness to fractions intervention on *calculations*: working memory, listening comprehension, reasoning, processing speed, and behavioral attention. Further investigation of these relations is warranted because only a few studies have investigated the cognitive processes related to fraction calculations learning. Fewer still have considered responsiveness to intervention, specifically. Further, conflicting findings exist for several cognitive processes (e.g., processing speed in Seethaler et al., 2011). Thus, in the present study, I focused on these five cognitive processes while also expanding on prior working memory findings by including two measures of working memory (i.e., counting recall and listening recall) in the analyses.

In terms of ordering of fractions, to my knowledge, the present study is the first to examine cognitive processes that underline fractions ordering. However, examining predictors and moderators of a similar measure of understanding of fraction magnitudes, number line estimation, should provide insights on the cognitive processes that underlie fraction magnitudes learning. The combined literature identifies different aspects of working memory, language, reasoning, processing speed, behavioral attention, and concept formation as related to number line estimation skill.

In the domain of word problems, only three studies were identified that examine cognitive predictors or moderators. Although limited, these studies provide preliminary insight into the cognitive processes related to responsiveness to fractions intervention on word-problem outcomes. Together, this evidence reveals working memory (i.e., counting recall), reasoning, processing speed, and behavioral attention may play important roles in fraction word problem learning. Because less is known about the roles of cognitive processes in word-problem solving involving fractions, it is important to include multiple cognitive processes in analyses of word-problem learning.

Purpose of the Present Study

The purpose of this study was to explore the cognitive processes involved in responsiveness to fractions intervention for third-grade students on three fractions outcomes: calculations, ordering, and word problems. The focus on this young age group stems from the Career- and College-Ready Standards adopted across the United States which establish a strong emphasis on fractions learning beginning in third grade. These standards set forth expectations that third graders develop understanding of fraction magnitudes. Thus, this study intends to inform future work with the objective of earlier prevention rather than later remediation of difficulty with responsiveness to fractions intervention. To this end, developing deeper understanding of the cognitive processes involved will serve to inform the development of effective intervention for inadequate responders.

The first research question was: On which cognitive processes do third-grade inadequate responders demonstrate substantially higher or lower performance relative to their own performance on the other cognitive processes? The second research question was: Does performance on cognitive processes differ between adequate responders and inadequate responders and, if so, on which outcomes? Accordingly, as in Krowka and Fuchs (2017), the first analytic procedure relied on a more common approach: examining mean-level differences *between* adequate versus inadequate responder groups. The second procedure involves a *within-group* profile analysis, in which the shape of the cognitive profile for each inadequate responder group (i.e., one for each outcome) is examined.

This study aimed to extend the previous study on cognitive profiles of inadequate responders to fractions intervention (Krowka & Fuchs, 2017) in three major ways. First, it addressed a younger population by focusing on third graders, the grade level when fractions appear as a major component of the mathematics curriculum. Second, it explored whether the cognitive processes associated with responsiveness differ as a function of fractions ordering, an outcome not examined in Krowka and Fuchs (2017). Finally, it examined the role of vocabulary in the cognitive profiles of adequate and inadequate responders.

CHAPTER 2

METHOD

Participants in the present study were from two larger randomized control trials investigating the efficacy of intervention designed to improve at-risk third-grade students' understanding of fractions. In the parent studies, students were drawn from a southeastern metropolitan school district during the 2015-2016 and 2016-2017 school years. Data were collected using a multi-step process. First, during August and September of each school year, examiners screened students for whom parental consent was obtained during a whole-class testing session to identify students who met at least one of two criteria: performance at or below the 21st percentile on a broad-based mathematics calculation assessment (Wide Range Achievement Test-4 [WRAT-4]; Wilkinson & Robertson, 2006) or WRAT performance at the 30th percentile *paired with* a raw score of two or lower on the Minuends to 18 subtest of the *Second-Grade Calculations Battery* (Fuchs, Hamlett, & Powell, 2003). Because the parent studies were not about intellectual disability, students were excluded if they earned T-scores below the 9th percentile on both the *Vocabulary* and *Matrix Reasoning* subtests of the Wechsler Abbreviated Scales of Intelligence (WASI; Wechsler, 2011). Remaining students who met the risk criterion were individually administered a battery of mathematics and cognitive measures. Then, they were randomly assigned to a business-as-usual control group or one of two variants of fractions intervention. Students were posttested on fractions measures that included calculations, ordering, and word problems.

Assessments were administered by graduate research assistants, blind to testing conditions, who had demonstrated acceptable fidelity during training prior to administration. All testing sessions were audio recorded and 20% of sessions, stratified by examiner, were randomly sampled and checked for fidelity by an independent scorer. Agreement on test administration and scoring exceeded 92%. Two independent research assistants scored tests and resolved scoring discrepancies. They also double-entered scores into the electronic database, with all discrepancies resolved.

For the present analyses, we relied on study participants who completed intervention and for whom complete sociodemographic information, cognitive process data, and posttesting data were available. Sample sizes were comparable across fractions outcomes: 124 students in the Calculations Sample, 120 in the Ordering Sample, and 120 in the Word Problem Sample, all drawn from the same pool of 124 students. The differences in sample size reflected more complete posttesting data for the calculations than the ordering and word-problem measures.

Operationalizing Response to Intervention

To determine which students require more intensive remediation, teachers and researchers must establish a criterion for distinguishing adequate versus inadequate responsiveness to intervention. Although responsiveness to intervention exists on a continuum (Fuchs, Compton, Fuchs, Bryant, & Davis, 2008), schools must allocate services based on a binary outcome for identifying discrete groups of students requiring advancement into interventions of increasing intensity and frequency.

In the literature, the methods for designating and defining “adequate response” and “inadequate response” are variable. Some methods include the use of benchmarks at the end of intervention (final outcome); others use students’ rate of progress (growth) across intervention. Prior research comparing these methods suggests that final outcome produces a greater proportion of inadequate responders, which may forecast long-term mathematics success more realistically (see Fuchs, Fuchs, & Gilbert, 2019). For this reason, in this study, I relied on final outcome at the end of intervention.

Final outcome maps well onto the primary goal of intervention with at-risk students: to close the performance gap between at-risk and typically developing learners. Thus, I used posttreatment status on each of the three target outcomes. Responsiveness was thus operationalized using the posttest mean of the control group on each relevant measure as the cut point. Students who fell above this cut-point were designated adequate responders; those below this cut-point, inadequate responders. See Table 1 for student demographics and screening data (i.e., WRAT-4 *z*-scores) for each analytic sample by responder group.

Table 1

Student Demographics and Screening Data by Analytic Sample and Responder Group

	Calculations Sample (*n* = 124)		Ordering Sample (*n* = 120)		Word Problems Sample (*n* = 120)	
	Adequate Responders (*n* = 96)	Inadequate Responders (*n* = 28)	Adequate Responders (*n* = 109)	Inadequate Responders (*n* = 11)	Adequate Responders (*n* = 89)	Inadequate Responders (*n* = 31)
Variable	*n* (%)		*n* (%)		*n* (%)	
Gender						
Female	55 (57%)	14 (50%)	60 (55%)	6 (55%)	50 (56%)	16 (52%)
Male	41 (43%)	14 (50%)	49 (45%)	5 (45%)	39 (44%)	15 (48%)
Race						
Black	50 (52%)	13 (46%)	55 (50%)	6 (55%)	43 (64%)	18 (58%)
Hispanic	23 (24%)	9 (32%)	29 (27%)	1 (9%)	24 (27%)	6 (19%)
White	19 (20%)	4 (14%)	21 (19%)	2 (18%)	18 (20%)	5 (19%)
Asian	2 (2%)	1 (4%)	2 (2%)	1 (9%)	2 (2%)	5 (1%)
Other	2 (2%)	0 (0%)	2 (2%)	0 (0%)	2 (2%)	0 (0%)
Subsidized Lunch	64 (67%)	19 (68%)	74 (68%)	10 (91%)	62 (70%)	22 (79%)
English Language Learner	12 (13%)	10 (36%)	20 (18%)	1 (9%)	16 (18%)	5 (16%)
Special Education	10 (10%)	3 (11%)	12 (11%)	1 (9%)	7 (8%)	6 (19%)
	M (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Screening Measure						
WRAT-Calculations	0.11 (0.98)	-0.43 (1.31)	0.06 (0.98)	-0.72 (1.62)	0.22 (0.79)	-0.66 (1.47)

Note. Special education status denotes qualification for special education services. WRAT-Arithmetic means and standard deviations are reported as *z*-scores.

Measures

Screening. The mathematics screening measure was the *WRAT 4-Math Computation* (Wilkinson & Robertson, 2006) in which students complete calculation problems of increasing difficulty. This measure taps the individual's ability to perform basic math skills through counting, identifying numbers, solving simple oral problems, and calculating written math problems in addition, subtraction, multiplication, division, fractions, decimals, and algebra. Test-retest reliability is .88. Alpha on this measure ranges from .85 to .93 (Powell et al., 2015). The subtraction measure consists of 25 single-digit problems with minuends up to 18. Students have 1 min to complete the measure.

Fractions outcomes. The calculation measure was from the *Fraction Battery-2015-revised* (Malone, Fuchs, & Schumacher, 2015). *Fraction Battery* includes seven addition items and seven subtraction items presented horizontally. Seven items have like denominators (e.g., 1/8 + 3/8 = 4/8 = 1/2; 3/4 − 2/4 = 1/4) and seven items have one fraction with an even denominator and one fraction equal to one half (e.g., 4/10 + 1/2 = 9/10). One point is awarded for each correct numerical answer for a maximum of 14. Alpha on the parent studies' samples ranged from .91 to .93.

Ordering Fractions, from the *Fraction Battery-2015-revised* (Malone et al., 2015) measures students' magnitude understanding with six items. Each item shows three fractions, which students to order from least to greatest. Three items have the same numerator and different denominators, three have different numerators and different denominators. The score is the number of correct answers; the maximum score is six. Alpha on the parent studies' samples ranged from .74 to .76.

Word Problems includes five problems. Two problems requiring students to compare fraction quantities (e.g., "2/3 of the school's soccer teams run during practice. During practice, the orange team ran 7/10 of a mile and the blue team ran 4/5 of a mile. Who ran the shorter distance?") and three problems requiring students to calculate an ending amount when there is a change in a starting amount (e.g., "Juan had 6/8 of a pizza in his fridge. He ate 1/8 of the pizza for dinner. How much pizza is left after dinner?"). The tester reads each item aloud while students follow along on paper. Students can ask for one rereading of each item. For each problem, students earn 1 point for the correct numerical answer and 1 point for the correct label (e.g., "pieces of pizza"). The maximum score is 10. Alpha on the parent studies' samples ranged from .48 to .68.

Cognitive Processing Measures Used for Investigating Differences Between and Among Responder Groups

Reasoning. *WASI Matrix Reasoning* (Wechsler, 2011) measures reasoning with pattern completion, classification, analogy, and serial reasoning tasks. Students select among five response options to complete a matrix with a missing section. At age 9, internal consistency reliability is .94.

Processing speed. *Cross Out* from the WJ-III (Woodcock et al., 2001) measures processing speed by asking students to locate and draw a line through five pictures that match a target picture in that row. Students have 3 min to complete 30 rows. Reliability is .91.

Working memory. To assess the central executive component of working memory, the parent studies used two subtests from the *Working Memory Test Battery for Children* (WMTB-C; Pickering & Gathercole, 2001). Both include six dual-task items at span levels from 1-6 to 1-9. Passing four items within a level moves the child to the next level. At each span level, the number of items to be remembered increases by one. Failing three items terminates the subtest. Scores are the number of correct trials. With *Listening Recall*, the child determines if each sentence in a series is true; then recalls the last word of each sentence. Test-retest reliability ranges from .84 to .93. With *Counting Recall*, the child determines how many objects are in an array and then recalls the series of counts in the trial. Test-retest reliability ranges from .82 to .91.

Vocabulary. The WASI *Vocabulary* subtest (Wechsler, 2011) measures vocabulary knowledge, expressive vocabulary, verbal knowledge, and foundation of information. The first four items require students to identify an object in a picture. For the remaining items, the examiner says a word and the child verbally defines the word. Responses are scores 0, 1, or 2 points, dependent on response quality. Testing is discontinued after five consecutive scores of 0. Scoring is based on total number of points earned. Split-half reliability is .86 to .87 at ages six to seven (Zhu, 1999).

Behavioral attention. The *Strength and Weaknesses of ADHD-Symptoms and Normal-Behavior* (SWAN; Swanson et al., 2004) samples items from the Diagnostic and Statistical Manual of Mental Disorders-IV (4th ed., text rev.; DSM–IV–TR; American Psychiatric Association [APA], 2000) criteria for Attention Deficit Hyperactivity Disorder for inattention (9 items) and hyperactivity-impulsivity (9 items), but scores are normally distributed. Teachers rate items on a 1-7 scale. We report data only for the inattentive subscale, as the average rating across the nine items. The SWAN correlates well with other dimensional assessments of behavior related to attention. Alpha for the inattentive subscale on the parent studies' samples was .96.

Intervention

All students who entered the parent studies received a base intervention delivered to pairs of students for 30-35 min per day, three days per week, for 13 weeks beginning late October to early February. Intervention was delivered by employees of a research grant, most of whom were master's students in the college of education (only some of whom were teachers or pre-service teachers).

Base intervention. The base intervention, which was common across the two parent studies, provided students with explicit instruction using a multi-component fractions intervention referred to as *Third-Grade Super Solvers* (Fuchs, Malone, Wang, Abramson, & Krowka, 2016). With this program, interventionists introduced new topics using worked examples, while explaining each step using simple, direct language. They then gradually faded use of worked examples as students practiced applying learned strategies to solve problems during guided and independent practice. Instruction incorporated efficient solution strategies to support understanding and mastery of fraction concepts and skills and to minimize learning challenges for students. Students had many opportunities to use strategies to generate correct responses during practice and were provided immediate feedback. Further, *Super Solvers* incorporated systematic, cumulative review throughout all lesson components. The base intervention portion of each lesson included four activities consistent across conditions: Problem Quest, Fraction Action, Math Blast, and Power Practice.

Problem Quest (5 min) provided instruction on basic multiplication facts (1s through 10s) and word problems. For the *Multiplication* topic, students learned strategies for solving the 1s, 2s, 3s, 4s, 5s, 9s, and 10s multiplication facts (e.g., skip counting). For solving the 6s, 7s, and 8s facts, students learned decomposition strategies (i.e., using an easy or known fact to solve a hard fact), and worked toward memorization. *Word-problem* instruction taught three types of word problems (compare, change, splitting) and relied primarily on schema theory (Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004) with which students attend to the structure and narrative of different word-problem types. Students learned to categorize word problems as belonging to a problem type based on their underlying mathematical structure. For each problem type, instruction began with conceptual orientation using concrete representations.

Fraction Action (20 min) included explicit instruction on the measurement interpretation of fractions and understanding fraction magnitudes. Instruction was designed to build upon classroom instruction and students' prior knowledge to develop more sophisticated strategies for evaluating fraction magnitudes. Strong emphasis was placed on comparing, ordering, placing fractions on number lines, and equivalencies. For example, students were taught to differentiate between the number of parts of a fraction (the numerator) and the size of the parts in a fraction (the denominator), and to use benchmarks such as one half when assessing fraction magnitudes. Fraction tiles, fraction circles, and number lines were used to introduce and review fraction concepts across the 39 lessons.

Math Blast (2 min) was a game-like fluency-building activity that alternated each week between multiplication and fraction magnitudes activities. Students used flashcards to work toward building fluency on different types of foundational skills such as comparing two fractions or solving basic multiplication problems. With a two-minute time limit, students took turns solving as many flashcards as possible, trying to beat the Math Blast score earned during the previous lesson.

The last portion of each lesson is *Power Practice* (7 min)*,* during which students independently and individually practiced the skills learned during the previous components of each lesson. Power Practice provided opportunities for students to demonstrate their knowledge on both newly introduced and previously taught content.

Differences Between Intervention and Control

In the parent studies, teachers provided information about their fraction instruction. They reported whether they based instruction primarily on the Common Core State Standards (n = 49), the district's mathematics curriculum: *enVisionMATH* (Scott Foresman-Addison Wesley, 2011; n = 0), or a combination of both (n = 8). In terms of content, there were three major distinctions between classroom and Super Solvers fraction instruction. First, classroom instruction focused primarily on part-whole understanding of fractions, whereas Super Solvers emphasized the measurement interpretation of fractions. For example, compared to the classroom fraction instruction, Super Solvers instruction placed greater emphasis on comparing fractions to a benchmark fraction (e.g., one half) and understanding the numerator-denominator relationship. Second, the control group did not restrict the range of fractions taught, whereas the intervention conditions limited the pool of denominators to 2, 3, 4, 5, 6, 8, 10, and 12. Third, for word problems, classroom instruction focused more on operational procedures and key words while Super Solvers focused more on identifying problem types and using schema-based instruction to reach a correct solution. The amount of total mathematics instructional time was similar for intervention and control students.

CHAPTER 3

DATA ANALYTIC STRATEGY AND RESULTS

Demographic Comparability of Adequate and Inadequate Response Groups

Chi-square tests were run to detect significant differences between adequate and inadequate responders by gender, ethnicity, subsidized lunch, or English-language learner status (see Table 2). Within the Calculations Sample, only the relation between responder grouping and English-language learner status was significant, χ^2 (1, N = 124) = 8.64, p = .003, where students for whom English was a second language were significantly more likely to be designated as an inadequate responder. In the Ordering Sample, students designated inadequate responders were more likely to receive subsidized lunch than were adequate responders, χ^2 (1, N = 120) = 4.17, p = .041. Within the Word Problems Sample, no differences between responder groups were significant.

Table 2

Chi-Square Values by Responder Group by Analytic Sample

	Calculations Sample						Ordering Sample						Word Problems Sample					
	Adequate Responders (*n* = 96)		Inadequate Responders (*n* = 28)				Adequate Responders (*n* = 109)		Inadequate Responders (*n* = 11)				Adequate Responders (*n* = 89)		Inadequate Responders (*n* = 31)			
Demographic Factor	*n*	%	*n*	%	$\chi^2(df)$	*p*	*n*	%	*n*	%	$\chi^2(df)$	*p*	*n*	%	*n*	%	$\chi^2(df)$	*p*
Gender					0.47(1)	0.494					0.001(1)	0.975					0.19(1)	0.660
Female	55	57	14	50			60	55	6	55			50	56	16	52		
Male	41	43	14	50			49	45	5	45			39	44	15	48		
Race					1.84(4)	0.764					3.77(4)	0.438					1.96(4)	0.744
Black	50	52	13	46			55	50	6	55			43	64	18	58		
Hispanic	23	24	9	32			29	27	1	9			24	27	6	19		
White	19	20	4	14			21	19	2	18			18	20	5	19		
Asian	2	2	1	4			2	2	1	9			2	2	5	1		
Other	2	2	0	0			2	2	0	0			2	2	0	0		
Subsidized Lunch	64	67	19	68	0.001(1)	0.973	74	68	10	91	4.17(1)	0.041	62	70	22	79	0.70(1)	0.403
English Language Learner	12	13	10	36	8.639(1)	0.003	20	18	1	9	0.44(1)	0.507	16	18	5	16	0.03(1)	0.871
Special Education	10	10	3	11	0.01(1)	0.917	12	11	1	9	0.01(1)	0.922	7	8	6	19	3.40(1)	0.065

Note. Chi-square values are reported as Pearson's chi-square.

Cognitive Processes Associated with Response

To examine the cognitive processes associated with responsiveness, I conducted two types of analyses for each of the three fractions outcome samples. The first was to examine between-group differences that test mean-level differences between adequate versus inadequate responder groups. The second analytic procedure was within-group profile analysis, in which the shape of the cognitive profiles for each responder groups for each fractions outcome sample was examined for patterns of cognitive strengths and weaknesses.

Between-group mean differences.

To find whether the performances of the adequate responder groups differed significantly from the performances of the inadequate responder groups, I examined the between-group mean differences separately for each fractions outcome. In Table 3, I provide the means and standard deviations of the raw scores and *z*-scores for the adequate and inadequate responder groups. The following formula was used to calculate *z*-scores: $z = \frac{X-\mu}{\sigma}$. Then I provide the *F* values and effect sizes calculated on the sample-based *z*-scores.

To do this, I tested whether the performance of the adequate responder group was significantly different from the performance of the inadequate responder group. I did this for each cognitive process, separately for each fractions outcome sample. As shown in Table 3, significant differences between adequate and inadequate responders were found primarily in the Calculations Sample, where reasoning, vocabulary, counting recall, and behavioral attention differed significantly between responder groups with inadequate responders scoring significantly lower than adequate responder on each measure. No significant differences were found between groups in the Ordering Sample. In the Word Problems Sample, inadequate responders scored significantly lower on behavioral attention, only.

Table 3

Means, Standard Deviations, F-Values, and Effect Sizes of Raw Scores and Sample-Based z-Scores of Cognitive Process Performance by Responder Group by Analytic Sample

	Calculations Sample						Ordering Sample						Word Problems Sample					
	Adequate Responders ($n = 96$)		Inadequate Responders ($n = 28$)				Adequate Responders ($n = 109$)		Inadequate Responders ($n = 11$)				Adequate Responders ($n = 89$)		Inadequate Responders ($n = 31$)			
Cognitive Process	Raw Score *M(SD)*	*z*-Score *M(SD)*	Raw Score *M(SD)*	*z*-Score *M(SD)*	*F*	*ES*	Raw Score *M(SD)*	*z*-Score *M(SD)*	Raw Score *M(SD)*	*z*-Score *M(SD)*	*F*	*ES*	Raw Score *M(SD)*	*z*-Score *M(SD)*	Raw Score *M(SD)*	*z*-Score *M(SD)*	*F*	*ES*
Reasoning	10.2 (3.7)	0.10 (1.08)	8.7 (1.8)	-0.35 (0.54)	9.69	-0.53^*	9.9 (3.4)	0.04 (1.01)	8.5 (2.5)	-0.41 (0.77)	1.89	-0.50	9.9 (3.3)	0.16 (0.98)	9.6 (3.5)	-0.05 (1.06)	0.37	-0.21
Processing Speed	12.8 (2.8)	0.09 (0.95)	11.6 (3.3)	-0.30 (1.13)	0.32	-0.37	12.5 (2.9)	0.02 (0.99)	11.7 (3.2)	-0.24 (1.08)	0.01	-0.25	12.7 (2.6)	0.09 (0.89)	11.7 (3.7)	-0.25 (1.25)	2.70	-0.31
Vocabulary	19.2 (4.9)	0.12 (1.03)	16.8 (3.8)	-0.39 (0.79)	2.35	-0.56^*	18.7 (4.9)	0.03 (1.02)	17.2 (3.4)	-0.29 (0.71)	2.27	-0.36	18.9 (4.7)	0.08 (0.99)	17.5 (4.8)	-0.22 (1.01)	0.21	-0.30
Listening Recall	10.3 (3.8)	0.09 (0.95)	8.8 (4.5)	-0.30 (1.12)	0.87	-0.38	9.8 (4.0)	-0.01 (1.01)	10.0 (3.9)	0.05 (0.96)	0.58	-0.06	10.0 (3.5)	0.07 (0.86)	9.0 (5.2)	-0.20 (1.31)	6.67	-0.24
Counting Recall	15.3 (4.4)	0.11 (1.00)	13.0 (4.1)	-0.39 (0.92)	0.66	-0.52^*	14.6 (4.4)	0.002 (1.01)	14.5 (3.9)	-0.14 (0.90)	0.22	-0.15	14.8 (3.9)	0.05 (0.91)	14.0 (5.4)	-0.13 (1.23)	3.79	-0.17
Behavioral Attention	34.7 (10.3)	0.17 (0.98)	26.8 (9.1)	-0.58 (0.87)	0.19	-0.81^*	33.0 (10.6)	0.04 (1.01)	28.6 (9.2)	-0.38 (0.87)	0.01	-0.45	33.9 (9.6)	0.12 (0.91)	28.9 (12.4)	-0.35 (1.18)	2.91	-0.45^*

Note. Reasoning is-WASI Matrix Reasoning. Processing Speed is from the Woodcock-Johnson III. Listening Recall and Counting Recall are from the Working Memory Test Battery for Children. Behavioral Attention is from the Strength and Weaknesses of ADHD-Symptoms and Normal-Behavior. Effect sizes are reported as Cohen's *d*. Significant effects are denoted with an asterisk ($p < 0.05$)

Within-group cognitive profile analysis. Conducting a between-group analysis of mean differences is the more common approach for examining distinctions between responder groups in comparison to the cognitive profile analysis. It is, however, a less stringent method due to the elevated performance across cognitive processing variables for the adequate responder group. Profile analysis, however, removes this elevation.

I assessed whether the cognitive profile of adequate responders versus inadequate responders had a distinctive shape on the six cognitive process measures at the beginning of third grade. The three outcome samples and six cognitive process measures were used to conduct a series of multivariate profile analyses to determine whether the responder groups in each sample could be differentiated.

Profile analysis compares patterns of group test performances by separating differences among groups and differences among measures *within* groups into three independent dimensions that represent *elevation*, *flatness*, and *shape*. This separation then allows for the comparison of performance patterns in two or more groups. The *elevation* effect represents between-group differences averaged across all cognitive measures. *Flatness* effects represent between-measures differences averaged across responder groups. Flatness is minimized in this study given that each measure was *z*-standardized against the full sample. The primary focus of this profile analysis, however, is neither elevation nor flatness but rather *shape*, which is discussed in detail below. For further information on methods and applications of profile analysis, see Bernstein, Garbin, and Teng (1988), Fletcher et al. (1994), and Francis, Espy, Rourke, and Fletcher (1991).

I conducted the profile analysis in four steps. First, because profile analysis requires comparability of scaling, I transformed data from each cognitive process onto the same scale by calculating *z*-scores across the participants separately for each of the three fractions outcome samples. Second, for each responsiveness group within each sample, I calculated the *elevation* of the group's cognitive performance by deriving the grand mean across the *z*-scores for the six cognitive process measures. As expected, for each type of fractions knowledge, the grand mean was higher for the adequate responder group than for the inadequate responder group.

Shape shows whether the pattern of performance of the responder groups differs across the cognitive process variables. Specifically, I want to explore which cognitive process performances differ within the inadequate responder group. If the profiles of each group differ in their shape, this suggests that the group differences vary over the measures. To isolate the shape effect for each of the two responder groups for each of the three fractions outcome samples, my third step was to remove the elevation effect by subtracting the grand mean from the mean performance level of each of the six means, thereby reducing the mean elevation of each group's residual score to zero. As a result, any variation among group means on the resulting residual scores for the cognitive process variables in entirely the result of the shape effect (i.e., the responder status by cognitive process variable interaction profile). Therefore, the plotted cognitive profiles reveal the variability for each group.

Figure 1 shows the cognitive profiles resulting from these analyses for each fractions knowledge outcome. The flatness effects of the adequate responder groups for each sample, although minimized, when compared to the inadequate responder suggests that the adequate responders performed more similarly across the cognitive process measures than did the inadequate responders. The shape effects for each sample reveal greater variability in the cognitive profiles for the inadequate responder groups.

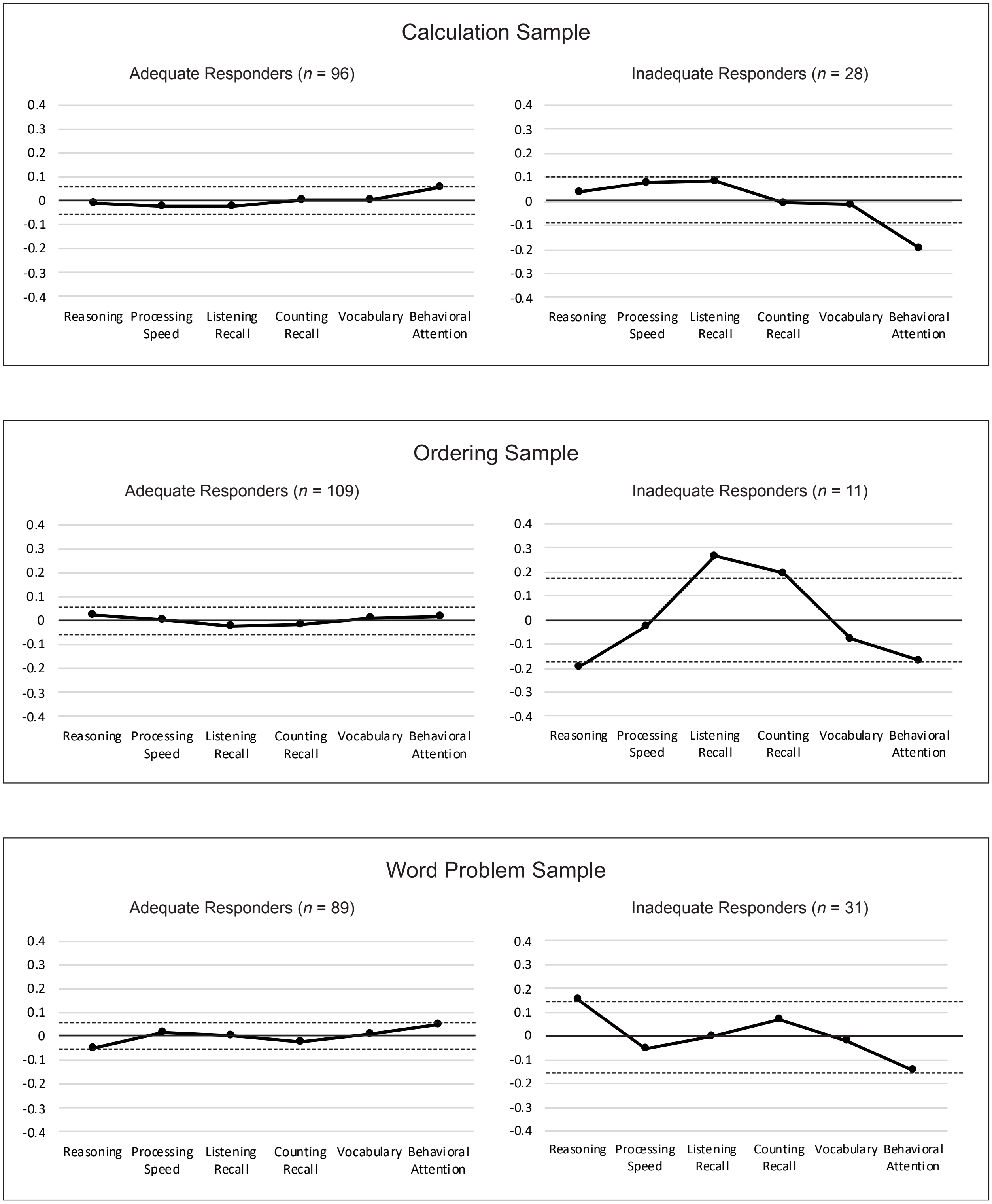

Figure 1. Cognitive profiles by responsiveness to fractions intervention.

Finally, for the adequate and inadequate responder groups separately for each fractions outcome sample, I identified which cognitive processes were more than one standard error of measurement above or below each group's grand mean. For the adequate responder groups, none of the mean values was discrepant from the group's grand mean in any of the three samples. In the Calculations Sample, inadequate responder performance was more than one standard error of measurement *below* the group's grand mean on behavioral attention. This indicates that inadequate responders were low relative to their other cognitive abilities on behavioral attention. In the Ordering Sample, inadequate responders were low on reasoning and high on listening recall and counting recall relative to their other cognitive abilities. For the Word Problems Sample, inadequate responders were high relative to their other cognitive abilities on reasoning.

CHAPTER 4

DISCUSSION

In the present study, I explored differences in demographic variables and cognitive processes for at-risk third graders who were adequately versus inadequately responsive to generally effective fractions intervention. To designate responsiveness, the cut-point was the control-group mean on posttest performance on each relevant measure. I extended prior work in this area by focusing on a lower grade level (third grade), by considering an additional type of fractions knowledge (ordering of fractions), and by examining the role of vocabulary.

I began by exploring the extent to which demographic factors were associated with responsiveness to generally effective fractions intervention. The percentage of students designated as inadequate responders for each analytic sample was 23 for the Calculations Sample, 9 for the Ordering Sample, and 26 for the Word Problems Sample. With the exception of English-language learner status, which was associated with inadequacy of responsiveness on the calculations outcome, and subsidized lunch, which was associated with inadequacy of responsiveness on the ordering outcome, demographic patterns were similar across samples.

Corroborating prior work (Krowka & Fuchs, 2017), there was no association found between adequate and inadequate responders for any fractions outcome sample for gender or race. However, we did find that responsiveness was associated with special education. So the lack of association for special education status with inadequacy of response in the present study was unexpected. This would be anticipated for students struggling to make improvement in mathematics skills. The present study's divergent finding may be the result of grade-level differences in identification of students for special education services. That is, fewer students are identified for special education at third than at fourth grade (National Center for Learning Disabilities, 2017).

In terms of cognitive processes, I conducted two types of analyses: an analysis of between-group mean differences and a within-group cognitive profile analysis. Unlike the *between*-group means comparison, which explores mean-level differences between adequate and inadequate responder groups, the *within*-group profile analysis reveals significantly low (or high) performance on cognitive processes relative to each groups' own performance on the other cognitive processes. Whereas the between-group means comparison is less stringent due to the elevated performance for the adequate responder groups across the cognitive process variables, the within-group profile analysis is a more conservative approach due to the removal of this main effect of elevation. It is typically the case therefore that the more conservative within-group analyses reveal fewer differences than univariate tests of differences between adequate and inadequate responders However, the within-group analyses may be more meaningful (see for example Fuchs, Fuchs, Steubing et al., 2008).

As expected in light of Krowka and Fuchs (2017), with respect to cognitive processes, results of the between-group differences approach provide support for the hypothesis that significant differences exist between inadequate and adequate responder groups. First, results suggest that behavioral attention plays a role in responsiveness to fractions intervention for both calculations and word-problem learning. Inadequate responders scored significantly lower than adequate responders on the measure of behavioral attention with an effect size of 0.81 for the calculations outcome and 0.45 for the word-problem outcome. Behavioral attention during math classroom instruction has been identified in prior work as a robust predictor of growth in both fraction calculations and fraction word-problem solving (Fuchs, Fuchs, & Compton, 2013; Hecht et al., 2003; Hecht & Vagi, 2010; Namkung & Fuchs, 2016). Further, students' tendency to engage in on-task attentional behavior has been associated with measures of general academic achievement (e.g., Mckinney & Speece, 1986; Hecht & Greenfield, 2001). Presumably, students who are better equipped to engage in more attentive behavior in the classroom are in a better position to benefit from instruction and intervention and are in turn better able to acquire and use conceptual and procedural understandings about fractions to solve fraction calculations and fraction word problems than students with less attentive behavior. In this way, behavioral attention seems transparently involved in supporting students' calculations and word-problem learning.

Conversely, students with weaker mathematics skills may be more likely to demonstrate behavioral inattentiveness in the classroom. For example, a student with mathematics learning difficulties who is unable to follow fractions instruction may become more distractible in the math classroom. In this study, behavioral attention was operationalized with a nine-item teacher-rating scale of inattention, with which teachers rate students from 1-7 based on their observations of student attentiveness. The scale includes items such as how well a student "sustains attention on tasks or play activities," "ignores extraneous stimuli," and "modulates verbal activity (controls excess talking)." Inattentiveness on these types of behaviors may be a student's response to experiencing learning difficulties. Instruction that fails to address the needs of students with mathematics learning difficulties may contribute to the behavioral inattentiveness teachers observe.

Specifically in terms of calculations, one potential explanation for the difference in behavioral attention between adequate and inadequate responder groups may be the number and complexity of procedures students are expected to learn and keep track of to successfully execute fraction calculations. For example, when adding or subtracting fractions, students must make sure the fractions have the same denominators, but this is contra-indicated when multiplying or dividing fractions. Calculations procedures require the ability to keep track of multiple numbers and steps that differ between problem types, thus requiring considerable attention both when learning of the procedures and when independently solving fraction calculations problems. Students with low behavioral attention may struggle more than typical students when attempting to learn and employ these overlapping and confusable procedures. Thus, when carrying out fraction calculations procedures, less attentive students may make more errors than their more attentive peers.

Results identifying between-group differences in behavioral attention on the word-problem outcome, in which inadequate responders scored significantly lower than adequate responders, were consistent with findings from fourth-grade students (Krowka & Fuchs, 2017). This finding in the present study therefore provides an important extension of this research to third-grade children. One explanation may be that during administration of the word-problems outcome, examiners read each problem aloud while students followed along, then allowed students time to complete the problem. Ability to sustain attention is likely required during a task of this nature to actively listen to the problem being read aloud and then to solve each word problem. A student with deficits in behavioral attention may also have difficulty inhibiting impulsivity during learning or assessment activities.

It is however unclear why behavioral attention failed to distinguish between adequate and inadequate responders for the Ordering Sample. It may also be the result of power issues given the small number of inadequate responders (*n* = 11) in the Ordering Sample: The effect size difference between adequate and inadequate responders for this sample was 0.45. In contrast to the present findings, Krowka and Fuchs (2017) found behavioral attention to be associated with word problem difficulty, and *not* calculations. In Krowka and Fuchs, the lack of significance was also attributed to power issues but for the Calculations Sample (a small number of inadequate responders; $n = 19$), a concern not present for the Calculations Sample in the present study.

With respect to the calculations outcome, between-group differences implicated three cognitive processes in addition to behavioral attention. They also scored significantly lower than adequate responders on reasoning, vocabulary, and counting recall. Corroborating evidence across the present findings and Krowka and Fuchs (2017) underscores the salience of reasoning for responsiveness to fraction calculations intervention. In the present study, reasoning was measured using WASI (Wechsler, 2011) Matrix Reasoning, which assesses the ability to apply fluid reasoning to analyze novel problems and to identify patterns and relationships. This task may map onto skills needed to solve fraction calculations such as the ability to identify the relationships between fractions in a calculations problem or between the numerator and denominator in a single fraction. For example, when solving the problem ½ + ¾ =, a student must identify and interpret the relationship between the denominators “2” and “4” to make decisions about whether to find a common denominator. Deficits in the aspects of reasoning ability tapped by Matrix Reasoning may make certain procedural steps such as this difficult. Identification of patterns, for which students likely rely on relational knowledge, is recognized as a central component of early mathematics understanding (National Council of Teachers of Mathematics, 2000). Embedding training of patterning and relational knowledge in fractions intervention may improve outcomes for students with reasoning deficits.

At the same time, vocabulary was associated with response to intervention on the calculations outcome. This corroborates prior work showing language ability (e.g., vocabulary, listening comprehension) supports fraction calculations (e.g., Fuchs, Schumacher, et al., 2013; Krowka & Fuchs, 2017; Namkung & Fuchs, 2016; Seethaler et al., 2011). Language is thus emerging as a key ability for fraction calculations and lends further support to the idea that language systems are involved in learning mathematics (e.g., Miura & Okamoto, 2003; Spelke & Tsivkin, 2001). Related prior work has demonstrated that aspects of general fractions understanding (i.e., fraction identification) may be influenced by the language used to refer to the part-whole relationship (e.g., Miura et al., 1999; Paik & Mix, 2003). Thus language ability appears to facilitate deeper learning of fractions concepts, and future fractions intervention might productively be designed to reduce demands on language or remediate language deficits.

Inadequate responders on the calculations outcome also scored significantly lower on working memory than adequate responders, specifically on counting recall but not on listening recall. This finding corroborates the literature identifying aspects of working memory as predictive of fraction calculations (e.g., Hecht, Close, & Santisi, 2003; Krowka & Fuchs, 2017; Siegler & Pyke, 2013) and a smaller literature that has more frequently associated numerical measures of working memory than non-numerical measures (Passolunghi & Cornoldi, 2008; Passolunghi & Siegel, 2001, 2004), with the exception of word problems (e.g., Fuchs et al., 2010, 2016). During the counting recall task, the student determines how many objects are in an array by counting the number of dots on a page and then recalls the series of counts. The ability to maintain numeric information, while also processing new information, is necessary for successful performance of fraction calculations. In this way, counting recall's role in responsiveness for the calculations outcome (effect size of 0.52) was expected.

Given that the between-group difference analysis revealed four distinctive cognitive processes for the Calculations Sample, it is interesting to consider the more conservative results of the second analytic procedure, the within-group profile analysis. Based on Krowka and Fuchs (2017), I had hypothesized that inadequate responders across all three samples would manifest a distinctive profile characterized by specific deficits in behavioral attention relative to this group's own performance on the other five cognitive measures. By contrast, while behavioral attention was a relative weakness for inadequate responders in all three outcome samples, it is only within the Calculations Sample that their performance on behavioral attention fell more than one standard error of measurement below the mean. In other words, unlike the findings of the between-group differences analysis which revealed several distinctive cognitive processes for the inadequate responder group, from the within-group profile analysis only behavioral attention emerged as a key limitation for students who responded inadequately to the generally effective fractions intervention only on the calculations outcome. It is not surprising that the more stringent profile analyses' focus on within-group shape identified fewer cognitive weaknesses compared to the less conservative, between-group differences approach.

In terms of the ordering outcome, the percentage of students who met the benchmark for inadequate response in the Ordering Sample (9% or 11 of 120 students) was small in comparison to the Calculations (23% or 28 of 124 students) and Word-Problem Samples (26% or 31 of 120 students). It is important to note this group size makes sense because it closely reflects the special education population within U.S. public schools (approximately 13%; Kena et al., 2014). Likely due to this small group size, however, the between-group analytic method revealed no significant differences between adequate and inadequate responder groups. This may be a power issue, as already discussed, and additional research with a larger sample size may yield a larger inadequate responder group. Alternatively, the lack of differences may indicate that the skills necessary to respond adequately to intervention on fraction calculations are not tapped by the cognitive processes assessed in the present study. Further still, because the intervention's major focus was fraction magnitude, it is possible that the intervention effects effectively compensated for participants' cognitive limitations. Future work should continue to explore more varied cognitive domains.

On the other hand, the within-group profile analysis of the Ordering Sample, revealed distinctly low reasoning ability relative to inadequate responder group's own performance on the other five cognitive processes, even as they manifested distinctly high listening recall and counting recall. These results mirror those found by Krowka and Fuchs (2017) whose cognitive profile analysis revealed reasoning and counting recall to be a relative weakness and strength, respectively, for a sample of students who responded inadequately on the similar number line estimation outcome. These findings support the arguments put forth by Krowka and Fuchs suggesting that determining fraction magnitudes requires the use of visual and relational reasoning and that the subset of students with this combination of relative strength and weakness may have unsuccessfully relied on strengths in working memory to place fractions in order from least to greatest. Thus, it is expected that inadequate responders in this sample demonstrated low reasoning ability and identifying ways to extend intervention to address these students' limitations in reasoning ability remains crucial. What is unexpected, though, is that the more rigorous within-group profile analysis would reveal more distinctive cognitive processes than the between-groups analysis. One explanation may be increased variability due to the small group size for the inadequate responders. Future work in this area should aim to conduct similar analyses with a larger sample.

A surprising finding for the word-problems outcome was that the between-group differences analysis did not reveal reasoning as associated with response to intervention. Behavioral attention, as already discussed, was the only cognitive process identified as significantly different between adequate and inadequate responders for this sample. Finding of a lack of association with reasoning diverges from previous work (Fuchs, Malone, et al., 2016; Krowka & Fuchs, 2017). One explanation for this difference may be the variation in reasoning skills by age. Fuchs, Malone, et al. (2016) and Krowka and Fuchs (2017) examined fourth- and fifth-grade students, respectively, while the present study focused on third graders. Considering the rapid development of reasoning as young children age (McArdle, Ferrer-Caja, Hamagami, & Woodcock, 2002), third-grade students' reasoning abilities may differ considerably from and provide less variance than at fourth and fifth grade.

Even more surprising finding for the word-problems outcome was that the within-group profile analysis revealed that inadequate response was associated with *strong* performance on the reasoning task. That is, unlike the between-group differences analysis which identified only behavior as associated with inadequate response, profile analysis identified reasoning as a key *strength* for inadequate responders in the Word Problem Samples relative to the group's own performance on the other cognitive processes. This is further unexpected because prior work (Fuchs, Malone et al., 2016; Krowka & Fuchs, 2017) provides the basis for expecting poor performance on fraction word problems as associated with *low* reasoning ability. Research is needed to explore other ways in which relative strength in reasoning affects responsiveness to generally effective fractions intervention on word-problem outcomes. Taken together, these findings underscore the need to further explore the role of reasoning ability in the learning of word problems.

It is important to consider the findings of this study in the context of its limitations. First, as in Krowka and Fuchs (2017), we identified adequate and inadequate responder groups by dichotomizing participants' responsiveness using a predetermined cut point. Measures of response used within this study provide continuous variables for which dichotomization using a seemingly arbitrary cut point is not recommended. Such methods of dichotomization can provide misleading results (e.g., MacCallum, Zhang, Preacher, & Rucker, 2002) by masking potentially notable differences present in the data.

On the other hand, it is a universal, necessary practice within schools to designate students dichotomously as either in need of intervention or not in need of intervention. Dichotomization is also necessary to designate students as adequate or inadequate responders in formulate decisions about movement between tiers of intervention within the response-to-intervention framework. Even so, further investigation is required to identify alternative strategies for identifying responsiveness that are based on the continuous nature of performance data.

A second limitation of the present study is that different findings may emerge if a wider range of outcome measures were used to identify analytic samples. The present study relied on a single, researcher-created measure for each sample to represent students' performance on each type of fractions knowledge. Future research should incorporate multiple measure to represent student performance. A third and related limitation concerns the use of a single measure for identifying the at-risk samples used in the present analyses. Different findings may also emerge if multiple screening measures were used to determine risk status within the parent studies, a consideration for future work.

Finally, as already discussed, the potential issue of power is present for the ordering outcome given the small number of inadequate responders in the sample ($n = 11$), which may have masked nominal effects. In light of these limitations, additional research is warranted on the efficacy of fractions intervention (and mathematics intervention in general) for third-grade students with specific patterns of cognitive strengths and weaknesses. Findings have implications for design and testing of interventions that take into consideration certain cognitive processes.

In sum, the combined results of the two analytic approaches underscore the importance of behavioral attention, reasoning, and aspects of working memory in responsiveness to fractions intervention. As a broad direction, future research should employ moderation analysis and include other cognitive factors that may uniquely influence adequacy of responsiveness to generally efficacious fractions intervention. Identifying cognitive sources of variability in responsiveness to intervention remains important for identifying strategies for expanding the efficacy of fractions intervention, even as it may contribute to the development of a comprehensive theory of fraction competence.

REFERENCES

American Psychiatric Association. (2000). *Diagnostic and statistical manual of mental disorders (revised 4th ed.)*. Washington, DC: Author. https://doi.org/10.1176/appi.books.9780890423349

Badian, N. A. (1999). Persistent arithmetic, reading, or arithmetic and reading disability. *Annals of Dyslexia*, *49*(1), 43. https://doi.org/10.1007/s11881-999-0019-8

Bailey, D., Hoard, M., Nugent, L., & Geary, D. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, *113*(3), 447–455. https://doi.org/10.1016/j.jecp.2012.06.004

Bailey, D., Siegler, R., & Geary, D. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, *17*(5), 775–785. https://doi.org/10.1111/desc.12155

Berch, D., & Mazzocco, M. (2007). *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities.* Baltimore: Brookes Publishing. https://doi.org/10.1097/dbp.0b013e31817aefe8

Bernstein, I., Garbin, C., & Teng, G. (1988). *Applied multivariate analysis.* New York, NY: Springer-Verlag. https://dx.doi.org/10.1007/978-1-4613-8740-4.

Bright, G., Behr, M., Post, T., & Wachsmuth, I. (1988). Identifying fractions on number lines. *Journal for Research in Mathematics Education*, 215–232. https://doi.org/10.2307/749066

Bryan, T. (1974). An observational analysis of classroom behaviors of children with learning disabilities. *Journal of Learning Disabilities*, *7*(1), 26-34. https://doi.org/10.1177/002221947400700106

Bull, R., & Johnston, R. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology*, *65*(1), 1-24. https://doi.org/10.1006/jecp.1996.2358

Bynner, J., & Parsons, S. (1997). *Does numeracy matter? Evidence from the National Child Development Study on the impact of poor numeracy on adult life*. London, England: The Basic Skills Agency.

Case, R. (1985). *Intellectual development: Birth to adulthood.* San Diego, CA: Academic Press. https://doi.org/10.1111/j.2044-8279.1986.tb02666.x

Cramer, K., Post, T., & delMas, R. (2002). Initial fraction learning by fourth- and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education*, *33*(2), 111–144. https://doi.org/10.2307/749646

Empson, S., & Levi, L. (2011). Extending children’s mathematics: Fractions and decimals. *Mathematics Education*, *27*(4), 403-434.

Fletcher, J., Shaywitz, S., Shankweiler, D., Katz, L., Liberman, I., Stuebing, K., … Shaywitz, B. (1994). Cognitive profiles of reading disability: Comparisons of discrepancy and low achievement definitions. *Journal of Educational Psychology*, *86*(1), 6. https://dx.doi.org/10.1037/0022-0663.86.1.6

Fletcher, J., Shaywitz, S., & Shaywitz, B. (1999). Comorbidity of learning and attention disorders: Separate but equal. *Pediatric Clinics of North America*, *46*(5), 885–897.

Francis, D., Espy, K., Rourke, B., & Fletcher, J. (1991). Validity of intelligence test scores in the definition of learning disability: A critical analysis. *Developmental Cognitive Neuroscience Laboratory-Faculty and Staff Publications*, 32.

Fuchs, D., Compton, D., Fuchs, L., Bryant, J., & Davis, G. (2008). Making "secondary intervention" work in a three-tier responsiveness-to-intervention model: Findings from the first-grade longitudinal reading study of the National Research Center on Learning Disabilities. *Reading and Writing; Dordrecht*, *21*(4), 413–436. https://doi.org/10.1007/s11145-007-9083-9

Fuchs, D., & Fuchs, L. (2019). On the importance of moderator analysis in intervention research: An introduction to the special issue. *Exceptional Children, 85*(2), 126-128. https://doi.org/10.1177/0014402918811924

Fuchs, L., Fuchs, D., & Compton, D. (2013). Intervention effects for students with comorbid forms of learning disability: Understanding the needs of nonresponders. *Journal of Learning Disabilities*, *46*(6). https://doi.org/10.1177/0022219412468889

Fuchs, L., Fuchs, D., Finelli, R., Courey, S., & Hamlett, C. (2004). Expanding schema-based transfer instruction to help third graders solve real-life mathematical problems. *American Educational Research Journal*, *41*(2), 419–445. https://doi.org/10.3102/00028312041002419

Fuchs, L., Fuchs, D., & Gilbert, J. (2019). Does the severity of students' pre-intervention math deficits affect responsiveness to generally effective first-grade intervention?. *Exceptional Children*, *85*(2), 147–162. https://doi.org/10.1177/0014402918782628

Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition?. *Journal of Educational Psychology*, *100*(1), 30. https://doi.org/10.1037/0022-0663.100.1.30

Fuchs, L., Geary, D., Compton, D., Fuchs, D., Hamlett, C., Seethaler, … Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology*, *46*(6), 1731. https://doi.org/10.1037/a0020662

Fuchs, L., Geary, D., Compton, D., Fuchs, D., Schatschneider, C., Hamlett, C., … Craddock, C. (2013). Effects of first-grade number knowledge tutoring with contrasting forms of practice. *Journal of Educational Psychology*, *105*(1), 58. https://doi.org/10.1037/a0030127

Fuchs, L., Hamlett, C. L., & Powell, S. (2003). *Second-Grade Calculations Battery*. Available from L.S. Fuchs, 228 Peabody, Vanderbilt University, Nashville, TN 37203.

Fuchs, L., Malone, A., Schumacher, R., Namkung, J., & Wang, A. (2017). Fraction intervention for students with mathematics difficulties: Lessons learned from five randomized controlled trials. *Journal of Learning Disabilities*, *50*(6), 631-639. https://doi.org/10.1177/0022219416677249

Fuchs, L., Malone, A., Wang, A., Abramson, R., & Krowka, S. (2016). *Super Solvers!* Available from L.S. Fuchs, 228 Peabody, Vanderbilt University, Nashville, TN 37203.

Fuchs, L., Schumacher, R., Long, J., Namkung, J., Hamlett, C., Cirino, P., … Changas, P. (2013). Improving at-risk learners' understanding of fractions. *Journal of Educational Psychology*, *105*(3), 683–700. https://doi.org/10.1037/a0032446

Fuchs, L., Schumacher, R., Long, J., Namkung, J., Malone, A., Wang, A., et al. (2016). Effects of intervention to improve at-risk fourth graders' understanding, calculations, and word problems with fractions. *The Elementary School Journal, 116*(4), 625–651. https://doi.org/10.1086/686303

Fuchs, L., Sterba, S., Fuchs, D., & Malone, A. (2016). Does evidence-based fractions intervention address the needs of very low-performing students? *Journal of Research on*

Educational Effectiveness, *9*(4), 662–677. https://doi.org/10.1080/19345747.2015.1123336

Gabriel, F., Coché, F., Szucs, D., Carette, V., Rey, B., & Content, A. (2013). A componential view of children's difficulties in learning fractions. *Frontiers in Psychology*, *4*(715), 1–12. https://doi.org/10.3389/fpsyg.2013.00715

Geary, D. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, *37*, 4–15. https://doi.org/10.1177/00222194040370010201

Geary, D., Hoard, M., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child development*, *78*(4), 1343-1359. https://doi.org/10.1111/j.1467-8624.2007.01069.x

Gross, J., Hudson, C., & Price, D. (2009). *The long term costs of numeracy difficulties. Every child a chance trust and KPMG*. East Sussex, UK: National Numeracy.

Gross-Tsur, V., Manor, O., & Shalev, R. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine & Child Neurology*, *38*(1), 25–33. https://doi.org/10.1111/j.1469-8749.1996.tb15029.x

Hansen, N., Jordan, N., & Rodrigues, J. (2017). Identifying learning difficulties with fractions: A longitudinal study of student growth from third through sixth grade. *Contemporary Educational Psychology*, *50*, 45–59. https://doi.org/10.1016/j.cedpsych.2015.11.002

Hawkins, E., Stancavage, F., & Dossey, J. (1998). *School policies and practices affecting instruction in mathematics: Findings from the National Assessment of Educational Progress*. Washington, D.C.: National Center for Education Statistics.

Hecht, S. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology*, *90*(3), 545. https://doi.org/10.1037//0022-0663.90.3.545

Hecht, S., Close, L., & Santisi, M. (2003). Sources of individual differences in fraction skills. *Journal of Experimental Child Psychology*, *86*(4), 277–302. https://doi.org/10.1016/j.jecp.2003.08.003

Hecht, S. A., & Greenfield, D. B. (2001). Comparing the predictive validity of first grade teacher ratings and reading-related tests on third grade levels of reading skills in young children exposed to poverty. *School Psychology Review*, *30*(1), 50-50.

Hecht, S., & Vagi, K. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*, *102*(4), 843–859. https://doi.org/10.1037/a0019824

Hiebert, J. (1985). Children's knowledge of common and decimal fractions. *Education and Urban Society*, *17*(4), 427–437. https://doi.org/10.1177/0013124585017004006

Hoffer, T., Venkataraman, L., Hedberg E., Shagle, S. (2007) *Final report on the national survey of algebra teachers for the National Math Panel.* Washington, DC: U.S. Department of Education.

Jitendra, A., Hoff, K., & Beck, M. (1999). Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. *Remedial and Special Education*, *20*(1), 50–64. https://doi.org/10.1177/074193259902000108

Jordan, N., Hansen, N., Fuchs, L., Siegler, R., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology*, *116*(1), 45–58. https://doi.org/10.1016/j.jecp.2013.02.001

Jordan, N., Levine, S., & Huttenlocher, J. (1995). Calculation abilities in young children with different patterns of cognitive functioning. *Journal of Learning Disabilities*, *28*(1), 53–64. https://doi.org/10.1177/002221949502800109

Kena, G., Aud, S., Johnson, F., Wang, X., Zhang, J., Rathbun, A., ... & Kristapovich, P. (2014). The Condition of Education 2014. NCES 2014-083. *National Center for Education Statistics*.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press. https://doi.org/10.17226/9822

Krowka, S., & Fuchs, L. (2017). Cognitive profiles associated with responsiveness to fraction intervention. *Learning Disabilities Research & Practice*, *32*(4), 216–230.

Landerl, K., & Moll, K. (2010). Comorbidity of learning disorders: prevalence and familial transmission. *Journal of Child Psychology and Psychiatry*, *51*(3), 287-294. https://doi.org/10.1111/j.1469-7610.2009.02164.x

Lipkus, I., Samsa, G., & Rimer, B. (2001). General performance on a numeracy scale among highly educated samples. *Medical Decision Making*, *21*(1), 37-44. https://doi.org/10.1177/0272989x0102100105

Lortie-Forgues, H., Tian, J., & Siegler, R. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review*, *38*, 201–221. https://doi.org/10.1016/j.dr.2015.07.008

MacCallum, R., Zhang, S., Preacher, K., & Rucker, D. (2002). On the practice of dichotomization of quantitative variables. *Psychological Methods*, *7*(1), 19. https://doi.org/10.1037//1082-989x.7.1.19

Malone, A., Fuchs, L., & Schumacher, R. (2015). *Fraction battery-2015-revised*. Available from L.S. Fuchs, 228 Peabody, Vanderbilt University, Nashville, TN 37203.

Malone, A., & Fuchs, L. (2017). Error patterns in ordering fractions among at-risk fourth-grade students. *Journal of Learning Disabilities*, *50*(3), 337–352. https://doi.org/10.1177/0022219416629647

Murphy, M., Mazzocco, M., Hanich, L., & Early, M. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities*, *40*(5), 458-478. https://doi.org/10.1177/00222194070400050901

Mazzocco, M., & Myers, G. (2003). Complexities in identifying and defining mathematics learning disability in the primary school-age years. *Annals of Dyslexia*, *53*(1), 218-253. https://doi.org/10.1007/s11881-003-0011-7

McArdle, J., Ferrer-Caja, E., Hamagami, F., & Woodcock, R. (2002). Comparative longitudinal structural analyses of the growth and decline of multiple intellectual abilities over the life span. *Developmental Psychology*, *38*(1), 115.

McKinney, J., & Speece, D. L. (1986). Academic consequences and longitudinal stability of behavioral subtypes of learning disabled children. *Journal of Educational Psychology*, *78*(5), 365. https://doi.org/10.1037/0022-0663.78.5.369

Miura, I., & Okamoto, Y. (2003). Language supports for mathematics understanding and performance. In A. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 229–242). Mahwah, NJ: Erlbaum.

Miura, I., Okamoto, Y., Vlahovic-Stetic, V., Kim, C., & Han, J. (1999). Language supports for children's understanding of numerical fractions: Cross-national comparisons. *Journal of Experimental Child Psychology*, *74*(4), 356-365. https://doi.org/10.1006/jecp.1999.2519

Montague, M., Enders, C., & Dietz, S. (2011). Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. *Learning Disability Quarterly*, *34*(4), 262–272. https://doi.org/10.1177/0731948711421762

Murnane, R., Willett, J., Braatz, M., & Duhaldeborde, Y. (2001). Do different dimensions of male high school students' skills predict labor market success a decade later? Evidence from the NLSY. *Economics of Education Review*, *20*(4), 311–320. https://doi.org/10.1016/S0272-7757(00)00056-X

Murphy, M., Mazzocco, M., Hanich, L., & Early, M. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities*, *40*(5), 458-478. https://doi:10.1177/ 00222194070400050901

Namkung, J., & Fuchs, L. (2016). Cognitive predictors of calculations and number line estimation with whole numbers and fractions among at-risk students. *Journal of Educational Psychology*, *108*(2), 214–228. https://doi.org/10.1037/edu0000055

Namkung, J., Fuchs, L., & Koziol, N. (2018). Does initial learning about the meaning of fractions present similar challenges for students with and without adequate whole-number skill?. *Learning and Individual Differences*, *61*, 151-157. https://doi.org/10.1016/j.lindif.2017.11.018

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematic*s. Reston, VA: Author

National Math Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, D.C.: U.S. Department of Education.

National Center for Education Statistics. (2007). *National Assessment of Educational Progress*. Washington, D.C.: National Center for Education Statistics, Institute of Education Sciences, U.S. Dept. of Education.

Paik, J., & Mix, K. (2003). US and Korean children's comprehension of fraction names: A reexamination of cross–national differences. *Child Development*, *74*(1), 144–154. https://doi.org/10.1111/1467-8624.t01-1-00526

Parmar, R., Cawley, J., & Frazita, R. (1996). Word problem-solving by students with and without mild disabilities. *Exceptional Children*, *62*(5), 415-429. https://doi.org/10.1177/001440299606200503

Passolunghi, M., & Cornoldi, C. (2008). Working memory failures in children with arithmetical difficulties. *Child Neuropsychology*, *14*(5), 387-400. https://doi.org/10.1080/09297040701566662

Passolunghi, M., & Siegel, L. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology*, *80*(1), 44-57. https://doi.org/10.1006/jecp.2000.2626

Passolunghi, M., & Siegel, L. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology*, *88*(4), 348-367. https://doi.org/10.1016/j.jecp.2004.04.002

Peng, P., Wang, C., & Namkung, J. (2018). Understanding the cognition related to mathematics difficulties: A meta-analysis on the cognitive deficit profiles and the bottleneck theory. *Review of Educational Research*, *88*(3), 434-476. https://doi.org/10.3102/0034654317753350

Pickering, S., & Gathercole, S. (2001). *Working memory test battery for children (WMTB-C)*. Psychological Corporation.

Powell, S., Fuchs, L., Cirino, P., Fuchs, D., Compton, D., & Changas, P. (2015). Effects of a multitier support system on calculation, word problem, and prealgebraic performance among at-risk learners. *Exceptional Children*, *81*(4), 443–470. https://doi.org/10.1177/0014402914563702

Reyna, V., & Brainerd, C. (2007). The importance of mathematics in health and human judgment: Numeracy, risk communication, and medical decision making. *Learning and Individual Differences*, *17*(2), 147-159. https://doi.org/10.1016/j.lindif.2007.03.010

Scott Foresman-Addison Wesley. (2011). *enVisionMATH*. San Antonio: Pearson.

Seethaler, P., Fuchs, L., Star, J., & Bryant, J. (2011). The cognitive predictors of computational skill with whole versus rational numbers: An exploratory study. *Learning and Individual Differences*, *21*(5), 536–542. https://doi.org/10.1016/j.lindif.2011.05.002

Siegler, R., Duncan, G., Davis-Kean, P., Duckworth, K., Claessens, A., Engel, M., … Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*(7), 691–697. https://doi.org/10.1177/0956797612440101

Siegler, R., & Pyke, A. (2013). Developmental and Individual Differences in Understanding of Fractions. *Developmental Psychology*, *49*(10), 1994–2004. https://doi.org/10.1037/a0031200

Siegler, R., Thompson, C., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Spelke, E., & Tsivkin, S. (2001). Language and number: A bilingual training study. *Cognition*, *78*(1), 45-88. https://doi.org/10.1016/s0010-0277(00)00108-6

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction*, *14*(5), 503–518. https://doi.org/10.1016/j.learninstruc.2004.06.015

Swanson, J., Schuck, S., Mann, M., Carlson, C., Hartman, K., Sergeant, J., … McCleary, R. (2004). *Categorical and dimensional definitions and evaluations of symptoms of ADHD: The SNAP and the SWAN rating scales.* Retrieved from https://www.adhd.net.

Tian, J., & Siegler, R. S. (2017). Which type of rational numbers should students learn first? *Educational Psychology Review, 30*(2), 351-372. https://doi.org/10.1007/s10648-017-9417-3

Wechsler, D. (2011). *Wechsler Abbreviated Scale of Intelligence*. San Antonio, TX: The Psychological Corporation.

Wilkinson, G. S., & Robertson, G. J. (2006). *Wide Range Achievement Test, 4th Edition.* Wilmington, DE: Wide Range.

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock-Johnson tests of achievement*. Itasca, IL: Riverside Publishing.

Zhu, J. (1999). Wechsler abbreviated scale of intelligence manual. *San Antonio, TX: Author*.