PROBABILISTIC DESIGN OF MULTIDISCIPLINARY SYSTEMS

By

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INTRODUCTION

Initiatives in the aerospace industry are continuously presenting engineers with challenges requiring the design, integration, and adaptation of systems to meet a wide variety of future missions. Consider, for example, the 2004 U.S. Vision for Space Exploration, which sets goals for NASA to return to the Moon by 2020 in preparation for follow-on missions to Mars and beyond (NASA 2004). In addition to achieving lofty goals for system performance, government and industry leaders as well as the general public demand the space program take substantial leaps to improve safety and reliability. After 40 years of experience in manned space exploration, there exists a considerable knowledge base to build upon. However, the needed advancements in performance and reliability cannot be realized by making mere incremental changes to existing systems. The reliability track record for the Space Shuttle is a good case in point; the catastrophic failure rate is over 1%, more than anticipated during the shuttle design and much more than that desired for next generation systems. (Prior to the 2004 Vision, NASA's reliability goals for the next generation launch vehicle included a less than 1 in 10,000 probability of crew loss. NASA 2000). It is clear that a more effective means for designing for reliability is needed; program requirements must include reliability standards and design processes must incorporate methods for assessing and engineering to these requirements.

This research proposes methods to this end, namely the probabilistic[1] (i.e., reliability-based) design of systems. In particular, these methods address design problems with three common elements: (1) reliability requirements given in probabilistic terms, (2) a mathematical 'design' formulation that ensures both performance and reliability requirements are met, and (3) some degree of system integration or analysis. The first element indicates that reliability requirements are given as standards for probability of success or failure. Thus, assessing reliability 'probabilistically' necessitates that failure versus success be defined by a distinct boundary. Furthermore, this assumes that a probability density function exists which describes the probability that system performance falls at any given point on either side of that boundary. Although the definition of *design* can be quite broad, this research deals specifically with single-objective optimization problems (i.e., problems having the second element), which can be mathematically formalized to minimize (or maximize) one particular performance attribute while satisfying constraints (requirements) on others. Finally, the aim of this research is to address the probabilistic design of *systems*. The key idea is that the *system* must be a "collection" of some components aimed at a "common objective" (Buede, 2000). This "collection" can take a number of forms, a few of which will be highlighted by example. In general, however, the methods proposed address the *communication* of information (e.g., design information and/or probabilistic information) between the components of a system (i.e., lateral communication), and between components and the overall system (i.e., vertical communication). The following sections present a review of the three elements as they build upon one another. This review is followed by an outline

[1] Although the term "*probabilistic* design" may suggest other connotations to readers in some fields, its use throughout this dissertation is synonymous with "*reliability-based* design" as described in this chapter.

of the proposed probabilistic system design methods and applications included in subsequent chapters.

Reliability Assessment using Probabilistic Analysis

Throughout this dissertation, the term reliability[2] refers to the degree of certainty to which a system will perform successfully. "Success" is defined as performance "as intended" which includes satisfying design requirements as well as avoiding catastrophic failure. Uncertainty in system performance arises from numerous fronts. Sources of uncertainty may be divided into two types: aleatory and epistemic (Oberkampf et. al., 2004). Aleatory uncertainty is irreducible. Examples include phenomena that exhibit natural variation like environmental conditions (temperature, wind speed, etc.). Manufacturing variations due to limited precision in tools and processes also result in this type of uncertainty. In contrast, epistemic uncertainty results from a lack of knowledge about the system, or due to approximations in the system behavior models; it can be reduced as more information about the system is obtained. Epistemic uncertainty is introduced at several levels. First, understanding a system's behavior begins with a physical model based on laws of physics. At this stage, assumptions are made (factors are neglected, ideal properties assumed, etc), introducing uncertainty. The physics-based model is later reduced to a mathematical model which is in turn converted to a computational model (for example, computational algorithms developed to solve partial differential equations.) At each step, model error is introduced which adds to the uncertainty associated with a predicted performance. This epistemic uncertainty could be

[2] *Reliability* is distinguished from *robustness*. Reliability is a measure of probability of success, while robustness is a measure of the (in)variability in performance over a range of conditions.

reduced by increasing the accuracy of the computation algorithm (e.g., reducing step size, etc.), reducing the number of simplifying assumptions, and generally improving the knowledge of a system's physics. One special kind of epistemic uncertainty involves having limited data to properly define the distribution parameters of the random variables. This type of uncertainty may be reduced by collecting more data.

Assessing the reliability of large, complex systems can be extremely difficult. Historically, in engineering, a probabilistic perspective of reliability has been inherently linked to a frequentist perspective. In other words, the predicted probability of future events was extrapolated from the historical frequency of past events. However, translating this perspective to assess reliability for modern aerospace systems presents a special challenge since historical databases based on legacy systems are few, new systems continue to expand the horizon in terms of both their operational environments and performance requirements, and engineers continue to use novel materials in design. A common, pseudo-analytical approach to designing for reliability is to employ factors of safety. Load and Reduction Factored (LRFD) steel design guidance, for example, specifies a combination of safety factors which reduce allowable strength from nominal material strength and increase required strength based on estimated loading conditions (AISC, 2006). These factors are used to assure reliability but can only be related directly to a corresponding probability of success or failure if sufficient empirical data is available. In other words, the degree of safety provided by the factor is only understood from the context of experience; a different factor would be appropriate for different kinds of systems under different operating conditions employing different materials. For systems with which engineers lack sufficient experience, a more rigorous analytical approach for assessing reliability is needed. In general, probabilistic methods provide such rigor by defining the uncertainty associated with a system at a primitive level and propagating that uncertainty through the system performance analysis.

The first step in probabilistic reliability analysis is to define probability density functions (pdfs) to describe input uncertainty. This includes aleatory and epistemic variables. (To capture epistemic uncertainty arising from a lack of accuracy or confidence in analysis methods, *model error* variables may be introduced. This idea is treated in detail in Chapter VI.) Input uncertainty is propagated through the system performance analysis model in order to characterize output uncertainty. Probabilistic reliability analysis is based on the concept of a limit state that defines the boundary between success and failure for a system (Haldar and Mahadevan, 2000). The limit state function, g, is derived from a system performance criterion and formulated such that $g < 0$ indicates failure. If the input parameters in the system analysis are uncertain, so will be the predicted value of g. The probability of system failure $P(g < 0)$ may be obtained from the volume integral under the joint probability density function of the input random variables over the failure domain, as shown in Eq. (1) and graphically in Fig. 1.

$$P_f = \int \ldots \int_{g \le 0} f_X(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n \qquad (1)$$

In Eq. (1), P_f is the probability of failure, f_X is the joint probability density of a random variable vector $\boldsymbol{X}$ with n elements; vector $\boldsymbol{x}$ represents a single realization of $\boldsymbol{X}$. Note that the integral is taken over the failure domain, or where $g \le 0$, so $P_f = P(\text{g} \le 0)$.

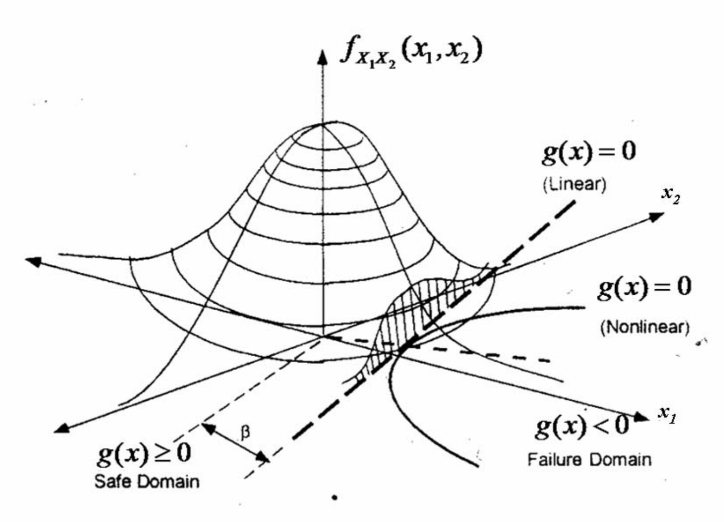


Figure 1. Limit State Modeling

Two types of methods have been used to evaluate this integral: (1) simulation methods and (2) analytical approximations. The most elementary of the first type of methods, Monte Carlo simulation, tends to be accurate only with a large number of simulations, especially for high reliability systems. Monte Carlo simulation is often impractical for real systems when a single analysis requires a significant amount of computational effort. However, simulation does hold some key advantages over other probabilistic analysis techniques. For one, basic simulation is not sequential and can therefore take maximum advantage of parallel processing. Another benefit is that simulation does not require gradient information, which is often difficult to obtain for real systems. Finally, the same set of simulation runs can be used to evaluate multiple limit states simultaneously, as opposed to analytical methods that construct approximations to one limit state at a time.

Analytical methods include first-order and second-order approximation techniques, which are well documented in literature (for a review, see Haldar and

Mahadevan, 2000). In first-order methods, a linear approximation of the limit state is used to estimate the failure probability from Eq. (1) as depicted by the dashed line in Fig. 1. The accuracy of the first-order method depends on the curvature of the limit state and the point from which the linear approximation is based. The First Order Second Moment (FOSM) method (Ang and Amin, 1967), for example, is based on a first-order Taylor Series approximation of the mean and standard deviation of a limit state function:

$$\mu_g \approx g(\mu_x) \tag{2}$$

$$\sigma_g^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial g}{\partial x_i}\frac{\partial g}{\partial x_j}\mathrm{Cov}(x_i, x_j)$$

where Cov is the covariance indicating the correlation between variables x_i and x_j. Assuming the limit state function g is linear, the cumulative distribution function (CDF) for the standard normal variable may be used to estimate the probability of failure, P_f:

$$P_f = P(g < 0) = \Phi\left(\frac{0 - \mu_g}{\sigma_g}\right) \tag{3}$$

FOSM requires minimal computational effort but sacrifices accuracy for non-linear limit states or systems with non-normal input variables.

The First Order Reliability Method (FORM), a more accurate analytical approach than FOSM, estimates the failure probability as $P_f = \Phi(-\beta)$ where β is the minimum distance from the origin to the limit state in the uncorrelated reduced normal space (Hasofer and Lind, 1974). The minimum distance point on the limit state is referred to as the most probable point or MPP, and β is referred to as the reliability index. (A graphical representation of the FORM concept is given in Fig. 2). The FORM method is able to handle correlated, non-normal random variables and nonlinear limit states;

however, the probability estimate is based on a first-order approximation of the limit state at the MPP.

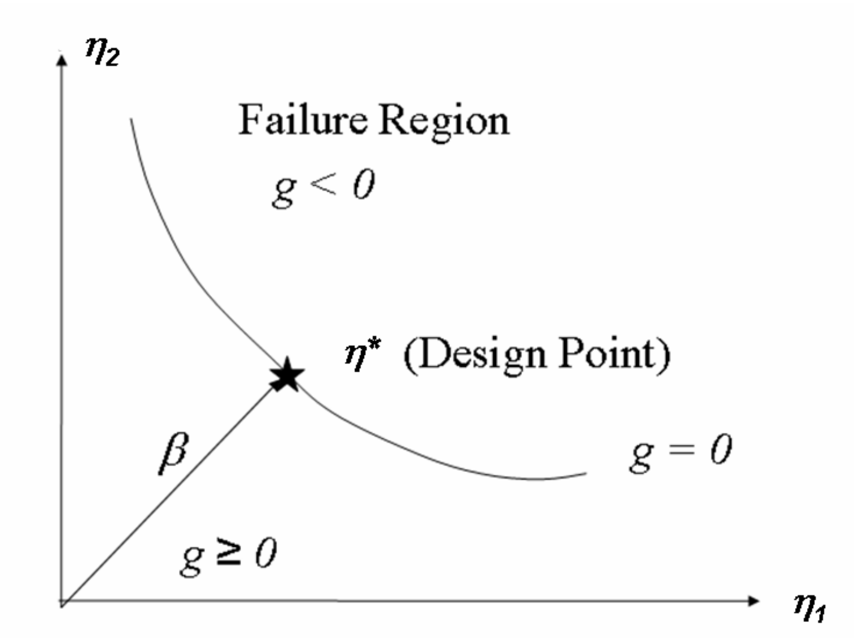


Figure 2. First - Order Reliability Method

Finding the most probable point (MPP) is an optimization problem for which any of several optimization algorithms may be used:

$$\text{Minimize} \quad \beta = \|\eta\| \tag{4}$$

subject to

$$g_\eta(\boldsymbol{\eta}) = 0$$

In Fig. 2 and Eq. (4), $\boldsymbol{\eta}$ is the vector of random variables in uncorrelated standard normal space and $\|\boldsymbol{\eta}\|$ denotes the norm of that vector. In other words, the mean and standard deviation of $\boldsymbol{\eta}$ is zero and one, respectively. In general, a set of random variables $\boldsymbol{x}$ may be non-normal and correlated, but these may be transformed to an uncorrelated standard normal space via a transformation T, i.e $\boldsymbol{\eta} = T(\boldsymbol{x})$. Rackwitz and Fiessler (1976), for example, propose a two-parameter transformation to equate the standard normal PDF and CDF values at a checking point to the respective PDF and CDF values of the original variable. The transformations from correlated to uncorrelated space are also well established (for a review, see Haldar and Mahadevan, 2000). The limit state function is also transformed to uncorrelated standard normal space so that $g_\eta(\boldsymbol{\eta}) = g(T^{-1}(\boldsymbol{\eta}))$. The solution to Eq. (4) is denoted $\boldsymbol{\eta}^*$ in standard normal space or $\boldsymbol{x}^*$ in original space; it is commonly called the Most Probable Point (MPP). The reliability index, β, then is the norm of $\boldsymbol{\eta}^*$ (Hasofer and Lind, 1974).

Probabilistic Design using Reliability-based Optimization

Assessing reliability, in and of itself, is not useful for design. Instead, the design process must ensure reliability requirements are met. Reliability based design optimization (RBDO) is a useful means to accomplish this. In general, optimization is a common method used to find some 'best' set of design variables while ensuring that performance requirements are met. A standard optimization problem is given in Eq. (5) where $\boldsymbol{d}$ is a set of design variables. The function, $f(\boldsymbol{d})$ is the objective, and $g(\boldsymbol{d}) \leq 0$ and $h(\boldsymbol{d}) = 0$ are inequality and equality constraints, respectively.

$$
\begin{aligned}
&\text{Minimize } f(\boldsymbol{d}) \\
&\text{subject to} \\
&g(\boldsymbol{d}) \leq 0 \\
&h(\boldsymbol{d}) = 0
\end{aligned}
\tag{5}
$$

A wide variety of algorithms are available to solve Eq. (5). (For a sample, see Nocedal and Wright, 1999). These algorithms provide either a local or global solution for the 'optimal' vector of design variables $\boldsymbol{d}$, denoted $\boldsymbol{d^*}$. The local solution is the point, $\boldsymbol{d^*}$ at which the objective function is smaller than at other points in its vicinity, while the global solution is the point at which the objective is smaller than at all other points in the design space. While global solutions are ideal, they are generally impossible to find. This dissertation uses local optimization methods.

RBDO problems include random input variables to either the objective or constraint functions to account for uncertainty in the analysis. They typically optimize a deterministic (i.e., non random) objective subject to probabilistic constraints, as shown in Eq. (6).

$$\text{Minimize } f(\boldsymbol{d}) \tag{6}$$

$$\text{subject to}$$

$$P(g_{\eta}(\boldsymbol{d}, \boldsymbol{\eta}) \leq 0) \leq P_{\text{acceptable}}$$

$$h_{\text{ineq}}(\boldsymbol{d}) \leq 0$$

$$h_{\text{eq}}(\boldsymbol{d}) = 0$$

Here, the vector of design variables, $\boldsymbol{d}$ can include both deterministic variables and parameters for random variables. Parameters for random variables, for example, may include the mean value or standard deviation, indicating the variable can be controlled somewhat but uncertainty cannot be eliminated. The variable $\boldsymbol{\eta}$ is a vector of standard normal variables transformed from a set of random variables. The use of $\boldsymbol{\eta}$ in the formulation allows the effect of variability of a random variable to be isolated from the random distribution parameters (e.g., mean and standard deviation) which may be also be design variables so that in evaluating the limit state, $g_{\eta}(\boldsymbol{d}, \boldsymbol{\eta})$, $\boldsymbol{d}$ and $\boldsymbol{\eta}$ may be treated as independent variables. The most important distinctive feature of RBDO problems is the probabilistic constraint, which is formulated to ensure an acceptable probability that some performance criterion is met (i.e., that the probability of failure is less than a maximum acceptable level). Note that non-probabilistic inequality and equality constraints may also be included in an RBDO formulation, but must be functions of deterministic variables only. Applications in this dissertation will include primarily probabilistic inequality constraints.

Optimization algorithms are iterative. They provide either gradient or non-gradient based searches for an optimum solution based on values of the objective and constraints for successive guesses. In the context of RBDO, since each probabilistic constraint must be evaluated several times, first order analytical approximations are the most common due to their efficiency. Two options are available for reformulating the RBDO problem with approximate, first-order constraints. The first uses a direct first-order reliability method (FORM), often referred to in the literature as the Reliability Index Approach or RIA (Yu et al, 1997), which is given in Eq. (7).

$$\begin{gathered}\text{Minimize } f(\boldsymbol{d}) \\ \text{subject to} \\ \beta \geq \beta_{\text{target}}\end{gathered} \tag{7}$$

Here $\boldsymbol{d}$ is the vector of design variables and the acceptable probability, $P_{\text{acceptable}}$ is transformed to a target reliability index, β_{target}, using the inverse of the standard normal cumulative distribution, $\beta_{\text{target}} = -\Phi^{-1}(P_{\text{acceptable}})$. The reliability index, β, is defined by Eq. (8).

$$\text{Minimize } \beta = \|\boldsymbol{\eta}\| \tag{8}$$

$$\text{s.t. } g_{\eta}(\boldsymbol{d}, \boldsymbol{\eta}) = 0$$

Note that Eq. (8) is the same definition for reliability index given in Eq. (4) for reliability assessment, except that the design variable vector, ***d*** is shown explicitly to relate back to the optimization formulation given in Eq. (7). As stated in the previous section, the solution to Eq. (8), $\boldsymbol{\eta}^*$, is called the MPP. Thus, RBDO based on direct FORM is, in its basic form, a nested optimization. The outer optimization conducts a search for the optimal design, ***d***, while calling an inner-loop which conducts an MPP search in order to evaluate probabilistic constraints (i.e., $\beta \geq \beta_{\text{target}}$).

Alternatively, an *inverse* FORM method, also called the Performance Measure Approach (PMA, Tu et al, 1999) is often used for RBDO as given by Eq. (9).

$$\text{Minimize } f(\boldsymbol{d}) \tag{9}$$

$$\text{s.t. } g^* \geq 0$$

where g^* is defined by Eq. (10).

$$\text{Minimize } g^* = g_{\eta}(\boldsymbol{d}, \boldsymbol{\eta}) \tag{10}$$

$$\text{s.t. } \|\boldsymbol{\eta}\| = \beta_{\text{target}}$$

The solution to Eq. (10) will be referred to as the PMA point, $\boldsymbol{\eta}'$, to distinguish it from the MPP, $\boldsymbol{\eta}^*$, of the direct FORM method. Figure 3 depicts graphically, for a two dimensional random variable vector, the equivalence of the direct and inverse FORM methods in ensuring that a probabilistic constraint is satisfied.

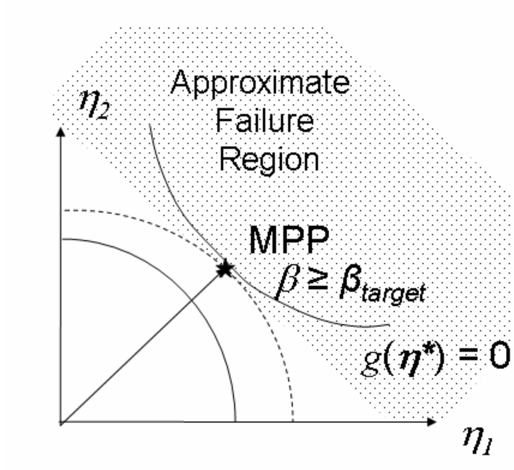


Figure 3(a). Direct FORM

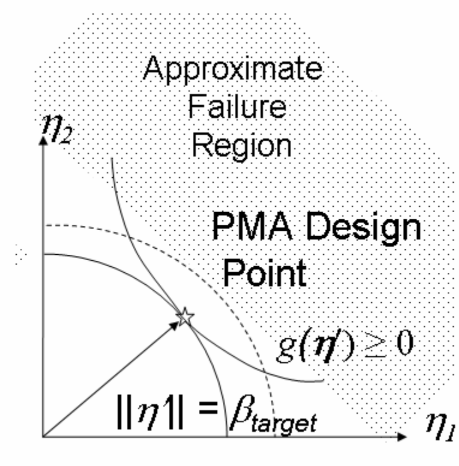


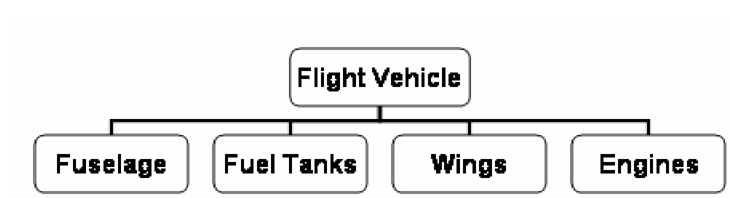
Figure 3(b). Inverse FORM

As with the direct FORM method, the inverse FORM RBDO is a nested optimization, in this case with an 'inner-loop' PMA search nested inside the primary 'outer-loop' optimization. This results in a multiplicative effect on the computational effort (i.e., the number of performance analyses required for optimization is multiplied by the number of analyses required for reliability assessment.) For real design problems with significant computational effort for a single performance analysis, simple direct or inverse FORM RBDO could be intractable. However, recent advances have led to significant improvements in the computational efficiency of RBDO methods (Tu et al, 1999, Royset et al, 2001, Wu et al, 2001, Zou et al, 2002, Du and Chen, 2003, Jiang and Mourelatos, 2004). These methods are described in more detail in Chapter III as they are incorporated specifically into reliability-based optimization methods for multidisciplinary systems.

Probabilistic Design of Systems

As stated earlier, the term *system* is used here to describe a collection of components aimed at a common objective. *Systems engineering* typically begins with a top-down design and decomposition stage (to progress from high-level requirements to a detailed design of components sufficient for production) followed by a bottom-up integration and qualification stage (which verifies that components as assembled meet requirements, Forsberg and Mooz, 1992). This process is associated with a set of hierarchical architectures. Consider for example, the simplistic bi-level physical and discipline hierarchies for a flight vehicle in Fig. 4. The physical architecture (Fig. 4(a)) is a product of top-down design decomposition. Note that at the higher level, the scope is broad but detail is limited while at lower levels the scope decreases and detail increases. The disciplinary hierarchy, on the other hand, results from bottom-up integration. Engineering expertise and design and analysis tools have developed primarily along specific disciplinary lines; these tools must be intentionally combined in order to analyze system performance at the higher level. System decomposition and integration has implications for reliability analysis as well as for reliability-based design. This research specifically addresses the communication of probabilistic information across system architectures.

Physical Architecture



Disciplinary Architecture

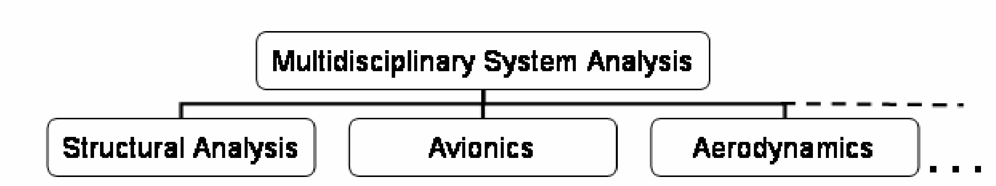


Figure 4. Launch Vehicle Physical and Disciplinary System Architectures

There are many different models for a systems design process such as the System Engineering VEE, spiral, and waterfall models to name a few. (For an overview, see Buede, 2000.) All systems engineering processes share a progression from designs which are broad in scope and limited in detail to those of increasing detail and decreasing scope (for example, from conceptual design of a whole system to preliminary design of subsystems to detailed design of individual components). In addition, all systems engineering models iterate between design levels. A final critical characteristic of the systems design process is that it is highly dependent on the interfaces (or connections) between elements of a system.

This research is motivated by the need to incorporate reliability requirements in system design and the conviction that probabilistic methods are particularly suitable for this purpose. Probabilistic methods effectively translate reliability assessment into terms easily understood by managers, operators, and the general public. They also provide more rigor than traditional methods for reliability-based design and build upon engineering analyses already needed to design for performance. However, rather than a study of the formal systems engineering process, this dissertation has a narrower focus:

developing methods and exploring concepts associated with iterative probabilistic design and the flow of probabilistic information across system interfaces. For example, the applications included here demonstrate the effectiveness of reliability-based optimization for design problems across disciplines, physical components, and design levels.

The goal of this dissertation is to develop and apply efficient methods for the probabilistic design of systems, considering system integration from two fronts. The first front considers the integration of analyses at a single level (i.e., lateral integration across the system architecture). The second front considers the synthesis of probabilistic design information from a system to component designs (i.e., vertical integration across the system architecture.) Four objectives are pursued to this end: (1) the development and study of efficient algorithms for multidisciplinary reliability analysis; (2) the extension of these methods to reliability-based optimization of multidisciplinary systems; (3) the development of a method for integrating system and component designs using model error propagation; and (4) the demonstration of these methods to two real world applications, the design of a power system for an unmanned aerial vehicle and the integrated design of a reusable launch vehicle and component liquid hydrogen tank. For the first objective, two methods are presented for the reliability analysis of multidisciplinary systems. Next, twelve algorithms for reliability-based multidisciplinary optimization are developed by synthesizing these concepts with established reliability-based design (RBDO) and multidisciplinary optimization (MDO) strategies. These methods are applicable to both lateral and vertical integration in probabilistic system design; they provide a means to integrate analyses at a single level but may also be adapted for integration across levels. To satisfy the third objective, model error assessment and propagation is investigated as an alternative means for integrating system and component designs. In accordance with the final objective, reliability-based multidisciplinary optimization methods are applied to the system design of an unmanned aerial vehicle, demonstrating system integration at a single level. In addition, both reliability-based optimization and model error propagation strategies are applied as alternative methods for the system and component integration of a reusable launch vehicle and its liquid hydrogen fuel tank.

In Chapter II, reliability-based analysis methods and deterministic multidisciplinary optimization strategies are combined to develop two efficient computational algorithms for reliability analysis of multidisciplinary systems. These concepts are extended to optimization in Chapter III, which introduces twelve algorithms using various multidisciplinary and reliability-based optimization techniques; each of the algorithms is demonstrated on 3 examples to test and compare accuracy and efficiency. In Chapter IV, these methods are applied to a real world application, the design of the power supply system for an unmanned aerial vehicle.

Chapters V and VI address the incorporation of reliability-based design across levels as demonstrated for the coupling of a reusable launch vehicle conceptual design for geometry and the structural sizing of a component tank. As an alternative to fixed point iteration between the two designs, Chapter V integrates the two levels within a single reliability-based optimization. In Chapter VI, the same problem is addressed from a different perspective. In this case, the component design is used to aid in characterizing the model error in the vehicle optimization. This chapter also considers the system sensitivity to model error as a valuable metric for selecting disciplinary models at various stages of design. The dissertation concludes with a summary and brief synopsis of future research needs.

CHAPTER II

FIRST-ORDER RELIABILITY ANALYSIS FOR MULTIDISCIPLINARY SYSTEMS

Introduction

This chapter considers probabilistic analysis of multidisciplinary systems. This is a critical first step in achieving reliability-based system design as it addresses how probabilistic information may be propagated across a system architecture (in this case, a disciplinary architecture). According to most systems engineering models, integration comprises the second phase of design. In practice, integration is required at the earliest stages of design as well. During these earlier (i.e., conceptual) stages of design, the system is looked at as a whole, so the scope of performance analysis is the largest. However, engineering expertise and analysis tools are primarily developed along very specific disciplinary lines (e.g., aerodynamics, structural, propulsion, thermal, etc.) so that many conceptual design tools involve significant integration of these disciplinary analyses. The resulting multidisciplinary analysis is a 'bottom-up' approach, but one that is applied during almost every phase of design. In addition, the underlying process involved in integrating disciplinary design or analysis tools applies to integration along other system architectures.

One emerging method for the design of aerospace systems is optimization. Several different formulations have been developed in the literature for multidisciplinary optimization (e.g., Cramer, et al, 1994; Braun and Kroo, 1996; Sobieszczanski-Sobieski et al, 2000; Renaud and Gabriele, 1994). The type of formulation adopted for multidisciplinary analysis significantly affects the computational effort required for probabilistic analysis as well as optimization. The accuracy of derivative approximations (usually found through finite differences in a black-box approach) is also a concern in both cases (Sobieszczanski-Sobieski, 2000), since many optimization algorithms and the more efficient reliability analysis methods such as the first-order reliability method (FORM) and the second-order reliability method (SORM) require them. In addition, the synthesized global analysis may not be sufficiently differentiable to ensure convergence of these methods. Therefore, it is valuable to investigate how MDO formulations may be extended to multidisciplinary reliability analysis.

One specific challenge that surfaces for integrated multidisciplinary systems involves the computational expense of wrapping one iterative process (optimization) around another (multidisciplinary analysis). In this case, iterative convergence loops are needed to ensure multidisciplinary feasibility (i.e., consistency of disciplinary responses throughout the system). In a conventional or fully-integrated[3] approach, the multidisciplinary analysis (MDA) convergence loops are nested inside the loops for probabilistic analysis and/or optimization. The resulting computational effort is unacceptable for most high fidelity analyses. Therefore, this work explores alternatives to the fully-integrated approach, using methods that exploit a distributed formulation of multidisciplinary analysis. Distributed formulations for reliability analysis have already been proposed (Du and Chen, 2002) in the literature. This chapter focuses on specific, efficient computational algorithms that solve these formulations.

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[3] The term "fully-integrated optimization" was adopted from Alexandrov and Lewis, 2000, indicating that multidisciplinary analysis is required for every optimization (or in this case probabilistic analysis) iteration. This conventional approach is given other names by other authors.

The following section introduces the background on the coupled nature of multidisciplinary analysis and the implications this has for limit state-based reliability analysis. Next, distributed formulations commonly proposed in multidisciplinary optimization (MDO) research are discussed, highlighting their potential to improve multidisciplinary probabilistic analysis. Then, two distributed algorithms for multidisciplinary systems are presented. The first algorithm uses a first-order second moment (FOSM) method to characterize intermediate variables while applying more rigorous reliability analysis, such as Monte Carlo analysis or the first-order reliability method (FORM), to the system as a whole. The second method proposes a specific algorithm to solve a decoupled optimization formulation for the first order reliability method. Each method is demonstrated for a two-discipline mathematical example system. Results are compared against otherwise equivalent coupled algorithms for accuracy and efficiency.

Multidisciplinary System Analysis

Multidisciplinary analysis (MDA) involves integrating individual (or discipline-specific) analyses, which share input and output data. A 'feasible' multidisciplinary system requires the simultaneous solution of all disciplinary analyses. For analyses performed in a particular sequence, interdisciplinary coupling may be either a feed-forward or feedback type. For feed-forward coupling, the output of an earlier analysis feeds 'forward' as the input of a later analysis. Feedback occurs when a coupled analysis must be performed prior to the analysis that determines its input. For systems with feedback coupling, iteration is required to ensure consistency of discipline responses, requiring multiple 'runs' of a single set of analyses. Even strictly feed-forward MDA systems can be computationally expensive since performing analyses in sequence prevents the time-saving approach of parallel computing.

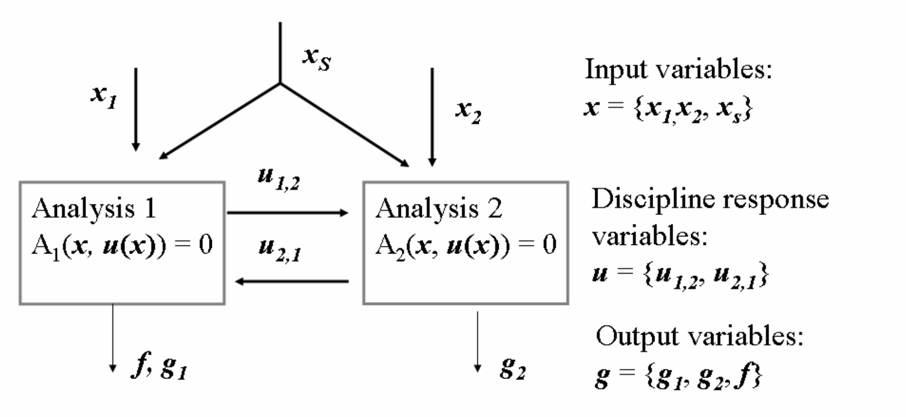


Figure 1. Example Two-Discipline Multidisciplinary System

Fig. 1 depicts coupling in a two-discipline system. Here $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ represent local input variables to analyses 1 and 2 respectively, while $\boldsymbol{x}_s$ indicates input variables common to both analyses. Variables $\boldsymbol{u_{1,2}}$ and $\boldsymbol{u_{2,1}}$ are disciplinary response variables that couple the two analyses (defined such that $\boldsymbol{u_{i,j}}$ is an output of analysis *i* and an input to analysis *j*). The system output variables are $\boldsymbol{f}$, $\boldsymbol{g_1}$, and $\boldsymbol{g_2}$; in the context of optimization, $\boldsymbol{f}$ may represent a system objective while $\boldsymbol{g_1}$, and $\boldsymbol{g_2}$ may represent limit states for reliability analysis. Multidisciplinary feasibility, then, may be found by simultaneously solving a set of non-linear equations represented disciplinary analyses as shown in Eq. (1).

$$A_i(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x})) = 0, \text{ for each i = 1,.. number of disciplines in system} \tag{1}$$

It may be seen that, regardless which analysis is performed first, an unknown variable (either $\boldsymbol{u_{1,2}}$ or $\boldsymbol{u_{2,1}}$) is needed, indicating a feedback condition. Systems with feedback coupling are typically solved with fixed-point iteration. In this process, assumed values for the unknown variables are initially used, and then updated by performing the analyses from which they are derived. The analyses are performed again with the updated values, and the process continues until convergence is reached. Fixed point iteration has major drawbacks. First, convergence is not guaranteed. The effectiveness of fixed-point iteration is sensitive to the starting point, and many systems will exhibit a divergent pattern for some starting points. Gradient-based algorithms for solving systems of equations can be more efficient. However, with complex multidisciplinary systems, analytical gradients are rarely available. Calculating numerical, finite difference-based gradients is another alternative but this requires additional system analyses.

Implications for Reliability Analysis

The purpose of multidisciplinary analysis is to predict the behavior of a complex, engineered system. These predictions are made with a degree of uncertainty, and this measure of performance uncertainty characterizes the reliability of the system. For coupled multidisciplinary systems, using fixed-point iteration for multidisciplinary analysis within probabilistic analysis algorithms may be inefficient. This effect is depicted in Fig. 2, which shows FORM being applied to a two-discipline analysis.

Another difficulty is in obtaining gradient information, usually required for the more efficient analytical approximation algorithms. If a finite difference method were used, the fixed-point iteration process for convergence would need to be repeated for each variable. Furthermore, one might select a less stringent convergence criterion to avoid unnecessary fixed-point iterations, but this introduces 'noise' that can interfere with finite difference estimates for the gradient. As a result, probabilistic analysis algorithms that simply use the system analysis as a black-box may not even converge in such situations. Similar problems arise for optimization, and these problems become worse when probabilistic analysis and optimization are simultaneously attempted for multidisciplinary systems. For example, Mahadevan and Gantt (1998) showed that a traditional probabilistic optimization approach for a coupled electronic packaging system did not converge for many starting points and required over 10,000 function evaluations for those starting points that did lead to convergence.

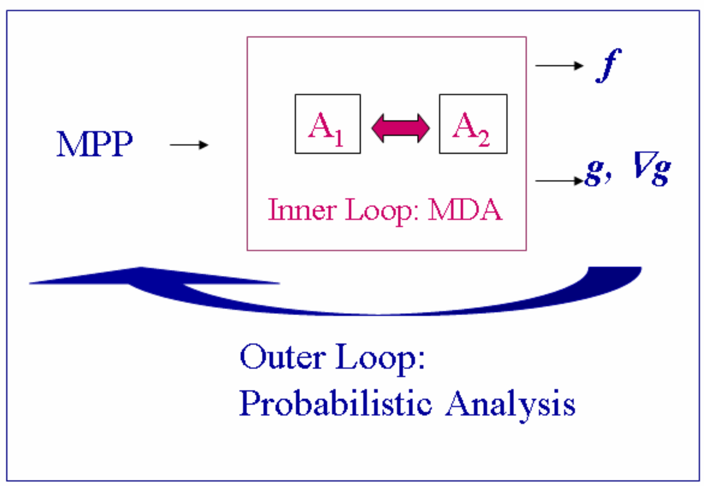


Figure 2. Probabilistic Analysis for Systems with Feedback

Distributed Analysis: A Strategy from Multidisciplinary Optimization (MDO)

The difficulties encountered in applying first order reliability analysis to multidisciplinary systems mirror those in multidisciplinary optimization (MDO), since FORM is an optimization problem (see Eqs. (7)-(10) in Chapter I). At a more basic level, reliability analysis, like optimization, is an iterative process which multiplies the cost of a single multidisciplinary analysis and compounds the computational expense of fixed-point convergence loops. Thus distributed strategies used in MDO may also be applicable for probabilistic analysis.

Consider the standard optimization problem formulated as follows:

$$\text{Minimize } f(\boldsymbol{x}) \tag{1}$$

$$\text{Subject to } h(\boldsymbol{x}) = 0 \text{ and } g(\boldsymbol{x}) \geq 0$$

Both gradient-based and non gradient-based non-linear programming algorithms are available to solve problems of this type (for an overview, see Nocedal and Wright, 1999). Obviously, gradient-based methods require at a minimum that the objective function be differentiable. When gradients cannot be obtained directly, approximation methods such as finite differencing are needed.

The optimization problem is expanded to include response variables, $\boldsymbol{u}(\boldsymbol{x})$, representing the output of disciplinary analyses:

$$\text{Minimize } f(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x})) \tag{2}$$

$$\text{Subject to } h(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x})) = 0 \text{ and } g(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x})) \geq 0$$

The response variables, $\boldsymbol{u}(\boldsymbol{x})$ must satisfy the multidisciplinary feasibility requirement as given by the set of disciplinary analysis equations, $A(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) = 0$. A direct approach is to reduce the MDO formulation to the standard optimization problem through variable reduction. In other words, the only independent optimization variables are the design variables $\boldsymbol{x}$; and the disciplinary response variables, $\boldsymbol{u}(\boldsymbol{x})$ must be solved for at every iteration in the system optimization. This is also known as the Multidisciplinary Feasible Method, or MDF (Cramer et al, 1994) or more generically as fully-integrated analysis and optimization (Alexandrov and Lewis, 2000). The limitation of this approach is that it can involve unnecessary computational expensive for some problems.

A reduction in the overall computation time may be accomplished if disciplinary analyses can be done in parallel in some cases. This approach requires an MDO problem formulation that decouples the disciplinary analyses from the multidisciplinary system optimization and from one another. Cramer, et al (1994) provide a review and taxonomy of MDO methods. More recently, Alexandrov and Lewis (2000) presented a review of MDO formulations from the perspective of the optimization algorithms used to solve them. In distributed analysis and optimization (DAO), also referred to in the literature as the Individual Discipline Feasible (IDF) method, auxiliary variables representing interdisciplinary flow are used to achieve autonomy for disciplinary analyses. Multidisciplinary feasibility is maintained in the system optimization through compatibility constraints that must be satisfied at the final solution. Other methods, such as Collaborative Optimization (Braun and Kroo, 1996) and Bi-Level Integrated System Synthesis, or BLISS, (Sobieszczanski-Sobieski et al, 2000) also use auxiliary variables but have optimizations at both the system and discipline levels. The methods adapted herein build upon distributed analysis and optimization.

Distributed analysis is also an effective strategy for probabilistic analysis of multidisciplinary systems. It enables probabilistic analysis without the fixed-point iteration convergence process required as an inner loop in the fully-integrated approach. In the following section, a distributed partial first-order second moment (FOSM) method is proposed and later in the chapter, an algorithm to extend FORM to multidisciplinary reliability analysis is developed. Both methods borrow from the DAO strategy used for multidisciplinary optimization.

Characterizing Random Auxiliary or Coupling Variables Using FOSM

From a probabilistic perspective, using auxiliary variables raises some interesting questions. The variables they represent depend on random input variables and will thus be random variables themselves. This begs the question, how does one select an appropriate probability density function for the auxiliary variables? Also, if assumptions are made regarding the PDFs of the auxiliary variables, how will any errors propagate to the system output variables? In other words, can the result be trusted?

One approximate option for characterizing the auxiliary variables is to apply FOSM on the multidisciplinary system. In this method, the integrated multidisciplinary analysis is considered estimating the mean, μ_g, and standard deviation, σ_g, of disciplinary response variables (see Eq. (2) of Chapter I). Thus, Eq. (1) is solved to find a feasible multidisciplinary system at the mean input. Then, using a finite difference process to calculate the gradient of g with respect to the random vector, $\boldsymbol{x}$, at least n+1 evaluations of the multidisciplinary system are required, where n is the dimension of $\boldsymbol{x}$. This leads to (n+1)*D disciplinary function calls, where D is the number of disciplinary analyses required in the iterative process to find a feasible multidisciplinary system. FOSM could be applied directly to system output variables (e.g., limit state functions) to determine the probability of failure according to Eq. (3).

$$P_f = P(g < 0) = \Phi\left(\frac{0 - \mu_g}{\sigma_g}\right) \tag{3}$$

The flow of probabilistic information for this direct, coupled FOSM method is depicted in Fig. 3. However, as discussed in the previous chapter, FOSM has limited accuracy, although the computational effort is relatively light, especially for non-linear limit states and non-Gaussian input variables.

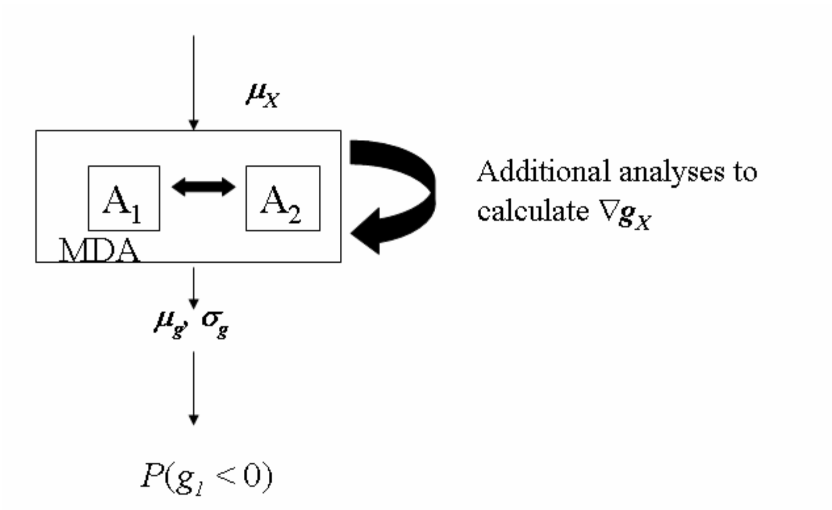


Figure 3. Direct, integrated FOSM for a multidisciplinary system

A better option is to combine FOSM with more sophisticated probabilistic analysis techniques through a distributed approach. Thus it is proposed in this chapter that FOSM be first applied to the coupled system to develop an initial statistical description of the auxiliary variables, and then a more accurate probabilistic analysis method be applied to the distributed system, treating the disciplinary analyses individually. (This approach may be referred to as a partial FOSM approach, as opposed to the direct FOSM approach described above). If a probability density function for $\boldsymbol{u}$ is assumed, then these variables can be easily incorporated in further probabilistic analysis (e.g., Monte Carlo, FORM, etc.) to be applied at the discipline level. The proposed algorithm is depicted in Fig. 4 with a corresponding pseudo code below.

Pseudo code for Partial FOSM

1. Find statistics of disciplinary responses using MDA to find $\boldsymbol{u}(\boldsymbol{x})$:

$$\mu_u \approx \boldsymbol{u}(\mu_x)$$

$$\sigma_u^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}_i}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}_j}\mathrm{Cov}(\boldsymbol{x}_i,\boldsymbol{x}_j)$$

2. Conduct probabilistic analysis to estimate $P(\mathrm{g}(\boldsymbol{x},\boldsymbol{u}')<0)$;

Use $\boldsymbol{u}' \sim N(\mu_u,\sigma_u)$ as independent random variable.

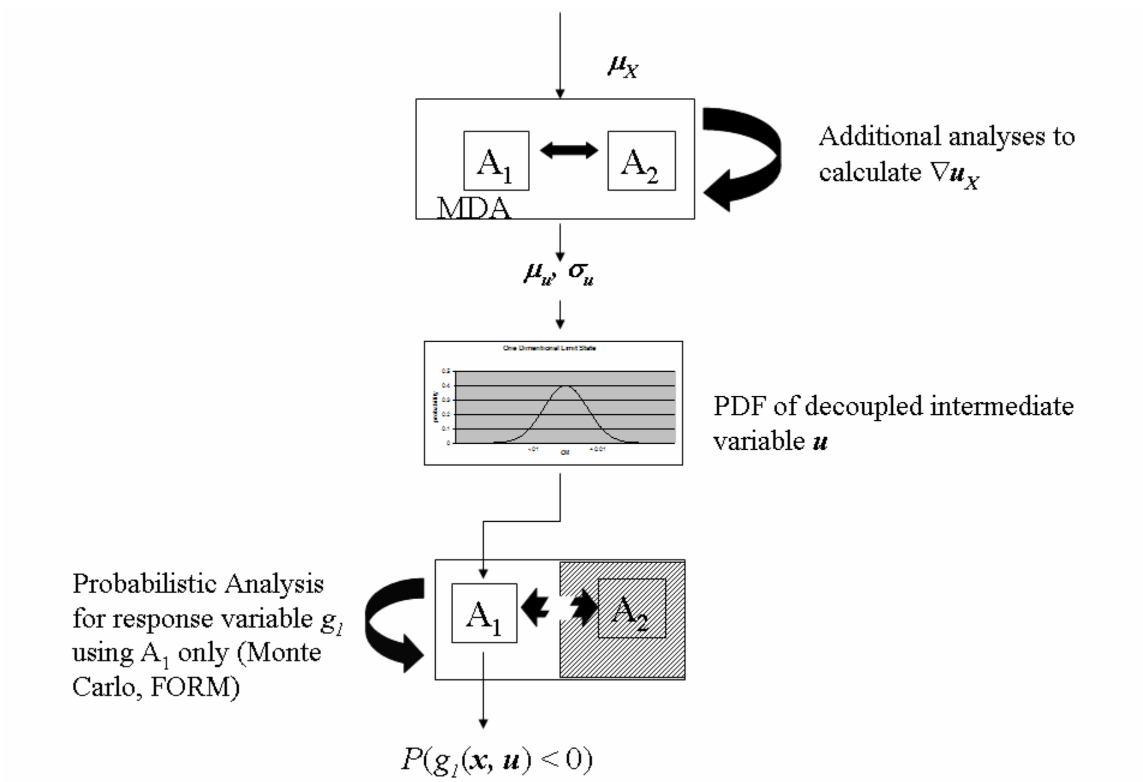


Figure 4. Partial FOSM Method (with Monte Carlo Analysis or FORM)

Note that the computational effort to calculate the mean, $\boldsymbol{\mu_u}$, and standard deviation, $\boldsymbol{\sigma_u}$, of the intermediate variable vector is the same as that required in the direct FOSM method to calculate the mean and standard deviation of the response variable, $\boldsymbol{\mu_g}$, and $\boldsymbol{\sigma_g}$. However, to improve accuracy, the partial FOSM approach conducts additional probability analysis for the discipline yielding the system response variable of interest, in this case discipline A_1 for response g_1. During the latter step, multidisciplinary analysis is not performed, saving computational effort for fixed point iteration or other feasibility search algorithm.

Both the direct FOSM and the partial FOSM approach rely on significant assumptions, namely that the first order approximation for the mean and standard deviation is adequate and that output and intermediate variables as well as input variables satisfy a normal (or other selected) distribution. However, with the partial FOSM approach, these assumptions only apply to the intermediate variables so non-normal probability distributions for system inputs and non-linearity in the response function may still be captured by a more sophisticated probabilistic analysis method.

In summary, the partial FOSM method combines FOSM (for characterization of intermediate variables) and more rigorous probabilistic analysis methods such as Monte Carlo Simulation or FORM. This technique recaptures some of the accuracy, which would otherwise be forfeited with direct FOSM, while avoiding repeated multidisciplinary analysis loops during probabilistic analysis. The advantages of the partial FOSM method are further illustrated via a numerical example at the end of this chapter.

Extension of FORM to Multidisciplinary Reliability Analysis

In the context of multidisciplinary systems, the First-Order Reliability Method can be given as an MDO formulation:

$$\text{Minimize} \quad \beta = \|\boldsymbol{\eta}\| \tag{4}$$

subject to

$$g_\eta(\boldsymbol{\eta}, \boldsymbol{u}_\eta(\boldsymbol{\eta})) = 0$$

As in the previous chapter, $\boldsymbol{\eta}$ denotes all the random input variables of the system in uncorrelated standard normal space. Functions g_η and $\boldsymbol{u}_\eta$ are transformed functions such that $g_\eta(\boldsymbol{\eta}) = g(T^{-1}(\boldsymbol{x}))$ where T is the transformation function from original space, $\boldsymbol{x}$, to standard normal space $\boldsymbol{\eta}$. Intermediate variables, $\boldsymbol{u}_\eta(\boldsymbol{\eta})$, are additionally included in the limit state function to indicate a multidisciplinary system. Though the end product is probabilistic information ($P(g \leq 0\ ) = \Phi(-\beta)$), solving the FORM formulation is a deterministic MDO problem. This is because the most probable point (MPP), $\boldsymbol{\eta}^*$, is a deterministic value; therefore, the disciplinary response variables at the MPP, $\boldsymbol{u}_\eta(\boldsymbol{\eta})$, are also deterministic. Given this fact, MDO methods may be used to find the solution. In fact, Du and Chen (2002), propose the FORM formulation below:

$$\text{Minimize} \quad \beta = \|\boldsymbol{\eta}\| \tag{5}$$

subject to

$$g_\eta(\boldsymbol{\eta}, \hat{\boldsymbol{u}}_\eta) = 0$$

$$\hat{\boldsymbol{u}} = \boldsymbol{u}_\eta(\boldsymbol{\eta})$$

In this formulation, the limit state g_η is a function of the input variables and auxiliary versions $\hat{\boldsymbol{u}}$ of the intermediate variables $\boldsymbol{u}_\eta(\boldsymbol{\eta})$. To ensure multidisciplinary system compatibility, the additional constraint $\hat{\boldsymbol{u}} = \boldsymbol{u}_\eta(\boldsymbol{\eta})$ is needed.

Optimization algorithms to solve Eq. (5) vary in efficiency, stability, and in the information needed, and their choice is often problem-dependent. Rackwitz and Fiessler (1978) proposed a specific direct FORM algorithm (to solve Eqs. 7 and 8 in Chapter I) based on a quadratic objective, $\frac{1}{2}|\boldsymbol{\eta}|^2$ and a linear approximation of the constraint $g = 0$:

$$\boldsymbol{\eta}_{k+1} = \frac{1}{|\nabla g_\eta(\boldsymbol{\eta})|^2} \left[\nabla g_\eta(\boldsymbol{\eta}_k)^t (\boldsymbol{\eta}_k) - g_\eta(\boldsymbol{\eta}_k) \right] \nabla g_\eta(\boldsymbol{\eta}_k) \tag{6}$$

where $\boldsymbol{\eta}_{k+1}$ is the standard normal MPP at the $(k + 1)^{\text{th}}$ iteration, and $\nabla g_\eta(\boldsymbol{\eta}_k)$ is the gradient vector (vector of derivatives of the limit state function with respect to each variable.) Fig. 5 depicts the use of the Rackwitz-Fiessler algorithm for a multidisciplinary system and a pseudo code follows below. Note that in order to calculate g_η and ∇g_η, as required for Eq. (6), multidisciplinary analysis is needed.

Sample Pseudo code for traditional FORM

Initiate: i = 0, $\boldsymbol{x}_0$, $\beta_0 = 100$

Repeat while i < 3 AND $|\beta_{\mathrm{i}} - \beta_{\mathrm{i\text{-}1}}| \geq .001$

$\boldsymbol{\eta}_{\mathrm{i}} = T(\boldsymbol{x}_i)$ transform MPP candidate to standard normal space

Find $\mathrm{g}_\eta(\boldsymbol{\eta}_i, \boldsymbol{u}_i(\boldsymbol{\eta}_i))$, $\nabla \mathrm{g}_\eta(\boldsymbol{\eta}_i, \boldsymbol{u}_i(\boldsymbol{\eta}_i))$ using MDA to evaluate $\boldsymbol{u}_i(\boldsymbol{\eta}_i)$

Solve Eq. 6 for $\boldsymbol{\eta}_{i+1} = \dfrac{1}{|\nabla g_\eta(\boldsymbol{\eta})|^2}\left[\nabla g_\eta(\boldsymbol{\eta}_i)^t(\boldsymbol{\eta}_i) - g_\eta(\boldsymbol{\eta}_i)\right] \nabla g_\eta(\boldsymbol{\eta}_i)$

$\beta_{\mathrm{i+1}} = \|\boldsymbol{\eta}_{i+1}\|$

Check $|\beta_{\mathrm{i}} - \beta_{\mathrm{i\text{-}1}}|$

$i = i + 1$

END

$P(\mathrm{g}<0) \approx \Phi(-\beta_{\mathrm{i\text{-}1}})$

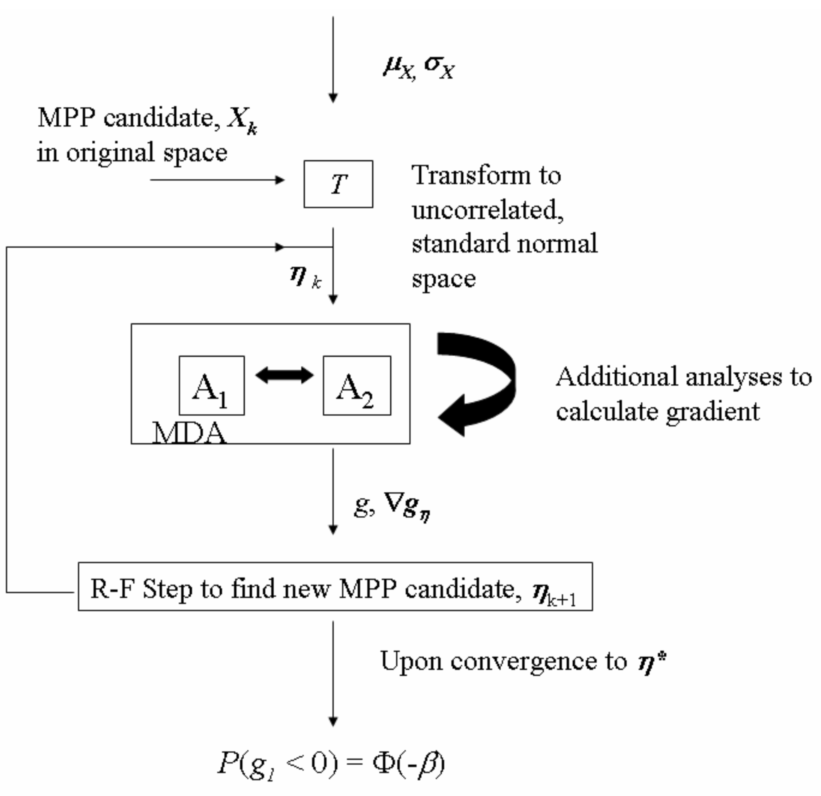


Figure 5. Rackwitz-Fiessler FORM for multidisciplinary system

Eq. (6) is usually quite efficient, but it may fail to converge for certain problems making alternative optimization algorithms necessary in those situations (Liu and DerKiureghian, 1991). For example, standard SQP algorithms use a line search to control the step size and thus ensure convergence (Nocedal and Wright, 1999). This line search requires additional analysis, however, and typically results in more computational effort than Eq. (6) for limit states that do converge. Notice that the MPP search algorithm using Eq. (6) does not satisfy the constraint $g = 0$ at every iteration; it only does so at a solution, thus providing the basis for a distributed FORM strategy for multidisciplinary systems.

Du and Chen (2002) demonstrate that the distributed FORM formulation of Eq. (5) provides an improvement in computational efficiency over the fully-integrated formulation using a standard SQP optimization algorithm. However, additional efficiency may be gained by using a more tailored algorithm in the spirit of the Rackwitz–Fiessler method. Note that Eq. (6) is only applicable to a single constraint problem, whereas the distributed formulation given in Eq. (5) has multiple constraints. Therefore, an algorithm is developed below to solve the multiple-constraint first-order reliability analysis formulation.

The distributed FORM algorithm proposed here uses linear approximations of the constraints, $g(\boldsymbol{\eta}, \hat{\boldsymbol{u}}) = 0$ and $\hat{\boldsymbol{u}} = \boldsymbol{u}_\eta(\boldsymbol{\eta})$, and minimizes of the Lagrangian L as

$$L = \sum \frac{1}{2}\boldsymbol{\eta}^2 + \lambda_1\left(a_0 + \boldsymbol{a}_x\boldsymbol{\eta} + \boldsymbol{a}_u\hat{\boldsymbol{u}}\right) + \lambda_2\left(b_0 + \boldsymbol{b}_x\boldsymbol{\eta} + \boldsymbol{b}_u\hat{\boldsymbol{u}}\right) \tag{7}$$

where $\boldsymbol{\eta}$ is the MPP vector, $\hat{\boldsymbol{u}}$ is an auxiliary variable vector (in standard normal space), and λ_1 and λ_2 are Lagrange multipliers. The coefficients a_0, $\boldsymbol{a_x}$ and $\boldsymbol{a_u}$ come from a first-order Taylor series approximation of the limit state g. (Note that $\boldsymbol{a_x}$ and $\boldsymbol{a_u}$ are vectors.)

$$g_\eta(\eta) \cong g_\eta(\eta_0, \hat{\boldsymbol{u}}) + [\nabla g_x(\boldsymbol{\eta}_0, \hat{\boldsymbol{u}})]^T(\boldsymbol{\eta} - \boldsymbol{\eta}_0) + [\nabla g_u(\boldsymbol{\eta}_0, \hat{\boldsymbol{u}})]^T(\boldsymbol{u}(\boldsymbol{x}) - \hat{\boldsymbol{u}}) = a_0 + \boldsymbol{a}_x\boldsymbol{\eta}_0 + \boldsymbol{a}_u\hat{\boldsymbol{u}} \quad (8)$$

so that

$$\begin{aligned} a_0 &= \mathrm{g}_\eta(\boldsymbol{\eta}, \hat{\boldsymbol{u}}) + [\nabla g_\eta(\boldsymbol{\eta}, \hat{\boldsymbol{u}})]^{\mathrm{T}}(\boldsymbol{\eta}) + [\nabla g_u(\boldsymbol{\eta}, \hat{\boldsymbol{u}})]^{\mathrm{T}}(\hat{\boldsymbol{u}}) \\ \boldsymbol{a}_x &= -[\nabla g_\eta(\boldsymbol{\eta}, \hat{\boldsymbol{u}})]^{\mathrm{T}} \text{ and} \\ \boldsymbol{a}_u &= [\nabla g_u(\boldsymbol{\eta}, \hat{\boldsymbol{u}})]^{\mathrm{T}} \end{aligned} \quad (9)$$

Similarly, the coefficients, b_0, $\boldsymbol{b}_x$, and $\boldsymbol{b}_u$ come from the first-order approximation of the compatibility constraint (between two disciplinary analyses), $\boldsymbol{u}_\eta(\boldsymbol{\eta})-\hat{\boldsymbol{u}}$.

$$\begin{aligned} b_0 &= \boldsymbol{u}_\eta(\boldsymbol{\eta}, \hat{\boldsymbol{u}}) + \nabla \mathrm{u}_\eta(\boldsymbol{\eta}, \hat{\boldsymbol{u}})(\boldsymbol{\eta}) + \nabla \mathrm{u}_u(\boldsymbol{\eta}, \hat{\boldsymbol{u}})(\hat{\boldsymbol{u}}) \\ \boldsymbol{b}_x &= -\nabla \boldsymbol{u}_\eta(\boldsymbol{\eta}, \hat{\boldsymbol{u}}) \text{ and} \\ \boldsymbol{b}_u &= -1 - \nabla \boldsymbol{u}_u(\boldsymbol{\eta}, \hat{\boldsymbol{u}}) \end{aligned} \quad (10)$$

Differentiating the Lagrangian with respect to $\boldsymbol{\eta}$ and setting the derivative to zero gives:

$$\boldsymbol{\eta} = -\lambda_1 \boldsymbol{a}_x - \lambda_2 \boldsymbol{b}_x \quad (11)$$

Differentiating with respect to, $\boldsymbol{u}$, λ_1 and λ_2, substituting with Eq. (10), and setting the partial derivatives to zero results in the following matrix equation:

$$\begin{bmatrix} \boldsymbol{a}_u & \boldsymbol{b}_u & \boldsymbol{0} \\ \boldsymbol{a}_x & \boldsymbol{a}_x\boldsymbol{b}_x & -\boldsymbol{a}_u \\ \boldsymbol{a}_x\boldsymbol{b}_x & \boldsymbol{b}_x & -\boldsymbol{b}_u \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \hat{\boldsymbol{u}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ a_0 \\ b_0 \end{Bmatrix} \quad (12)$$

For linear systems, solving Eqs. (11) and (12) gives a critical point to the Lagrangian. If this critical point is a minimum, the solution is the most probable point. For non-linear systems, the solution may be used iteratively to update the MPP, $\boldsymbol{\eta}^*$, and the auxiliary variables u. Eq. (12) may be considered an extension of Eq. (6), except that the solution of Eq. (12) accounts for the auxiliary variables, $\hat{\boldsymbol{u}}$, and the additional compatibility constraint, $\boldsymbol{u}(\boldsymbol{\eta})-\hat{\boldsymbol{u}}$. Fig. 6 demonstrates the multi-constraint FORM algorithm. A pseudo code for the algorithm follows:

Psuedo code for Multi-constraint FORM

Initiate: i = 0, $\boldsymbol{x}_0$, $\hat{\boldsymbol{u}}_0$, $\beta_0 = 100$

Repeat while i < 3 AND $|\beta_{\text{i}} - \beta_{\text{i-1}}| \geq .001$

$\boldsymbol{\eta}_{\text{i}} = T(\boldsymbol{x}_i)$ transform MPP candidate to standard normal space

Find $\text{g}_\eta(\boldsymbol{\eta}_i, \hat{\boldsymbol{u}}_i)$, $\nabla\,\text{g}_\eta(\boldsymbol{\eta}_i, \hat{\boldsymbol{u}}_i)$, $\boldsymbol{u}(\boldsymbol{x}_i, \hat{\boldsymbol{u}}_i)$, $\nabla(\boldsymbol{x}_i, \hat{\boldsymbol{u}}_i)$

Find $\boldsymbol{a}$ and $\boldsymbol{b}$ (Eqs. 9 and 10)

Solve Eq. 12 for $\hat{\boldsymbol{u}}_{i+1}, \boldsymbol{\lambda}$: $\begin{bmatrix} \boldsymbol{a_u} & \boldsymbol{b_u} & \boldsymbol{0} \\ \boldsymbol{a_x} & \boldsymbol{a_x b_x} & -\boldsymbol{a_u} \\ \boldsymbol{a_x b_x} & \boldsymbol{b_x} & -\boldsymbol{b_u} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \hat{\boldsymbol{u}}_{i+1} \end{Bmatrix} = \begin{Bmatrix} 0 \\ a_0 \\ b_0 \end{Bmatrix}$

Solve Eq. 11 for $\boldsymbol{\eta}_{i+1} = -\lambda_1 \boldsymbol{a_x} - \lambda_2 \boldsymbol{b_x}$

$\beta_{\text{i+1}} = \|\boldsymbol{\eta}_{i+1}\|$

Check $|\beta_{\text{i}} - \beta_{\text{i-1}}|$

$i = i + 1$

END

$P(\text{g}<0) \approx \Phi(-\beta_{\text{i-1}})$

Note that this algorithm is distinguished from the Rackwitz-Fiessler method in that iterative, multidisciplinary analysis in not required. Instead, auxiliary intermediate variables, $\hat{\boldsymbol{u}}$ are used to evaluate each discipline.

This method can be extended to include even more constraints by augmenting the Lagrangian in Eq. (11) and following the steps in Eqs. (8) to (12). Thus it is referred to as multi-constraint FORM. Multi-constraint FORM is a distributed method since the multidisciplinary feasibility conditions are simply added as constraints in the MPP search and only satisfied at convergence. The potential computational advantage of this approach is demonstrated on a numerical example in the following section.

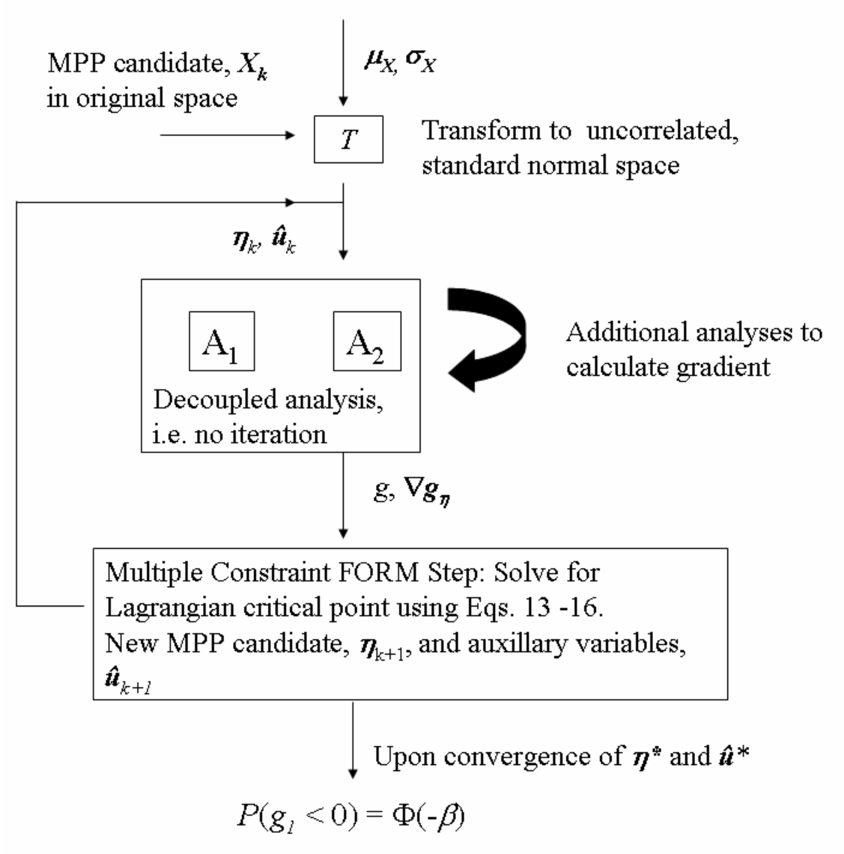


Figure 6. Multi-constraint FORM for multidisciplinary system

Numerical Example

Each of the proposed decoupled probabilistic analysis algorithms: (1) partial FOSM, and (2) decoupled, multi-constraint FORM, are applied to the two-discipline example system in Fig. 1, taken from Du and Chen (2002). In addition, as a baseline, the basic Monte Carlo method, fully-integrated FOSM (Fig. 3), and distributed FORM (using a standard sequential quadratic programming algorithm and the Rackwitz-Fiessler formula of Eq. 6) are applied to the same system to the integrated system. The results are compared with respect to accuracy and computational efficiency.

The functional relationships for the disciplinary analyses are as follows:

Analysis 1	Analysis 2
$\mathrm{x}_s = \{x_1\}, \mathrm{x}_1 = \{x_2, x_3\}$	$\mathrm{x}_s = \{x_1\}, \mathrm{x}_2 = \{x_4, x_5\}$
$\mathrm{u}_{1,2} = x_1^2 + 2x_2 - x_3 + 2\sqrt{\mathrm{u}_{2,1}}$	$\mathrm{u}_{2,1} = x_1 x_4 + x_4^2 + x_5 + \mathrm{u}_{1,2}$
$\mathrm{g}_1 = 4.5 - (x_1^2 + 2x_2 + x_3 + x_2 e^{-\mathrm{u}_{2,1}})$	$\mathrm{g}_2 = \sqrt{x_1} + x_4 + x_5(0.4x_1)$
f = N/A	

The limit state for the system is given by g_1, so that the probability of failure, P_f, is given by $P_f = P(g_1 < 0)$.

The system is undefined in a region for which $u_{2,1} < 0$ but is continuously differentiable over the region of interest. Although this system may be solved algebraically by variable reduction, the comparison is based on using fixed-point iteration to find the feasible system. (In other words, a trial value of $u_{2,1}$ is selected, next $u_{1,2}$ is computed from analysis 1, then $u_{2,1}$ is computed from analysis 2; and the process is repeated with the new value for $u_{2,1}$ until convergence.) This is done to simulate the behavior of large multidisciplinary systems that may not have closed-form solutions. For the same reason, finite differencing is used to approximate the gradients even though analytical derivatives could easily be derived for this particular example.

Partial FOSM

Partial FOSM techniques are employed on the example system, first combining FOSM with Monte Carlo Simulation and then with FORM using the Rackwitz-Fiessler algorithm in Eq. (6). Both techniques use FOSM on the multidisciplinary system to estimate the mean and standard deviations of the intermediate variables, and assume that they variables follow a normal distribution. The first method subsequently performs a Monte Carlo analysis on A_1 to determine the probability of failure, $P_f = P(g_1 \le 0)$. No attempt is made to assure a feasible system, rather the auxiliary variables are simply treated as random input variables with a normal distribution and mean and standard deviation as calculated via FOSM. The second method, alternatively, performs a Rackwitz-Fiessler (RF) FORM analysis to determine P_f. (In this case, Eq. (6) is solved iteratively for A_1 only, again treating intermediate variables as normally distributed random variables with the mean and standard deviation given by the previous FOSM analysis. These methods are compared with Monte Carlo, integrated RF-FORM analysis, and direct, integrated FOSM analyses on the multidisciplinary system. In other words, the system is considered a black box and multidisciplinary feasibility is required with each function call. The results are shown in Table 1.

As expected, the FOSM method, when applied directly on the coupled analysis of the system, is not very accurate. The linear approximation of the limit state is taken at the mean value rather than the MPP, resulting in poor estimation for the tail end of the joint probability distribution which is where failure occurs. However, using the partial FOSM method with RF-FORM increases the accuracy with very little additional computational effort. When RF-FORM alone is applied to the multidisciplinary analysis, 324 function calls are needed. This is due to the fixed-point iteration in both the evaluation of the limit state and the gradient. For the partial FOSM/R-F method, no system convergence loops are needed, resulting in significant computational savings. The same is true for Monte Carlo analysis. Although ten thousand iterations are called for, each system analysis call requires 18 function calls (on average) in the convergence process. This is not needed when the parameters of the coupling variable, $\boldsymbol{u}_{2,1}$, are approximated with FOSM; in this case, each Monte Carlo run calls for a single disciplinary analysis. Thus, using the partial FOSM may achieve significant computational savings in this example without significantly sacrificing accuracy.

Table 1: Comparison of Integrated Reliability Methods with Partial FOSM

Method	β	P_f	Number of disciplinary analyses*
Monte Carlo (integrated)	$\Phi^{-1}(.0557) = 1.5919$	.0557	180,186
Rackwitz-Fiessler (RF) FORM (integrated)	1.6252	.0521	324
FOSM (integrated)	$\frac{\mu_{g_1}}{\sigma_{g_a}} = \frac{.500}{.3001} = 1.6661$	.0478	126
FOSM with Monte Carlo	$\Phi^{-1}(.0550) = 1.5982$	.0550	10,126
FOSM with RF-FORM	1.6251	.0521	148

*The number of analyses includes finite difference runs to approximate the gradient in the case of FORM.

Using FOSM as the initial step may be particularly valuable when the limit state function is an output of a fairly simple analysis but relies on input from a more computationally intensive analysis. In this case, the difficult analysis need only be performed as a part of the FOSM process, while FORM or Monte Carlo can be used on the simpler analysis to improve accuracy. If the limit state function is highly dependent on the intermediate variable, and an incorrect distribution is used, this will obviously affect the accuracy of the partial FOSM method.

Distributed Multi-Constraint FORM

The proposed distributed, multi-constraint FORM algorithm is applied to the example system, and is compared with the integrated R-F algorithm as well as SQP for both integrated and distributed formulations. Results are shown in Table 2.

Table 2: Comparison of Integrated vs. Distributed FORM Methods

Formulation	Method	β	P_f	Number of disciplinary analyses*
Integrated (Eq. 6)	FORM with R-F Algorithm	1.6252	.0521	324
Integrated (Eq. 4)	FORM with SQP	1.6252	.0521	1840
Distributed (Eqs. 11-12)	Multi-constraint FORM	1.6252	.0521	69
Distributed (Eq. 5)	FORM with SQP	1.6252	.0521	370

*The number of analyses includes finite difference runs to approximate the gradient.

From Table 2, it may be seen that all FORM algorithms produced the same result, regardless of whether it was applied to the integrated system (i.e., conventional approach) or the distributed system and regardless of which optimization algorithm was used. (Note, in addition to reaching the same reliability index, all methods converged to the identical most probable point, $\boldsymbol{x'}$ = [2.3477, 1.9014, 0.9507, 0.0, 0.0].) However, using the distributed formulation netted a five-fold reduction in the total number of function evaluations over that of the integrated system. Du and Chen (2002) produced similar results in evaluating the distributed vs. integrated formulations using SQP. However, the proposed multi-constraint FORM algorithm is seen to result in another five-fold improvement in efficiency over the distributed SQP method as it does not implement a line search requiring several additional function evaluations for each iteration. Thus, the proposed multi-constraint FORM-based technique appears promising for dramatic savings in computational effort in the reliability analysis for certain multidisciplinary systems.

Of course, this example only points to the possibility of improvement. Further study is needed to identify specific characteristics of the class of problems for which this algorithm is best suited. At this point, one would anticipate poor performance of the algorithm in situations where the Newton-Raphson formula fails to find roots for either the (1) limit state or (2) disciplinary analysis equations. In the first case, Haldar and Mahadevan (2000) review well known convergence problems associated with using the Rackwitz-Fiessler step such as divergence at an inflection point or oscillation on either side of the most probable point. The multi-constraint FORM method is derived in the same way so one would expect similar limitations. In many cases, this problem may be circumvented by selecting a different starting point. In the second case, the multi-constraint FORM algorithm presumes that a gradient-based step is an efficient means of satisfying disciplinary analyses compared to multidisciplinary analysis (such as fixed point iteration.) Addition of a line search could also be implemented to facilitate convergence, but this would offset much of the savings in computational effort. A similar modification would be to implement a hybrid algorithm that defers to SQP at the first sign of divergence or oscillation with multi-constraint FORM. (This strategy is employed in the following chapter). Other areas which need to be explored include the effects of dimensionality of the vector of auxiliary variables (a measure of the 'tightness' of interdisciplinary coupling), the availability of analytical derivatives, the number of disciplines, and the dimension of the design variable vector.

Conclusion

This chapter developed two computational algorithms that take advantage of a distributed formulation to perform reliability analysis of multidisciplinary systems. A Partial FOSM method may be most useful for multidisciplinary systems where (1) the limit state failure probability is relatively insensitive to intermediate disciplinary response variables and (2) the expense of a single disciplinary analysis yielding the system limit state is much less than that of multidisciplinary analysis. In addition, the technique is particularly useful to marry with sampling methods to account for non-linearity in a system limit state with respect to random input variables. A multi-constraint FORM was also presented. In general, it is applicable for limit states and disciplinary analyses which are continuously differentiable; it also necessitates that a solution to the multidisciplinary system exists at the most probable point.

Each of these ideas has promise but needs to be examined in more detail. First, many additional examples are needed to identify system characteristics required for the algorithms to be effective. For the partial FOSM methods, this would ideally lead to a relative a priori prediction on accuracy based on failure sensitivity to auxiliary variables. The multi-constraint FORM method is expected to encounter convergence problems for particular systems; it would be valuable to determine characteristics of such systems and to develop early exit criteria so that minimal effort is expended before resorting to alternative algorithms. In addition, gradient approximations are a key factor when using either FORM or FOSM and usually dominate the computational effort. Decoupling the analysis needs to be exploited to a greater extent in this regard. It is obvious that first-order reliability estimates are approximate; typically efficient Monte Carlo schemes, such as importance sampling (Haldar and Mahadeven, 2000), are used subsequently to improve the accuracy of the FORM estimates. Future work needs to develop efficiency in important sampling in the context of multidisciplinary analysis. The techniques should also be evaluated for realistic multidisciplinary problems to evaluate their robustness.

Finally, many multidisciplinary systems are not continuous presenting problems for multidisciplinary analysis let alone reliability analysis. However, if the system limit state is differentiable in the vicinity of the MPP and the multidisciplinary system is feasible at the MPP, there is a solution to the FORM formulation. Further modification of the multi-constraint FORM method would be required for the MPP search in this situation.

The ultimate goal in sharing methodology between probabilistic multidisciplinary analysis and multidisciplinary optimization is to efficiently solve probabilistic multidisciplinary optimization problems or multidisciplinary optimization under uncertainty. If probabilistic constraints are given for an MDO problem, an outer optimization loop needs to be added to Fig. 1, further compounding the computational effort. The development of probabilistic MDO has been recently reported, by combining reliability-based design optimization (RBDO) and MDO methods, utilizing the decoupling concept (Chiralaksanakul and Mahadevan, 2004). These methods can be further enhanced by incorporating the proposed multidisciplinary reliability analysis techniques. To this end, the next chapter presents twelve algorithms for reliability-based optimization of multidisciplinary systems. These algorithms build upon the ideas presented in this chapter by combining efficient reliability analysis of multidisciplinary systems with methods for multidisciplinary optimization and reliability-based optimization.

CHAPTER III

RELIABILITY-BASED OPTIMIZATION OF MULTIDISCIPLINARY SYSTEMS

Introduction

This chapter extends the strategies employed in first-order reliability *analysis* of multidisciplinary systems for the development and study of algorithms for reliability-based *optimization* of multidisciplinary systems in accordance with the second research objective. The significance of this extension is that *optimization* provides a means for design, specifically for designing systems that meet predetermined standards for reliability. However, this capability comes at the expense of significant additional computational effort. The algorithms provided (referred to herein as MDO-RBDO methods) exploit efficiencies from existing deterministic multidisciplinary optimization (MDO) methods and single discipline reliability-based design optimization (RBDO) to mitigate the computational expense of probabilistic design of multidisciplinary systems.

The performance of various RBDO-MDO algorithms is investigated with three simple example problems to gain insight into the relative consistency, accuracy, and efficiency of each method. To this end, twelve basic algorithms are developed and tested on each example. Each algorithm combines an RBDO strategy (nested, sequential, or single-loop) using either a direct or inverse first order reliability method (FORM) with two common MDO formulations (fully integrated analysis or simultaneous analysis and design). The RBDO strategy uses first-order reliability analysis (FORM) to evaluate probabilistic constraints either directly or through an inverse formulation.

The following section reviews the concepts behind the twelve algorithms, providing a general formulation for the class of MDO-RBDO problems. Then, a brief review of MDO and RBDO concepts is given leading to a classification of the MDO-RBDO combination algorithms. This is followed with a detailed methodology discussion of each algorithm. The performance of the algorithms is then evaluated for three example problems. Comparative analysis of the algorithm's performance is based on efficiency (analysis count), accuracy, and consistency (ability to converge to an optimum regardless of starting point.)

MDO-RBDO Formulation

The class of RBDO-MDO problems is formulated as in Eq. (1) as defined in Chapter I.

$$\text{Minimize } f(\boldsymbol{d})$$

$$\text{s.t. } P\{\, g_{\eta(i)}\, [\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{u}(\boldsymbol{d}, \boldsymbol{\eta})] = 0\} \leq P_{\text{acceptable,}}\;\; i = 1 \ldots m_{\text{constraints}} \tag{1}$$

$$\text{where } A_{\eta(j)}\, [\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{u}(\boldsymbol{d}, \boldsymbol{\eta})] = 0,\;\; j = 1 \ldots m_{\text{disciplines}}$$

The most straight-forward approach to reliability-based optimization (RBDO) employs an 'outer' optimization loop with reliability analysis as an inner loop, multiplying the computational effort for each reliability constraint evaluation. Thus, for multidisciplinary systems, conventional reliability-based optimization involves three nested loops: two optimization loops (one for reliability analysis and one for the system optimization itself) and a multidisciplinary analysis loop. This effect is shown graphically in Fig. 1.

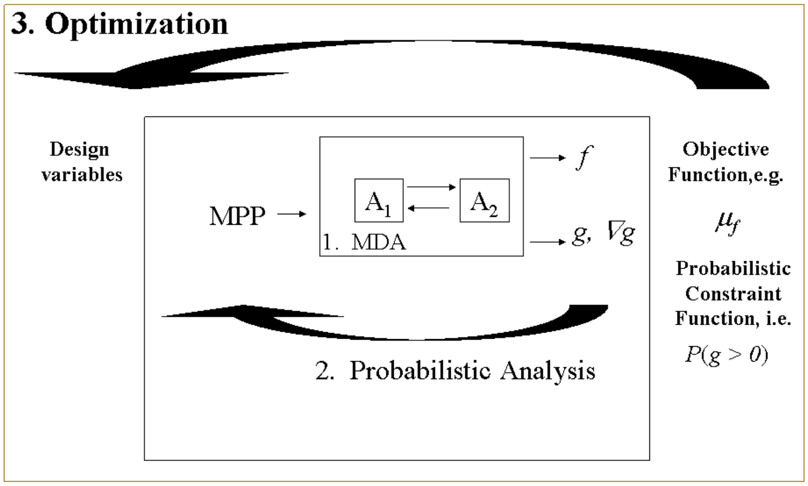


Figure 1. Reliability-based Optimization of Multidisciplinary Systems

Fortunately, as demonstrated in the previous chapter, using distributed strategies from MDO may reduce this effort by combining the inner two loops. At the same time, recent advances in RBDO have led to improvements in synthesizing the outer two loops. The algorithms presented in this chapter incorporate both of these techniques, aiming to provide alternatives for solving Eq. (1) most efficiently.

MDO-RBDO Concepts and Classification

Table 1: Synopsis of RBDO-MDO algorithms considered

	MDO Strategy			
RBDO Strategy	Fully-integrated Analysis		Simultaneous Analysis and Design	
Nested	1. Direct FORM	2. Inverse FORM	7. Direct FORM	8. Inverse FORM
Sequential	3. Direct FORM	4. Inverse FORM	9. Direct FORM	10. Inverse FORM
Single-loop	5. Direct FORM	6. Inverse FORM	11. Direct FORM	12. Inverse FORM

The algorithms employed in this chapter are classified in three ways yielding twelve combination algorithms as depicted in Fig. 1. The first classification is according to their underlying formulation, specifically how the probabilistic constraint is evaluated. Chapter I provided an overview of reliability-based design optimization, discussing in some detail two primary means to ensure satisfaction of a first order probabilistic constraint: a direct FORM method known as the reliability-index (RIA) approach, and the performance measure approach (PMA) based on an inverse formulation. The direct first-order reliability-based optimization formulation given in Eq. (7) in Chapter I is restated here in Eq. (2).

$$\text{Minimize } f(\mathbf{d}) \text{ s.t. } \beta \geq \beta_{\text{target,}}\ i = 1...m \tag{2}$$

where $\boldsymbol{d}$ is the vector of design variables and the acceptable probability, $P_{\text{acceptable}}$ is transformed to a target reliability index, β_{target} using the inverse of the standard normal cumulative distribution, i.e., $\beta_{\text{target}} = -\Phi^{-1}(P_{\text{acceptable}})$. The first-order reliability index, β is defined by Eq. (3).

$$\text{Minimize } \beta = \|\boldsymbol{\eta}\| \tag{3}$$

$$\text{s.t. } g_{\eta}(\boldsymbol{d}, \boldsymbol{\eta}) = 0$$

Alternatively, an "inverse" FORM method is often used for RBDO as given in Eq. (9) in Chapter I and restated here as Eq. (4).

$$\text{Minimize } f(\boldsymbol{d}) \tag{4}$$

$$\text{s.t. } g^* \geq 0$$

where g^* is defined by Eq. (5).

$$\text{Minimize } g^* = g_{\eta}(\boldsymbol{d}, \boldsymbol{\eta}) \tag{5}$$

$$\text{s.t. } \|\boldsymbol{\eta}\| = \beta_{\text{target}}$$

Thus, the RBDO algorithms used herein are distinguished first as either direct (i.e., based on Eqs. (2)-(3)) or inverse FORM methods (based on Eqs. (4)-(5)).

The second criterion for classifying the MDO-RBDO algorithms is based on the method used to ensure multidisciplinary feasibility. The first set of methods uses fully-integrated analysis and optimization. Consider again the optimization formulation as shown in Eq. (1). Each disciplinary analysis depends on both system inputs ($\boldsymbol{d}$) and responses ($\boldsymbol{u}$) from the other disciplines. A feasible system is defined as one in which inputs and discipline outputs simultaneously satisfy all disciplinary analysis equations, $\mathbf{A}[\boldsymbol{d},\boldsymbol{\eta},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta})] = 0$. A single objective or constraint function evaluation involves solving a system of non-linear equations to ensure that shared response values are compatible across all disciplines and that there are no disciplinary analysis residuals. This representation is trivially identical to the standard optimization formulation, but highlights the interdependency of disciplinary analyses on one another, the objective function, and the constraint functions. For fully-integrated analysis, the set of disciplinary equations must be solved completely each time disciplinary responses are required for either reliability analysis (to determine the limit state) or optimization.

However, as an alternative to fully-integrated analysis, one may opt for the simultaneous analysis and design approach, as employed in the context of the direct FORM algorithms for reliability analysis presented in Chapter II. With SAND, auxiliary variables representing the discipline response variables are employed as independent design variables. In other words, in the SAND formulation, both $\boldsymbol{d}$ and $\hat{\boldsymbol{u}}$ are design variables, where $\hat{\boldsymbol{u}}$ denotes the surrogate of the discipline response variables. Compatibility constraints are added to the optimization problem to ensure the disciplinary analysis equations are satisfied. For any feasible solution, these compatibility constraints also ensure that the surrogate response variables are the same as the true response variables (i.e., $\hat{\boldsymbol{u}} = \boldsymbol{u}(\boldsymbol{d})$). In this way, the system optimizer also ensures multidisciplinary compatibility but avoids spending significant effort to do so away from the optimal design point. Note that although the previous chapter demonstrated the value of using SAND to improve the computational efficiency of direct first order reliability analysis, the same process may apply to an inverse FORM formulation.

The third and final classification of MDO-RBDO algorithms is based on the technique for combining reliability analysis with optimization. As mentioned briefly in Chapter I, researchers have developed several techniques to streamline reliability-based optimization of single discipline systems (Royset et al, 2001; Du and Chen, 2002; Liang and Mourelatos, 2004; Zou et al). Based on these developments, RBDO algorithms can be generally classified by three fundamental approaches to combining optimization with reliability analysis (loops 2 and 3 from Fig. 1): nested, sequential, and single-loop.

The most straightforward method for performing reliability-based design involves nested optimization, as shown by the combinations of optimizations in Eqs. 2 and 3 for direct FORM or Eqs. 4 and 5 for inverse FORM. (Note: this would comprise the two outer loops of Fig. 1, i.e., loops 2 and 3). However, in order to alleviate the 'nested' effect of RBDO, one can either decouple (i.e., separate) the reliability analysis from the primary optimization problem (Royset et al, 2001; Du and Chen, 2002; Zou et al, 2002), or combine them into a single loop. The first approach borrows from a common optimization strategy, solving sequential subproblems. For RBDO, the optimization subproblem uses deterministic constraints to approximate probabilistic constraints.

Reliability analysis then follows full optimization to update these constraints. The process is repeated until convergence. Du and Chen apply this technique with inverse FORM in their "Sequential Optimization and Reliability Analysis (SORA)" method. Alternatively, Zou et al. (2002) presents a sequential RBDO algorithm which uses a first order approximation of the probabilistic constraint, enabling any reliability analysis technique (e.g., direct or inverse FORM, as well as simulation methods) to update the deterministic subproblem. The second strategy combines reliability analysis and optimization in a single-loop. For example, Liang and Mourelatos (2004) demonstrate computational improvement through a single loop RBDO method which imposes the Karush-Kuhn-Tucker optimality conditions of the reliability 'loop' for the representative deterministic constraints. In this case, each iteration includes both a step toward the optimal design, $\boldsymbol{d}$ and a step toward the MPP, $\boldsymbol{\eta}^*$ for direct FORM or the PMA point, $\boldsymbol{\eta}'$ for inverse FORM. Thus, the final classification divides the RBDO methods into three groups according to how optimization and reliability analysis are combined: nested, sequential, and single-loop.

The two RBDO formulations (direct and inverse FORM) combine with the two MDO approaches (fully-integrated and simultaneous analysis and design) and the three RBDO strategies for combining optimization with reliability analysis (nested, sequential, and single loop) to form twelve algorithms. Table 1 provides a summary of the MDO and RBDO techniques comprising each method. These 12 algorithms are founded upon the theory due to Chiralaksanakul and Mahadevan (2004) for integrating reliability-based design optimization (RBDO) with multidisciplinary optimization. This methodology specifically addresses how multidisciplinary feasibility is assured and disciplinary responses are tracked during both reliability analysis and optimization iterations as described in more detail in the following section.

Methodology

Method 1: Fully-integrated, Nested RBDO-MDO Using Direct FORM

Fully-integrated MDA: The first six methods are 'fully-integrated' with respect to the multidisciplinary analysis. In other words, each time the limit state $g_{\eta(i)}[\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{u}(\boldsymbol{d}, \boldsymbol{\eta})]$ is to be evaluated, the system of multidisciplinary equations given by $A_{\eta}[\boldsymbol{d}, \boldsymbol{\eta}, \boldsymbol{u}(\boldsymbol{d}, \boldsymbol{\eta})]=0$ must first be solved for $\boldsymbol{u}(\boldsymbol{d}, \boldsymbol{\eta})$. For this study, these non-linear equations are solved using Newton's method; analytical gradients (∇A) are calculated to avoid adding analysis evaluations for finite difference approximations. In practice, multidisciplinary analysis is most often done by fixed point iteration. However, since a gradient-based optimizer will be used to solve the system equations for the simultaneous analysis and design MDO methods, using Newton's method to solve them for the fully-integrated analysis provides the more equitable basis for comparison.

Nested RBDO: Fig. 2 outlines the pseudo code for nested optimization and reliability analysis as employed in Methods 1, 2, 7 and 8. All nested methods use sequential quadratic programming (SQP) for the outer optimization loop in this study; in this context k tracks the outer loop iterations. The inner loop algorithm is dependant on the reliability analysis technique employed. For candidate design points, the algorithm must evaluate the objective and constraint functions and their gradients; then based on the results, select an appropriate descent direction and step size.

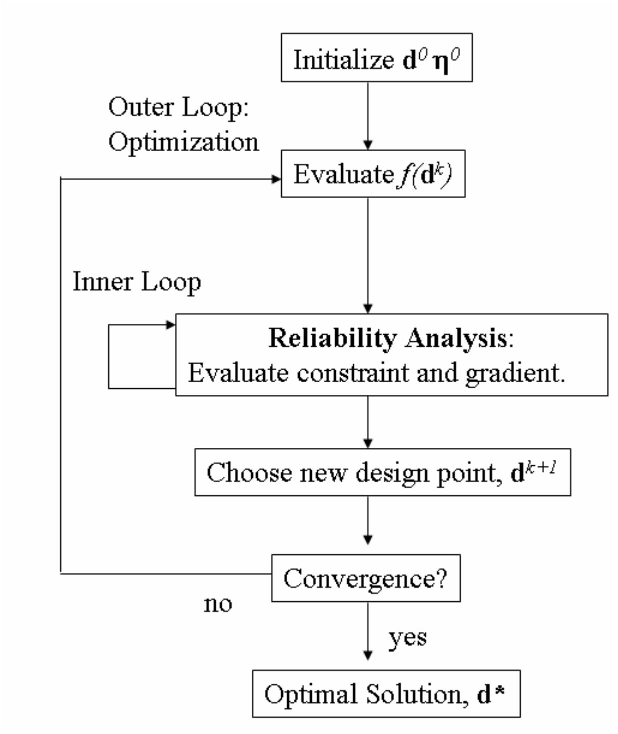


Figure 2. Pseudo Code for Nested Methods

Direct FORM Reliability Analysis: The first method uses direct FORM for reliability analysis (i.e., MPP search as in Eq. (3)). Since the reliability analysis is repeated for every optimization loop, the efficiency of this step is critical. Here, the Rackwitz-Fiessler (R-F) Newton step given in Eq. (6) is applied iteratively until convergence as the 'first choice' MPP search algorithm; in this case, q tracks the inner loop or reliability analysis iterations. Note that in order to evaluate the limit state $g_{\boldsymbol{\eta}}$ or its gradient, the discipline response variables, $\boldsymbol{u}(\boldsymbol{d}, \boldsymbol{\eta}^q)$ are needed where $\boldsymbol{\eta}^q$ is the random variable vector in standard normal space for the q^{th} iteration of Eq. (6).

$$\boldsymbol{\eta}^{q+1} = \frac{1}{\|g_\eta\|}\,[(\nabla_\eta g)^T \boldsymbol{\eta}^q - g_\eta]\nabla_\eta g \qquad (6)$$

The R-F step will typically find the MPP in fewer than 10 (inner loop, reliability analysis) iterations if successful. Unfortunately, however, this method may not converge for some limit states (Haldar and Mahadevan, 2000). For this reason, SQP is invoked when the RF algorithm fails to converge after 10 iterations.

Note from the pseudo code in Fig. 2 that the SQP optimizer requires gradients of all constraints, including probabilistic constraints. The gradient for the probabilistic constraint is calculated as in Eq. (7):

$$\frac{\partial \beta}{\partial \boldsymbol{d}} = \frac{\dfrac{\partial g_\eta[\boldsymbol{d},\boldsymbol{\eta}^*,\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^*)]}{\partial \boldsymbol{d}}}{\sqrt{\|\nabla_\eta g\|}} \qquad (7)$$

where β is the reliability index, $\boldsymbol{\eta}^*$ is the direct FORM MPP, and $\nabla_\eta g$ is the gradient of the limit state with respect to the MPP.

Method 2: Fully-integrated, Nested RBDO Using Inverse FORM

Inverse FORM Reliability Analysis: The second method only differs from Method 1 in that it uses inverse FORM for reliability analysis (i.e., optimization as in Eq. (4) coupled with PMA point search as in Eq. (5)). To improve the efficiency of this step, Eq. (8) below is applied iteratively as the 'first choice' MPP search algorithm as it will often converge with significantly fewer function evaluations than generic optimization algorithms. Sequential quadratic programming is invoked when Eq. (8) fails to converge after 10 iterations.

$$\boldsymbol{\eta}^{q+1} = \frac{\nabla_{\boldsymbol{\eta}}\, g}{\sqrt{\left\| \nabla_{\boldsymbol{\eta}}\, g \right\|}} \beta_{\text{target}} \tag{8}$$

The gradient for the probabilistic constraint using inverse FORM (Eq. (4)) is simply the gradient of the limit state with respect to the design variable vector, $\boldsymbol{d}$. In other words $\frac{\partial g^*}{\partial \boldsymbol{d}} = \frac{\partial g(\boldsymbol{d}, \boldsymbol{\eta}')}{\partial \boldsymbol{d}}$.

Method 3: Fully-integrated, Sequential RBDO Using Direct FORM

Sequential Optimization and Direct FORM Reliability Analysis: In sequential optimization and reliability analysis, deterministic subproblems are solved sequentially so as to decouple reliability analysis from optimization as depicted by the pseudo code given in Fig. 3. Since the optimization subproblem is deterministic, reliability analysis is not required to evaluate the constraints. Probabilistic analysis follows the optimization, updating the subproblem. The entire sequence is then repeated iteratively but usually converges quickly. The third method uses sequential deterministic optimization subproblems formed by linearizing the probabilistic constraint. Equation (10) gives the general form of the subproblem given by Zou and Mahadevan (2004).

$$\text{Minimize}\, f(\boldsymbol{d}) \tag{9}$$

$$\text{s.t.}\;\; P_f^k - \nabla_d P_f^k (\boldsymbol{d} - \boldsymbol{d}^k) \le P_{\text{acceptable}}$$

where P_f^k is the current estimate of the failure probability, $P(g \le 0)$. The solution to the subproblem, Eq. (9), gives the next iteration for the design variable vector, $\boldsymbol{d}^{k+1}$. Note that the linearization could just have easily been performed on the reliability index, β as in Eq. (2), given the FORM relationship $P_f = \Phi(-\beta)$. However, Zou and Mahadevan's subproblem is more generic in that it can also accommodate reliability analysis methods other than FORM (e.g., Monte Carlo Simulation, second order methods) to evaluate P_f. In this method, direct FORM is used as outlined in Method 1. In other words, the R-F step, Eq. (6), is applied iteratively to determine the most probable point, η^* and the probability of failure is subsequently determined from the reliability index, β. The chain rule of differentiation gives the probability gradient as in Eq. (10).

$$\nabla_{\boldsymbol{d}} P_f^k = -\phi(-\beta)\frac{\partial \beta}{\partial \boldsymbol{d}} \tag{10}$$

where $\frac{\partial \beta}{\partial \boldsymbol{d}}$ is given by Eq. (7) as described in the first method. Note that in implementation, this method is very similar to Method 1 if SQP is used as the optimization algorithm since SQP will also linearize the probabilistic constraint. However, this method is unique in a two ways. First, no representative model for the objective is given by Eq. (9); SQP uses a quadratic local model. Second, any non-probabilistic constraints would be included in Eq. (9) in their initial state. (Non-probabilistic constraints were excluded from Eq. (1) to simplify the MDO-RBDO formulation but could be present in many real applications.) A final practical distinction is that SQP algorithms typically require multiple constraint evaluations for each iteration during a line search while this method does not conduct a line search.

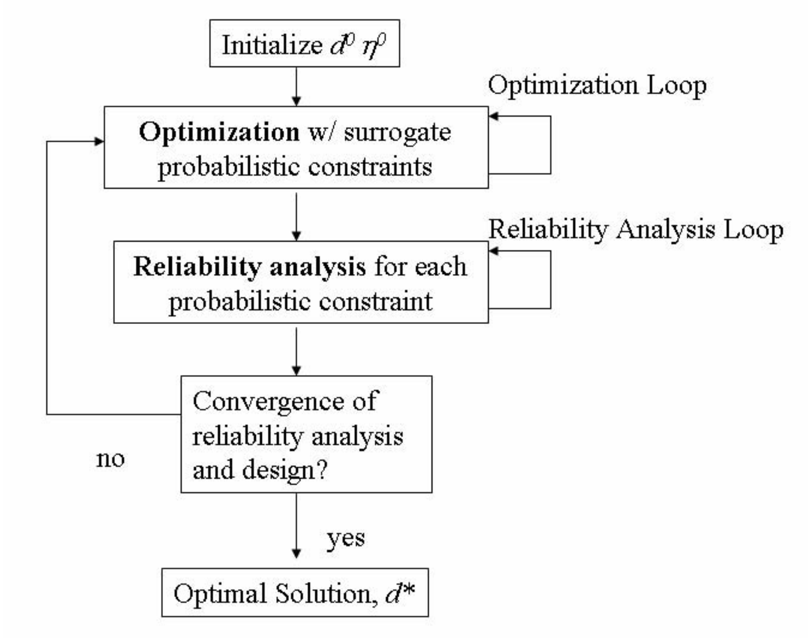


Figure 3. Pseudo Code for Sequential Methods

Method 4: Fully-integrated, Sequential RBDO Using Inverse FORM

Sequential Optimization and Inverse FORM Reliability Analysis: The fourth method employs sequential optimization with inverse reliability analysis. In this case, the deterministic subproblem is formed by fixing the realization of the standard normal variable, $\boldsymbol{\eta}^k$ (representing the current solution of Eq. (4) for the PMA point, $\boldsymbol{\eta}'$) for each constraint as in Eq. (11).

$$\text{Minimize } f(\mathbf{d}) \tag{11}$$

$$\text{s.t. } g_{\eta(i)}[\boldsymbol{d},\boldsymbol{\eta}_i^k,\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}_i^k)] \geq 0 \text{ for i} = 1\ldots m_{\text{constraints}}$$

Inverse FORM reliability analysis is performed as described in Method 2, i.e., by conducting an iterative search for the PMA point using Eq. (8). This provides a new estimate of the MPP, $\boldsymbol{\eta}^{k+1}$, which in turn is used in the next deterministic optimization subproblem. Note that each constraint is a limit state and thus will have its own PMA point. The process is repeated until convergence. Sequential RBDO using inverse FORM is the basis of the SORA method (sequential optimization and reliability analysis) proposed by Du and Chen (2003). However, the SORA method has several additional efficiency strategies. Here only the basic concept of decoupling is adopted in order to compare all the methods on equal footing.

Method 5: Fully-integrated, Single Loop RBDO Using Direct FORM

Single-loop optimization and Reliability Analysis: The final two fully-integrated MDO methods combine optimization and FORM in a single loop. Using direct FORM, the optimization given in Eq. (12) follows. In this approach, the design point, $\mathbf{d}^k$ and the MPP estimate, $\boldsymbol{\eta}^k$ are updated simultaneously in the same loop. The algorithm calculates a new MPP estimate $\boldsymbol{\eta}^k$ each time the optimizer calls on the constraint $\left\|\boldsymbol{\eta}^k\right\| \geq \beta_{\text{target}}$, based on the last calculated value using the R-F step. Both $\boldsymbol{d}$ and $\boldsymbol{\eta}^k$ must ultimately converge before a legitimate solution is reached.

$$\begin{aligned}
&\text{Minimize}\, f(\boldsymbol{d}) \\
&\text{s.t. } \left\|\boldsymbol{\eta}^k\right\| \geq \beta_{\text{target}} \\
&\text{where } \boldsymbol{\eta}^k = \frac{1}{\left\|\nabla g_{\boldsymbol{\eta}}\right\|}\left[\nabla g_{\boldsymbol{\eta}}^T \boldsymbol{\eta}^{k-1} - g_{\boldsymbol{\eta}}\right]\nabla g_{\boldsymbol{\eta}}
\end{aligned} \tag{12}$$

Method 6: Fully-integrated, Single Loop RBDO Using Inverse FORM

Single-loop optimization and Reliability Analysis: The final fully-integrated MDO method combines optimization and inverse FORM in a single loop as given by Eq.

(13). In this case, the PMA point estimate, $\boldsymbol{\eta}^k$ is updated each time the optimizer calls the constraint by enforcing the Karush-Kuhn Tucker solution of inverse FORM, Eq. (4). Just as with the previous method, the new estimate for the PMA point is based on the last calculated estimate and both $\boldsymbol{d}$ and $\boldsymbol{\eta}^k$ must ultimately converge for a final solution.

$$\begin{aligned}
&\text{Minimize } f(\boldsymbol{d}) \\
&\text{s.t. } g_{\eta}(\boldsymbol{d},\boldsymbol{\eta}^k,\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^k)) \geq 0 \\
&\text{where } \boldsymbol{\eta}^k = \frac{-\nabla_{\eta} g(\boldsymbol{d},\boldsymbol{\eta}^{k-1},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^{k-1}))}{\sqrt{\left\|\nabla_{\eta} g(\boldsymbol{d},\boldsymbol{\eta}^{k-1},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^{k-1}))\right\|}}
\end{aligned} \tag{13}$$

Method 7: SAND, Nested RBDO Using Direct FORM

Simultaneous Analysis and Design: The second set of six methods use simultaneous analysis and design (SAND) in lieu of fully-integrated multidisciplinary analysis. In other words, multidisciplinary analysis is not required for every optimization iteration or reliability analysis iteration; rather, multidisciplinary feasibility is enforced as a constraint. For the initial design ($\boldsymbol{d}^0$) and reliability MPP estimate ($\boldsymbol{\eta}^0$), multidisciplinary analysis is performed to find the feasible response variables, $\boldsymbol{u}(\boldsymbol{d}^0, \boldsymbol{\eta}^0)$. From this point on, however, independent auxiliary variables, $\hat{\boldsymbol{u}}$, are used in lieu of disciplinary response variables. The limit state may then be evaluated without requiring multidisciplinary analysis, i.e., $g(\boldsymbol{d}, \boldsymbol{\eta}, \hat{\boldsymbol{u}})$. To ensure the design is feasible, and that the reliability analysis is accurate, the multidisciplinary analysis equations, $A(\boldsymbol{d}, \boldsymbol{\eta}, \hat{\boldsymbol{u}}) = 0$, are added to the reliability analysis formulations (i.e., Eqs. (2) and (4) for direct and inverse FORM, respectively) as constraints. For example, Eq. (14) gives a SAND translation of Eq. (3), direct FORM, to find the reliability index.

$$
\begin{aligned}
&\text{Minimize } \beta = \|\boldsymbol{\eta}\| \\
&\text{s.t. } g_{\eta}(\boldsymbol{d},\boldsymbol{\eta},\hat{\boldsymbol{u}}) = 0 \\
&A(\boldsymbol{d},\boldsymbol{\eta},\hat{\boldsymbol{u}}) = 0
\end{aligned}
\tag{14}
$$

Nested RBDO: Method 7 employs nested RBDO in the same manner as Method 1 (Fig. 2). The distinction is that the MPP search uses the SAND reliability analysis as in Eq. (14) instead of the fully-integrated multidisciplinary analysis.

Direct FORM Reliability Analysis: Computational efficiency of direct FORM relies heavily on the reliability analysis algorithm. For example, the Rackwitz-Fiessler (R-F) Newton step given in Eq. (6) improves the efficiency of the fully-integrated methods described earlier. The R-F Newton step is based on a specific solution to the Karush-Kuhn Tucker (KKT) conditions for a quadratic program approximation of the MPP search optimization problem (i.e., Eq. (3) with a first-order approximation for the limit state). As demonstrated in Chapter II, the same technique may be applied to Eq. (14) to develop an efficient step for direct FORM using SAND. In other words, reliability analysis is performed by finding successive solutions to the quadratic subproblem given in Eq. (15).

$$
\begin{aligned}
&\text{Minimize } \beta = \|\boldsymbol{\eta}\| \\
&\text{s.t. } g_{\eta} + \nabla g_{\eta}(\boldsymbol{\eta} - \boldsymbol{\eta}^{q}) = 0 \\
&A_{\eta} + \nabla A_{\eta}(\boldsymbol{\eta} - \boldsymbol{\eta}^{q}) = 0
\end{aligned}
\tag{15}
$$

If this generally more efficient algorithm fails to converge in 10 iterations, the MATLAB ‘fmincon’ optimizer (which uses a line search to select the step size) is then applied. One other nuance with the SAND method relates to gradient evaluation. The gradient calculation for direct FORM is given in Eq. (7), which requires $\dfrac{\partial g_{\eta}(\boldsymbol{d},\boldsymbol{\eta}^*,\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^*))}{\partial \boldsymbol{d}}$. To avoid multidisciplinary analysis, this derivative may be evaluated via the chain rule as in Eq. (16).

$$\frac{\partial g_{\eta}(\boldsymbol{d},\boldsymbol{\eta}^*,\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^*))}{\partial \boldsymbol{d}} = \frac{\partial g_{\eta}(\boldsymbol{d},\boldsymbol{\eta}^*,\hat{\boldsymbol{u}})}{\partial \boldsymbol{d}} + \frac{\partial \boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^*)}{\partial \boldsymbol{d}} * \frac{\partial g_{\eta}(\boldsymbol{d},\boldsymbol{\eta}^*,\hat{\boldsymbol{u}})}{\partial \hat{\boldsymbol{u}}} \tag{16}$$

where $\dfrac{\partial \boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^*)}{\partial \boldsymbol{d}}$ is calculated by solving $\nabla_d A(\boldsymbol{d},\boldsymbol{\eta}^*,\boldsymbol{u}(\boldsymbol{d},\boldsymbol{\eta}^*)) = 0$.

Method 8: SAND, Nested RBDO Using Inverse FORM

Simultaneous Analysis and Design using Inverse FORM*:* As with the previous method, Method 8 avoids multidisciplinary analysis each time the limit state is evaluated by using independent auxiliary response variables, **û**, and adding multidisciplinary compatibility constraints to reliability analysis. For inverse FORM, this results in Eq. (17).

$$\begin{aligned} &\text{Minimize } g_{\eta}(\boldsymbol{d},\boldsymbol{\eta},\hat{\boldsymbol{u}}) = 0 \\ &\text{s.t. } \|\boldsymbol{\eta}\| = \beta_{\text{target}} \\ &A(\boldsymbol{d},\boldsymbol{\eta},\hat{\boldsymbol{u}}) = 0 \end{aligned} \tag{17}$$

For this study, a standard sequential quadratic programming algorithm is used to solve Eq. (17). This is in contrast to Method 2, which solves Eq. (8) iteratively as the first choice algorithm. No obvious counterpart to Eq. (8) has been found to date for the SAND formulation for inverse FORM.

Method 9: SAND, Sequential RBDO Using Direct FORM

Method 9 follows the same process as outlined in Fig. 3. The distinction between

Method 9 and Method 3, its fully-integrated counterpart, is that direct FORM is implemented using SAND via Eq. (14).

Method 10: SAND, Sequential RBDO Using Inverse FORM

Method 10 also follows the flow outlined in Fig. 3. It is executed in the same manner as Method 4, its fully-integrated counterpart, except that inverse FORM is implemented using SAND via Eq. (17).

Method 11: SAND, Single-loop RBDO Using Direct FORM

Method 11 combines optimization and direct FORM via SAND in a single loop as given by Eq. (18).

$$\begin{aligned}
&\text{Minimize} \, f(\boldsymbol{d}) \\
&\text{s.t.} \ \left\|\boldsymbol{\eta}^k\right\| \geq \beta_{\text{target}} \\
&A(\boldsymbol{d},\boldsymbol{\eta}^k,\hat{\boldsymbol{u}}) = 0 \\
&\text{where} \ \boldsymbol{\eta}^k = \frac{1}{\left\|\nabla g_\eta(\boldsymbol{d},\boldsymbol{\eta}^{k-1},\hat{\boldsymbol{u}})\right\|}\left[\nabla g_{\boldsymbol{\eta}}(\boldsymbol{d},\boldsymbol{\eta}^{k-1},\hat{\boldsymbol{u}})^T\boldsymbol{\eta}^{k-1} - g_{\boldsymbol{\eta}}\right]\nabla g_{\boldsymbol{\eta}}(\boldsymbol{d},\boldsymbol{\eta}^{k-1},\hat{\boldsymbol{u}})
\end{aligned} \tag{18}$$

Method 12: SAND, Single-loop RBDO Using Inverse FORM

The final method combines optimization and inverse FORM via SAND in a single loop as given by Eq. (19).

$$\begin{aligned}
&\text{Minimize} \, f(\boldsymbol{d}) \\
&\text{s.t.} \ g_{\boldsymbol{\eta}}(\boldsymbol{d},\boldsymbol{\eta}^k,\hat{\boldsymbol{u}}) \geq 0 \\
&A(\boldsymbol{d},\boldsymbol{\eta}^k,\hat{\boldsymbol{u}}) = 0 \\
&\text{where} \ \boldsymbol{\eta}^k = \frac{-\nabla_{\boldsymbol{\eta}} g(\boldsymbol{d},\boldsymbol{\eta}^{k-1},\hat{\boldsymbol{u}})}{\sqrt{\left\|\nabla_{\boldsymbol{\eta}} g(\boldsymbol{d},\boldsymbol{\eta}^{k-1},\hat{\boldsymbol{u}})\right\|}}
\end{aligned} \tag{19}$$

Numerical Examples

Example 1:

Each of the twelve RBDO-MDO algorithms was applied to three sample optimization problems. The first numerical example was studied by Chiralaksanakul and Mahadevan (2004). It is presented here as well, as a proof of concept example.

$$
\begin{aligned}
&\text{Maximize } f(\boldsymbol{d}) = \boldsymbol{d_2} \\
&\text{s.t. } P(g_1(\boldsymbol{d},\boldsymbol{x},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{x})) \le 0) \le .0013 \\
&P(g_2(\boldsymbol{d},\boldsymbol{x},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{x})) \le 0) \le .0013 \\
&\rule{10cm}{0.4pt} \\
&\text{where } \boldsymbol{d} = [d_1, d_2],\ \boldsymbol{x} = [x_1 \sim N(1,\ 1),\ x_2 \sim N(1,\ 1)] \\
&g_1(\boldsymbol{d},\boldsymbol{x},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{x})) = x_1 - u_1(\boldsymbol{d},\boldsymbol{x}) + 0.5 * (x_2 + 1) d_1 \\
&g_2(\boldsymbol{d},\boldsymbol{x},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{x})) = u_2(\boldsymbol{d},\boldsymbol{x}) \\
&\rule{10cm}{0.4pt} \\
&A_1(\boldsymbol{d},\boldsymbol{x},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{x})) = x_2 d_1 + 2d_2 - u_1 + u_2 = 0 \\
&A_2(\boldsymbol{d},\boldsymbol{x},\boldsymbol{u}(\boldsymbol{d},\boldsymbol{x})) = 3d_1 - u_1 - u_2 = 0
\end{aligned}
\tag{20}
$$

In this example, the vector, $\boldsymbol{x}$ consists of two random normal variables with a mean of 1 and standard deviation of 1. In this case, there is no direct relationship between the design variable vector, $\boldsymbol{d}$, and the random variable vector. In other words, the probability distribution for the random variable vector, $\boldsymbol{x}$, is independent of $\boldsymbol{d}$. For this reason, the first example is less complex than many RBDO problems. Response variables, $\boldsymbol{u}(\boldsymbol{d}, \boldsymbol{x})$, are determined by solving the disciplinary analysis equations, $A(\boldsymbol{d}, \boldsymbol{x}, \boldsymbol{u}(\boldsymbol{d}, \boldsymbol{x})) = 0$.

Table 2: Results for Example 1

Multidisciplinary Analysis		Fully-Integrated		
RBDO Method		Solution	Disciplinary Evals	Stability Rating
Nested	1. Direct FORM	[.3784 .3216]	76	Good
	2. Inverse FORM	[.3784 .3216]	88	Good
Sequential	3. Direct FORM	[.3784 .3216]	76	Good
	4. Inverse FORM	[.3784 .3216]	80	Good
Single Loop	5. Direct FORM	Does not converge		Poor
	6. Inverse FORM	[.3784 .3216]	39	Good
Multidisciplinary Analysis		**Simultaneous Analysis and Design**		
RBDO Method		Solution	Disciplinary Evals	Stability Rating
Nested	7. Direct FORM	[.3784 .3216]	55	Good
	8. Inverse FORM	[.3784 .3216]	331	Good
Sequential	9. Direct FORM	[.3784 .3216]	55	Good
	10. Inverse FORM	[.3784 .3216]	107	Good
Single Loop	11. Direct FORM	Does not converge		Poor
	10. Inverse FORM	[.3784 .3216]	15	Good

Algorithm performance was compared using three metrics: accuracy, efficiency, and consistency. Accuracy is defined as the ability to get a true local minimum. For this measure, Method 1 gives the baseline solution. The efficiency metric is the number of disciplinary analysis evaluations, $A(\boldsymbol{d}, \boldsymbol{\eta}^k, \boldsymbol{u}(\boldsymbol{d}, \boldsymbol{\eta}^k))$. The baseline point for this evaluation is the mean value optimum (i.e., the deterministic solution to the optimization if the random variable is fixed at its mean value.) To ascertain consistency, three different starting points were used to determine if the algorithm is consistently able to reach a local minimum: a lower bound, the mean value optimum, and an upper bound. A rating of "good" indicates the algorithm converged to the local minimum for all three starting points. A rating of "fair" indicates the algorithm converged to a local minimum using the mean value optimum as the starting point but did not converge for at least one other starting point. A rating of "poor" indicates that the algorithm did not converge to a local minimum at the mean value optimum. The results are given in Table 2.

There are a few interesting observations from this example. First, except for the single loop direct FORM method, all the fully-integrated methods (1-4 and 6) performed well. In this study the single-loop direct FORM methods (5 and 11) failed to converge for any of the examples; thus they are not considered viable MDO-RBDO methods. The single loop inverse FORM method was twice as efficient, which was expected since optimization and reliability analysis are conducted together. Since the multidisciplinary analysis for this example is fairly simple (typically only 2-3 disciplinary analysis calls were required for every fully-integrated multidisciplinary analysis), one would not expect a significant efficiency savings from going to simultaneous analysis and design. In fact, the direct FORM methods performed slightly better using SAND (e.g., methods 7 and 9 performed better than methods 1 and 3) while the inverse methods were less efficient (e.g., methods 8 and 10 were less efficient than methods 2 and 4). That the SAND nested, inverse FORM (Method 8) performed as inefficiently as it did (331 disciplinary evaluations), was somewhat surprising. On closer examination, the SAND inverse FORM inner loop required only 2 outer optimization loops, but each loop needed over 150 disciplinary analyses for the inverse FORM reliability analysis. Apparently, adding the additional multidisciplinary compatibility constraint significantly complicated the inverse FORM problem.

Example 2:

The second example, given by Eq. (21) was derived from a common example used to evaluate RBDO techniques (Liang and Mourelatos, 2004). The original problem did not require multidisciplinary analysis.

$$
\begin{aligned}
&\text{Minimize } f(\boldsymbol{d}) = d_1 + d_2 \\
&\text{s.t } P(g_i(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) \le 0) \le .0013,\ i = 1...3 \\
&\overline{\hspace{20em}} \\
&\text{where } \boldsymbol{d} = [d_1, d_2],\ \boldsymbol{x} = [x_1 \sim N(d_1, 0.3), x_2 \sim N(d_2, 0.2)] \\
&g_1(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) = \frac{u_1 x_1}{20} - 1 \\
&g_2(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) = \frac{u_2}{30} + \frac{(x_1 - x_2 - 12)^2}{120} - 1 \\
&g_3(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) = \frac{80}{x_1^{\,2} + 8x_2 + 5} - 1 \\
&\overline{\hspace{20em}} \\
&A_1(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) = u_1 - x_1(u_2 - x_1 + 5) = 0 \\
&A_2(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) = u_2 - \frac{u_1}{x_2} - x_2 + 5 = 0 \\
&\text{s.t. } x_1 \ne x_2
\end{aligned}
\tag{21}
$$

The results for Example 2 are given in Table 3. Again, a few interesting observations are found. For the fully-integrated analysis, the single-loop, inverse FORM method performs most efficiently but is not as consistent as the nested direct FORM (Method 1) or sequential inverse FORM methods (Method 4). For direct FORM, the SAND approach shows roughly a two-fold improvement over the fully-integrated methods. However, using SAND with the inverse FORM methods raises problems. For the nested method, the inner reliability analysis loops do not converge during the earlier optimization iterations. When this inner loop is truncated by imposing a maximum iteration limit, the algorithm does converge but not until a significant number of disciplinary analyses are required. The SAND, sequential inverse FORM (Method 10) performs much better. Nevertheless, it is still less efficient than the fully-integrated version (Method 4) since Eq. (8) used for fully-integrated inverse FORM provides an exact solution to the KKT conditions which is not available when the multidisciplinary compatibility constraints are added. Finally, we note that the most stable algorithms are the nested, direct FORM methods (method 1 and 7) and the sequential, inverse FORM methods (methods 4 and 10).

Table 3: Results for Example 2

Multidisciplinary Analysis		**Fully-Integrated**		
RBDO Method		Solution	Disciplinary Evals	Stability Rating
Nested	1. Direct FORM	[3.4391 3.2866]	825	Good
	2. Inverse FORM	[3.4391 3.2866]	828	Fair
Sequential	3. Direct FORM	[3.4391 3.2866]	644	Fair
	4. Inverse FORM	[3.4391 3.2866]	683	Good
Single Loop	5. Direct FORM	Does not converge		Poor
	6. Inverse FORM	[3.4391 3.2866]	261	Fair
Multidisciplinary Analysis		**Simultaneous Analysis and Design**		
RBDO Method		Solution	Disciplinary Evals	Stability Rating
Nested	7. Direct FORM	[3.4391 3.2866]	405	Good
	8. Inverse FORM	[3.4391 3.2866]	19345*	Fair
Sequential	9. Direct FORM	[3.4391 3.2866]	348	Fair
	10. Inverse FORM	[3.4391 3.2866]	1554	Good
Single Loop	11. Direct FORM	Does not converge		Poor
	12. Inverse FORM	Does not converge		Poor

* Results achieved by truncating inner reliability analyses that do not converge in a number of steps.

Example 3:

The final example is a three-bar truss optimization problem shown in Fig. 4.

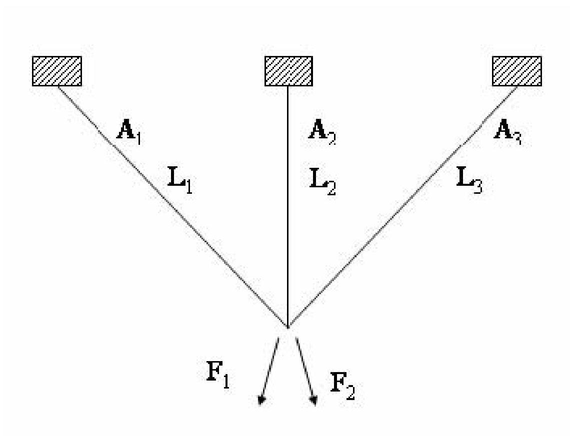


Figure 4. Three Bar Truss

The objective is to select appropriate cross-section areas for the three bars, in order to minimize the weight of the truss while sustaining loads under two load cases, $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ within allowable stresses. The bar areas, $\boldsymbol{A}$, lengths, $\boldsymbol{L}$, and the applied loads, $\boldsymbol{F}$ are all random normal variables with mean, $\boldsymbol{\mu}$ and standard deviation, $\boldsymbol{\sigma}$. The RBDO problem formulation is given by Eq. (22). The design variables are the mean values of the bar areas. The optimization is constrained by the allowable stress. Disciplinary analyses consist of static equilibrium equations and stress equations for each load case. Disciplinary response variables are the displacements, $\boldsymbol{u}$, and member stresses, $\boldsymbol{f_i}$ for each case. The results are given in Table 4.

$$\text{Minimize } f(\boldsymbol{\mu}_A) = \sum_{i=1}^{3} \mu_{A(i)} \mu_{L(i)}$$

$$\text{s.t } P(stress_{i(1)} \leq stress_{\text{allowable}}) \leq 0.0013, \; i = 1...3 \text{ (load case 1)}$$

$$P(stress_{i(2)} \leq stress_{\text{allowable}}) \leq 0.0013, \; i = 1...3 \text{ (load case 2)}$$

$$\text{where } A_i \sim N(\mu_{A(i)}, 1) \text{ in}^2, \; \boldsymbol{L} = [L_1, L_3 \sim N(141.4, \; 14.4), L_2 \sim N(100, \; 10)] \text{ in}$$

$$\boldsymbol{F_1} \sim N(-500\hat{i} - 1000\hat{j}, \; 150\hat{i} + 300\hat{j}) \text{ lb}, \; \boldsymbol{F_2} \sim N(500\hat{i} - 1000\hat{j}, \; 150\hat{i} + 300\hat{j}) \text{ lb}$$

$$stress_{i(1)} = \frac{\|\boldsymbol{f_{i(1)}}\|}{A_i} \text{ (load case 1)}, \; stress_{i(2)} = \frac{\|\boldsymbol{f_{i(2)}}\|}{A_i} \text{ (load case 2)}, \; stress_{\text{allowable}} = 100\text{psi}$$

$$\boldsymbol{F}_1 - \sum_{i=1}^{3} \boldsymbol{f_{i(1)}} = 0 \text{ (load case 1)} \quad \boldsymbol{F}_2 - \sum_{i=1}^{3} \boldsymbol{f_{i(2)}} = 0 \text{ (load case 2)}$$

$$\boldsymbol{f_{i(1)}} - k\boldsymbol{u_1} = 0, \; i = 1...3 \text{ (load case 1)}, \; \boldsymbol{f_{i(2)}} - k\boldsymbol{u_2}, \; i = 1...3 \text{ (load case 2)}$$

where k is the stiffness matrix (E = 29000 psi) and $\boldsymbol{u}$ is displacement (22)

For this problem, we note that inverse FORM was more efficient than direct FORM for the first six, fully-integrated methods. However, as in the other two examples, we again see difficulty in combining inverse FORM with the SAND multidisciplinary optimization approach. For the SAND, sequential inverse FORM algorithm (method 10), this issue was overcome by performing fully-integrated multidisciplinary analysis when the problem arose. The resulting, mixed fully-integrated/SAND method actually performed best of all for this example. As in the other two examples, we find that the single-loop, inverse FORM method is most efficient when combined with fully-integrated multidisciplinary analysis but does not do well with SAND. Again, the single-loop, direct FORM methods do not converge at all.

Table 4: Results from Three-Bar Truss Example

Multidisciplinary Analysis		**Fully-Integrated**		
RBDO Method		Solution (μ_A)	Disciplinary Evals	Stability Rating
Nested	1. Direct FORM	[11.0392 12.2814 11.0392]	217183	Good
	2. Inverse FORM	[11.0392 12.2814 11.0392]	66554	Good
Sequential	3. Direct FORM	[11.0392 12.2814 11.0392]	212821	Fair
	4. Inverse FORM	[11.0392 12.2814 11.0392]	59476	Good
Single Loop	5. Direct FORM	Does not converge		Poor
	6. Inverse FORM	[11.0392 12.2814 11.0392]	10738	Fair
Multidisciplinary Analysis		**Distributed**		
RBDO Method		Solution (μ_A)	Disciplinary Evals	Stability Rating
Nested	7. Direct FORM	[11.0392 12.2814 11.0392]	9342	Good
	8. Inverse FORM	Does not converge		Fair
Sequential	9. Direct FORM	[11.0437 12.2659 11.0437]**	9075	Fair
	10. Inverse FORM	[11.0392 12.2814 11.0392]	8476*	Good*
Single Loop	11. Direct FORM	Does not converge		Poor
	12. Inverse FORM	Does not converge		Poor

*Method 10 did not converge using the original algorithm. Reliability analysis for certain limit states failed in early stages; modified algorithm resorts to fully-integrated analysis when this occurred.
**Method 9 converged to a slightly different optimum than the other methods but is well within 3 significant digits which is acceptable for this application.

Although the three example problems are much less complex than typical multidisciplinary design problems, these results present some interesting observations that bear further analysis. Table 5 is provided to highlight the relative performance of each of the three components of the twelve algorithms: reliability analysis method, RBDO technique, and multidisciplinary analysis strategy.

Table 5: Summary of Results

Best Reliability Analysis Method				
	Example	1	2	3
Fully-integrated (FI)	Nested (N)	Inv/Direct	Inv/Direct	Inverse
	Sequential (Sq)	Inv/Direct	Inv/Direct	Inverse
	Single Loop (SL)	Inverse	Inverse	Inverse
SAND	Nested	Direct	Direct	Direct
	Sequential	Direct	Direct	Direct
	Single Loop	Inverse	X	X
Best RBDO Technique				
	Example	1	2	3
Direct FORM	Fully-Integrated	N/ Sq	Sq	N/ Sq
	SAND	N/ Sq	N/ Sq	N/ Sq
Inverse FORM	Fully-Integrated	SL	SL	SL
	SAND	SL	Sq	Sq
Best MDA Strategy				
	Example	1	2	3
Direct FORM	Nested (N)	SAND	SAND	SAND
	Sequential (Sq)	SAND	SAND	SAND
	Single Loop (SL)	X	X	X
Inverse FORM	Nested	FI	FI	FI
	Sequential	FI	FI	FI
	Single Loop	FI	FI	FI

First, with respect to the reliability analysis method, one can make a few early generalizations: (1) clearly SAND appears to be better coupled with direct than inverse FORM, (2) direct FORM single loop methods are not viable options as they fail for even the simplest examples, (3) for the lower dimensioned problems, there is little difference in the performance of direct vs. inverse form with fully-integrated MDA but example 3 suggests there may be an advantage to using inverse FORM for higher dimensioned problems. Analysis of the inverse FORM plus SAND methods (8, 10, and 12) reveals some insight into the first observation. For fully integrated methods (2, 4, and 6), Eq. (8) provides an exact solution to the KKT conditions at the performance measure approach (PMA) point; the PMA search uses it iteratively by calculating the gradient at successive guesses. However, for the SAND methods, an additional constraint (multidisciplinary feasibility) is added to the PMA formulation and Eq. (8) no longer satisfies the KKT conditions. Since no exact solution has been developed to date, a standard optimizer (SQP) was used for the PMA search. Sequential quadratic programming uses first order approximations of the constraints but the target reliability constraint of the PMA search, $\|\eta\| = \beta_{\text{target}}$, is very non-linear (it is in fact a hypersphere) so the successive linear approximations are particularly susceptible to cycling. In some cases (e.g., with Method 10 on Example 3), truncating the PMA search at a maximum number of steps away from the design solution provided acceptable performance. However, further work is needed to develop an inverse FORM, SAND algorithm. One tactic would be to find a critical point to the Lagrangian based on a linear approximation of the disciplinary analysis equations but retaining the second order form of the target reliability constraint. As for the second observation, one key difference is noted in the mechanism for the single loop, direct FORM algorithm and the inverse FORM counterpart. The limit state provides the equality constraint for direct FORM instead of an inequality constraint as in inverse FORM. This may hint to the reason single loop direct FORM encounters convergence problems more frequently, but further investigation is required to fully understand the problem. Although additional research may reveal a better approach, direct FORM with SAND is not recommended for any MDO-RBDO applications at this time. Finally, additional examples are needed to evaluate the effect of dimensionality on the performance of direct versus inverse FORM with fully-integrated MDA.

A second set of observations can be made about the selection of RBDO technique: (1) when they work, single loop methods (typically with inverse FORM and fully-integrated MDA) are particularly efficient and (2) there appears to be no computational advantage to using sequential methods over nested RBDO. The reason for the first observation is obvious: single loop methods make a single reliability analysis calculation for each optimization iteration. Of course, as discussed concerning direct FORM single loop methods, the fact that the candidate design variable is also changing introduces errors in the evaluation of the limit state and gradient. However, the inverse FORM step only requires the gradient so this error would be insignificant for approximately linear limit states. Performance on systems with non-linear limit states would be more dependent on starting point as the second and third example problems (which are both non-linear) reflect. The second observation is less surprising when one takes an incremental look at where computational effort is required. Let m be the number of optimization iterations required for nested methods and n be the average number of reliability analysis iterations. Similarly for sequential methods, let m' be the average number of deterministic optimization iterations, n' be the average number of reliability analysis iterations, and p be the total number of sequential loops. The computation effort for nested methods would then be of the order $m*n$ while sequential methods would be of the order $(p*m' + p*n')$. Assuming $m' \approx m$ and $n' \approx n$ and that n is significantly smaller than m (as is the case for all three examples), computational savings for sequential methods is possible if $p < n$. However, FORM (direct or inverse) often converges in n = 3 to 8 iterations while sequential methods need a minimum of three overall loops to ensure convergence and often require 4 to 5 (p = 3 to 5). Thus, in many cases sequential methods will not offer computational savings. It should be noted, nonetheless, that sequential methods have other benefits. For one, in isolating the optimization from reliability analysis, it would be easier to diagnose the source of problems such as cycling or divergence.

Finally, a study of the multidisciplinary analysis and optimization strategy re-iterates the observations of poor performance of SAND and inverse FORM for reasons already discussed.

Conclusion

The intent of this chapter was to develop several combinations of RBDO and MDO methods and to exercise the methods on a few simple problems in an effort to gain a better understanding of their relative strengths and weaknesses. With this in mind, a few conclusions may be drawn. First, the sequential RBDO strategies may not provide significant savings when measured on a 'level playing field' with nested methods (i.e., less than 50% improvement in efficiency, similar accuracy, and similar stability). One of the key elements in providing comparable nested RBDO algorithms is using the analytical gradient for the reliability and performance measure indices (β and g^*) so that the optimizer does not require repetition of the reliability analysis loop in order to obtain finite difference derivatives. The other key factor is using the most efficient MPP search algorithms (i.e., Eqs. 7 and 9). If both of these techniques are used, nested methods can be comparably efficient to other methods. The second significant finding is that using sequential quadratic programming to solve a SAND formulation of the inverse FORM problem, Eq. (18) frequently creates difficulty. Research is needed to develop a step which satisfies KKT conditions for a distributed inverse FORM formulation similar to the multi-constraint (direct) FORM methods developed in Chapter II. The fully-integrated single-loop, inverse FORM method performed very efficiently and appears suitable for nearly linear limit states. Finally, using SAND with direct FORM provided improvement over the fully-integrated methods for all three cases. Further research is needed to compare performance on a variety of problems to identify the effects of system characteristics such as dimensionality of the design variable vector, dimensionality of the random variable vector, degree of interdependence of disciplines, number of probabilistic constraints, presence of deterministic equality and inequality constraints, and conditioning of both limit states and disciplinary analyses. At this point, one may be cautiously optimistic regarding the performance of direct FORM SAND methods (7 and 9).

As industries continue to demand more complex systems, and as society continues to raise the bar in terms of performance and reliability expectations, engineers will continue to look for methods to design for reliability. Probabilistic analysis and reliability-based design optimization are promising approaches in this regard. However, implementation of RBDO has been limited to small scale problems to date; its applicability to large multidisciplinary systems will be instrumental in achieving more widespread use. Studying the effectiveness of various RBDO-MDO algorithms is a first step in this direction. Further study is needed in order to (1) determine the applicability of these methods to large scale problems and (2) fully characterize the system properties suitable to particular methods. However, based on the results to date, the top three methods include single-loop, inverse FORM with fully-integrated MDA (method 6); nested, direct FORM simultaneous analysis and design (method 7), and sequential, direct FORM SAND (method 9). Method 6 is would appear most appropriate for problems for which the limit states are close to linear. Method 9 (or its inverse FORM counterpart, method 10) shows promise for more complex systems where designers anticipate the need to diagnose problems by segregating the reliability analysis and optimization phases. Methods 7 and 9 appears particularly suitable for multidisciplinary systems amenable to being solved with gradient-based optimizers. (Additional research is needed to better define this caveat but one could test the system by comparing the effort required for multidisciplinary analysis using Newton's method versus that required for fixed-point iteration. If Newton's method is effective and efficient, there is a possibility SAND methods will perform favorably. The real world application treated in the following chapter provides an example where this is not the case.)

Naturally, all the methods presented in this chapter, require a well defined system for which requirements and interdependencies among disciplines are known. Communication between disciplines is an integral component for MDO-RBDO, though the SAND methods (6-12) offer a little more flexibility in terms of when all must be satisfied (or, 'come to agreement'). If disciplinary integration is prohibitive, a completely different strategy is needed. Chapter VI offers one alternative.

CHAPTER IV

APPLICATION OF PROBABILISTIC SYSTEM DESIGN: UNMANNED AERIAL VEHICLE

Introduction

This chapter investigates the reliability-based multidisciplinary optimization methods proposed in the previous chapter for the design of a solar power supply for a high altitude, long endurance (HALE) unmanned aerial vehicle (UAV). This problem presents several issues of interest with regard to how simulation-based design is performed in practice, in addition to being a suitable real world application of reliability-based MDO. The problem is not multidisciplinary in the strictest sense since the UAV performance analysis is accomplished through a single code which, though involving both aerodynamic and propulsion analyses, offers no mechanism for decoupling them. However, when coupled with two custom algorithms which apply an iterative process in order to find a feasible design solution, the UAV design suggests an optimization formulation with the basic properties of traditional MDO problems. In fact, as will be demonstrated, the optimization approach will offer much needed flexibility in achieving the desired design reliability. A second important property of the UAV design, is that it is tightly coupled, a feature common to many practical multidisciplinary design problems. Finally, gradient-based methods are ineffective for multidisciplinary analysis, leaving fixed-point iteration as the basic option for convergence algorithms. (Newton's method needed on average 30+ iterations for MDA while fixed point iteration typically converged in under 10). Thus, applying reliability-based MDO requires some modifications of the methods in the previous chapter to take full advantage of the most effective analysis methodology. Finally, to some extent, the UAV application blurs the distinction between design and analysis, a common feature for legacy algorithms for aerospace system design. This presents a challenge in finding a suitable limit state in order to re-formulate the problem for reliability-based design optimization.

The following section begins with an overview of the original system analysis and deterministic design approach provided by researchers at NASA Langley. This section explains the key input and output variables, describes the performance analysis and design algorithms, and shows how they are integrated in the original design process. Next, the design problem is reformulated, first as a deterministic optimization, to provide needed flexibility for reliability requirements that will follow. This section provides a revised integrated analysis that has the same mathematical structure as multidisciplinary analysis discussed in the previous chapters. Next, uncertainty is introduced in key variables and the final reliability-based design problem formulation is provided. This is followed by a section entitled "Practical Implementation of Reliability-Based MDO," which describes modifications to the twelve RBDO-MDO methods of Chapter 3 needed to take advantage of the fixed-point iteration strategy for multidisciplinary analysis. Finally, results are reported in terms of the effectiveness of each of the 12 methods in achieving a solution, and general conclusions are drawn.

System Analysis and Deterministic Design

This application is the design of a self-sustaining solar power supply for a high altitude, long endurance, unmanned aerial vehicle (UAV). The design goal is for the vehicle to accomplish reconnaissance independently over a long period of time (weeks or months). In order to be self-sustaining, the vehicle must store enough energy to continue powering the vehicle during the hours of darkness. During the day the solar cells power the vehicle and use extra capacity for electrolysis (i.e., to create electrical energy to break down H_2O into hydrogen and oxygen). At night, hydrogen and oxygen recombine in fuel cells creating energy to power the vehicle's motor. Key design variables include the number of solar cells, the number of electrolyzer cells (to break down water), the rated shaft power of the electric motors, the hydrogen storage capacity, the 'zero fuel' weight of the vehicle, and the wing area. Other important parameters include the latitude and time of year as these have a significant affect on the sun elevation and number of daylight hours. For this design, the latitude is fixed at 47 degrees North and the time of year is in the height of winter.

The primary analysis is accomplished by a performance analysis (PA) algorithm (Nickol et al., 2007), which calculates a take-off weight, power storage requirements, and a final 'state of charge' at the end of each day. At this stage we first begin to see a combination of analysis and design; key input variables to PA suggest a design (e.g., empty weight, rated motor power, number of fuel and electrolyzer cells, etc.), which the code analyzes, providing output that includes both new design information (e.g., required power output per fuel cell, remaining charge at the end of a cycle, etc) and revisions to the original input parameters (e.g., empty weight, rated motor power, and number of fuel and electrolyzer cells). In other words, the PA algorithm is structured such that a single execution does not guarantee a feasible design. Instead, it generates weight and power requirements that update initial design inputs. For example, the code uses a user 'guess' for zero fuel (i.e., empty vehicle) weight (*zfw*) as the basis for its calculations; however, the code subsequently calculates an updated value for zero fuel weight. To ensure feasibility based on weight, the code must be run iteratively until the *zfw* input value equals the *zfw* output value. This feature allows engineers to make adjustments outside the code to account for different assumptions. For one, the PA calculations are based strictly on cruise conditions and empirical weight predictions from present day technologies. Researchers at NASA Langley have linked the PA code to an external Power, Weight and Sizing (PWS) algorithm to (1) adjust weight predictions, (2) add a requirement to maintain a desired climb rate and (3) run PA iteratively to assure convergence of zero fuel weight. PWS adjusts weight predictions based on assumptions about technological advancements and adjusts power requirements to add maneuver capability (i.e., for climbing versus cruise/loiter). In addition, as current technology does not support a self-sustainable vehicle design for some missions, the PWS code allows for an input of external power, or power that must be added to that available from the regeneration cells. Thus the power-generated-to-power-required fraction is a measure of infeasibility at the given design variables. PWS runs the PA code iteratively to converge five key design variables: zero fuel (or empty) weight, motor rated shaft power, number of fuel cells, hydrogen storage capacity, and external power). A third code, the Electrolyzer Sizer (ES) provides a revised estimate of the number of electrolyzer cells (*na*) needed to ensure there is enough energy to get the vehicle through the night before needing to recharge. It also requires several iterations of PA, searching for the minimum number of electrolyzer cells to provide the required energy.

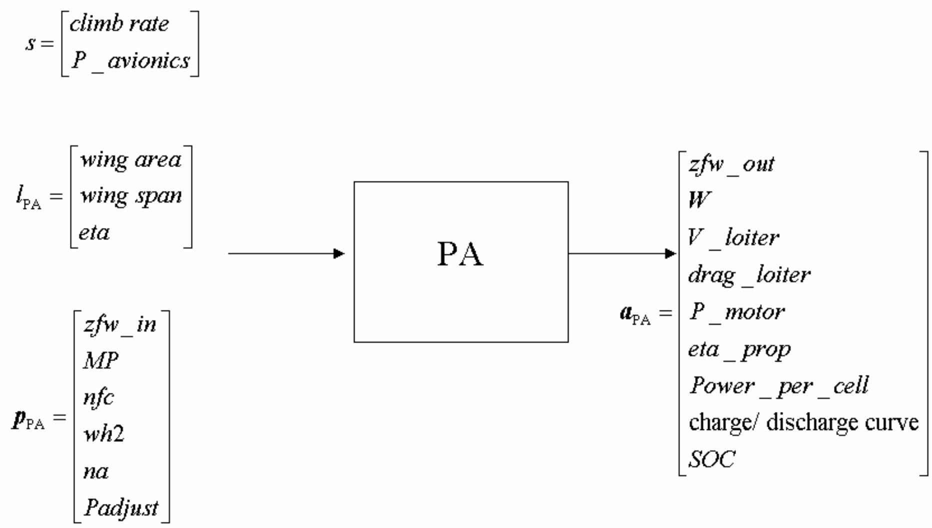


Fig 1(a): Performance Analysis (PA) Disciplinary Analysis

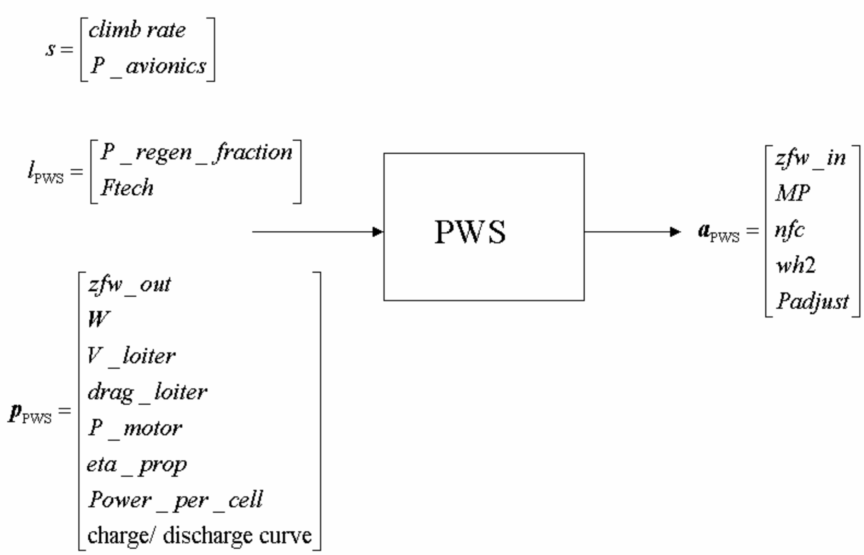


Fig 1(b): Power Weights and Sizing (PWS) Disciplinary Analysis

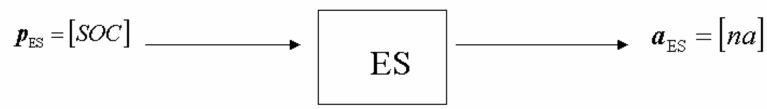


Fig 1(c): Electrolyzer Sizer (ES) Disciplinary Analysis

Fig. 1 depicts the inputs and outputs of interest for each of the three disciplines. Input variables shared between two or more disciplines, ***s***, include the required design climb rate and the power draw from avionics. For the Performance Analysis discipline (PA), additional inputs include local variables, *wing area*, *wing span*, engine efficiency (*eta*) and parameters from the other two disciplines: the input zero fuel weight (*zfw_in*), the rated shaft power (*MP*), the number of fuel cells (*nfc*), the weight of hydrogen storage capacity (*wh2*), the number of electrolyzer cells (*na*), and the external power adjustment (*Padjust*). Outputs of interest include the calculated zero fuel weight (*zfw_out*), a weight distribution by component (***W***), loiter velocity and drag, the power required by the motors, the propulsion efficiency (*eta_prop*), the power supplied by each fuel cell, charge/discharge curves for the electrolyzer cells, and a state of charge deficit at the end of a night cycle (*SOC*). The Power Weights and Sizing code (PWS) requires the same shared inputs, ***s***, as well as local variables including the fraction of power supplied by the regeneration system (*P_regen_fraction*) and weight reduction factors for technological advancement assumptions (***Ftech***). In addition, the PWS code requires inputs from the performance analysis as depicted if Fig. 1(b). The Electrolyzer Sizer code (ES) simply optimizes the number of electrolyzer cells (*na*) by driving *SOC* to zero.

The relationship among the disciplines, as defined by the original design process, is depicted in Fig. 2.

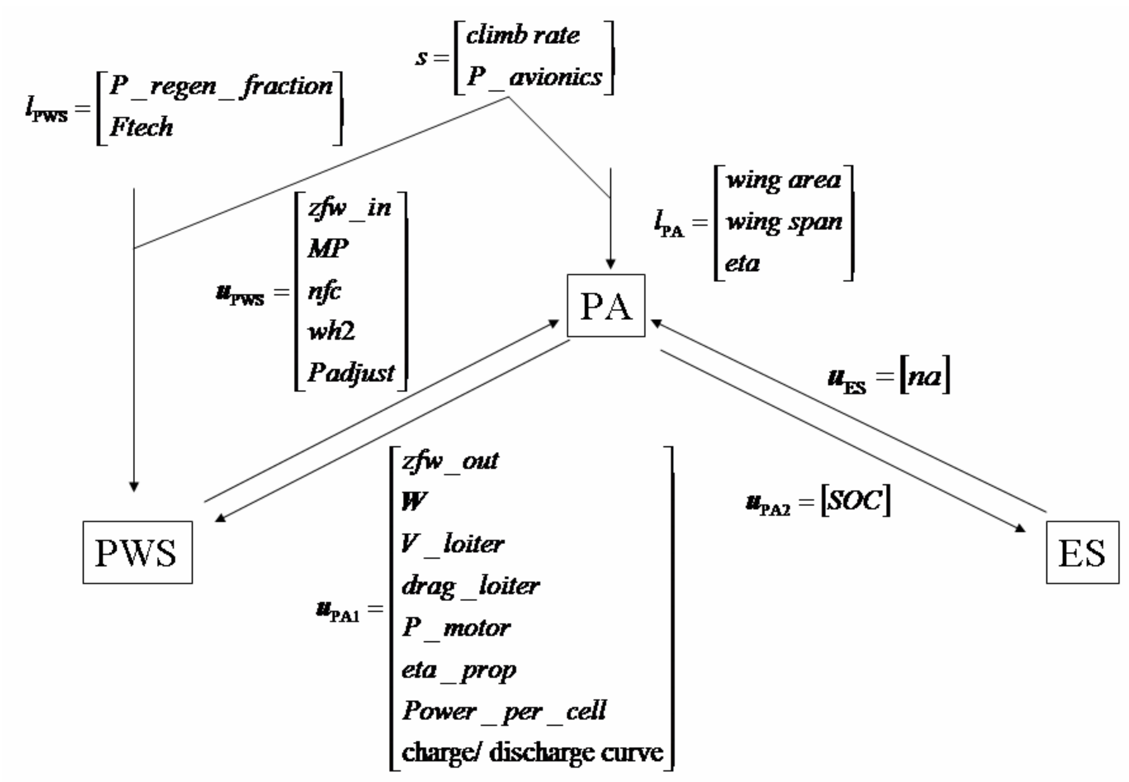


Figure 2. System Analysis and Deterministic Design

Note that the notation, $\boldsymbol{u}$, is used to represent disciplinary response variables that are outputs from one discipline and inputs to another. Response variables from PA are denoted $\boldsymbol{u}_{PA1}$ and $\boldsymbol{u}_{PA2}$ to distinguish between those needed as inputs for PWS and ES respectively. In this form, fixed point iteration (or some other convergence algorithm) is needed to find a single feasible design by solving a system of non-linear equations to find the disciplinary response variables ($\boldsymbol{u}_{PA1}$, $\boldsymbol{u}_{PA2}$, $\boldsymbol{u}_{\mathbf{PWS}}$, $\boldsymbol{u}_{\mathbf{ES}}$) as in Eq. (1). Note here that $\boldsymbol{u}$ and $\boldsymbol{l}$ without the subscript represent the entire set of discipline response variables and local variables for all three analyses.

$$\begin{aligned} &\boldsymbol{u}_{PA} - PA(\boldsymbol{s},\boldsymbol{l}\ ,\boldsymbol{u}) = 0 \\ &\boldsymbol{u}_{PWS} - PWS(\boldsymbol{s},\boldsymbol{l},\boldsymbol{u}\ \ ) = 0 \\ &\boldsymbol{u}_{ES} - PWS(\boldsymbol{s},\boldsymbol{l},\boldsymbol{u}\ \ ) = 0 \end{aligned} \tag{1}$$

There are some significant drawbacks to this approach. Small changes in the system can lead to large changes in the design solution; in other words, it is not very robust. The design solution provides no room for uncertainty since a small change will lead to an infeasible solution. (A preferable formulation would have inequality constraints to allow sufficient 'overdesign' to account for uncertainty.) In addition, the PWS and ES algorithms end up competing with one another to converge their disciplinary response variables, often failing to find a feasible solution to the integrated system even when one exists. In preparation for a probabilistic design formulation which considers uncertainty, an optimization problem with inequality constraints is needed. In the following section, an alternative deterministic optimization formulation is provided, which will be the basis for the reliability-based optimization problem to follow.

Revised Deterministic Optimization Formulation

As discussed, in its original form, the UAV design does not involve true optimization. Requirements that could be viewed as 'design' constraints are treated as equality constraints which have a unique solution; mathematically, they can be viewed as multidisciplinary feasibility criteria (in other words, the 'design' is found by solving the set of non-linear equations). For one, the state of charge (*SOC*) constraint is driven to zero by solving the ES analysis when in reality any value of *SOC* less than zero would provide a feasible design. In preparation for a probabilistic design formulation which considers uncertainty, an optimization problem with inequality constraints is needed. This way the inequality constraint may represent a probabilistic limit state and we can assure satisfaction to a desired reliability level. A revised, deterministic optimization is thus formulated as a bridge to probabilistic optimization. This formulation seeks to (1) establish a clear objective, (2) distinguish between optimization constraints and multidisciplinary feasibility requirements, and (3) provide inequality constraints as a basis for probabilistic limit states in the next step. Fig. 3 depicts the revised analysis.

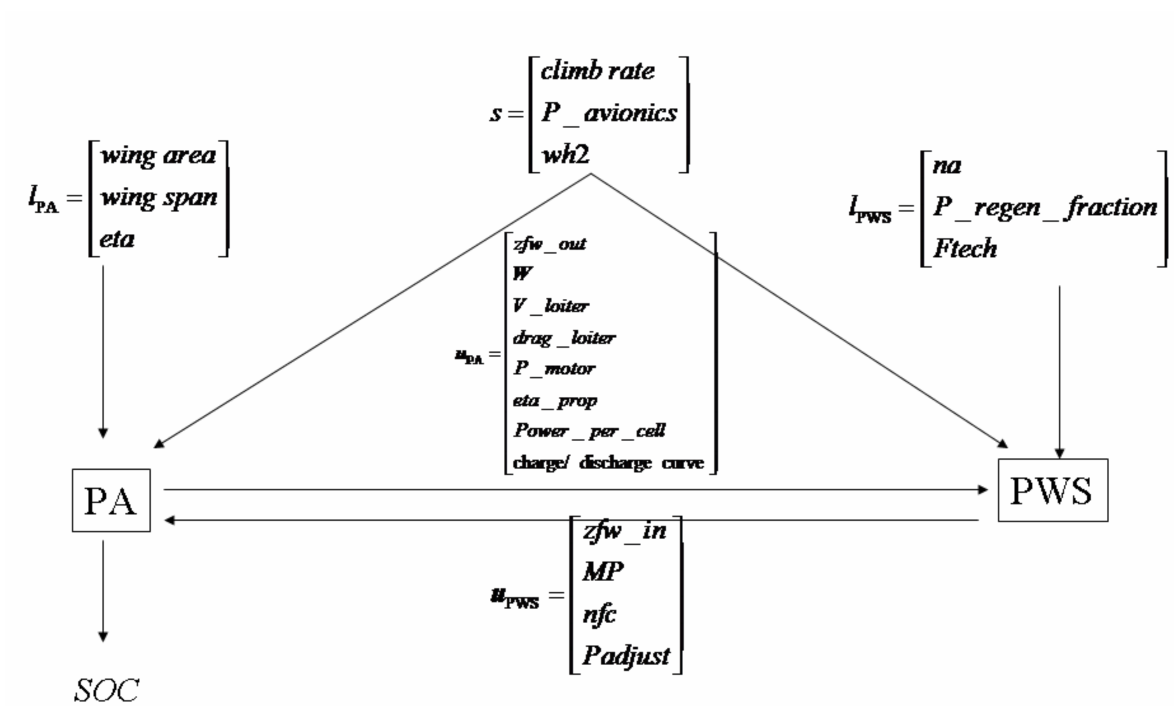


Figure 3. Revised Integrated Analysis

One of the major differences in Figs. 2 and 3 is that the ES code has been eliminated in its entirety. In the original design process, sizing of the electrolyzer cells was treated as an equality constraint; the ES code assured a size ‘just enough’ to meet the charge requirements and no more. The assumption was that this would provide an ‘optimal’ design. However, as already discussed, this methodology provides no room for uncertainty in the analysis and often interferes with the other analyses’ attempts to satisfy legitimate multidiscipline feasibility constraints. Sizing of the electrolyzer cells is now accomplished through the optimization formulation where the number of cells (*na*) is a design variable and the state of charge (*SOC*) is an inequality constraint. Similarly, sizing of the hydrogen storage is also now part of the optimization problem; *wh2* is now a design variable and no longer an output of the PWS code.

A baseline deterministic optimization formulation is given in Eq. (2) below. Here *P_regen_fraction* is the fraction of total energy needed to keep the UAV in the air under the given conditions that can be supplied by the regeneration system (i.e., solar energy, electrolyzer and hydrogen fuel cells). This is necessary because under certain conditions, the current technology and design concept cannot provide a fully self-sufficient system. Thus, by maximizing *P_regen_fraction*, the infeasibility (in terms of design constraint satisfaction) of the system is minimized.

$$\text{Maximize } P\_regen\_fraction$$
$$\text{subject to } SOC \le 0$$

$$\text{DesignVariables: } na,\ P\_regen\_fraction,\ wh2$$

$$\left.\begin{aligned} \boldsymbol{u}_{PA} - PA(\boldsymbol{s}, \boldsymbol{l}_{PA}, \boldsymbol{u}_{PWS}) = 0 \\ \boldsymbol{u}_{PWS} - PWS(\boldsymbol{s}, \boldsymbol{l}_{PWS}, \boldsymbol{u}_{PA}) = 0 \end{aligned}\right\} \text{Multidisciplinary Feasibility Constraints}$$

Discipline Response Variables:

$$\boldsymbol{u}_{\mathrm{PA}} = \begin{bmatrix} zfw\_out \\ \boldsymbol{W} \\ V\_loiter \\ drag\_loiter \\ P\_motor \\ eta\_prop \\ Power\_per\_cell \\ \text{charge/ discharge curve} \end{bmatrix} \text{ and } \boldsymbol{u}_{\mathrm{PWS}} = \begin{bmatrix} zfw\_in \\ MP \\ nfc \\ Padjust \end{bmatrix} \qquad (2)$$

Probabilistic Optimization Formulation

Equation (2) presents the UAV design problem in a traditional multidisciplinary optimization form (deterministic). From here, uncertainty in the analysis can be considered to provide reliability of performance to a desired level. Since this design is concerned with the power supply system, performance reliability is contingent upon the uncertainty associated with the remaining charge (*SOC*) at the end of the cycle. If *SOC* is greater than zero, the UAV will not be able to maintain operation through a complete daily cycle, indicated performance failure.

One of the most significant areas of uncertainty for this problem regards assumptions about technological advancement. In the PWS code, designers may choose factors to reduce the weight of various vehicle components (in other words, assume novel, light weight materials will become available during the design phase of the vehicle.) Since the current state of technology provides an upper limit to the technical advancement factor (*Ftech*) of one, uncertainty for this variable may be modeled as a lognormal distribution. Other potential sources include the engine efficiency (*eta*), *wing span* and *wing area*. Finally, there is considerable uncertainty surrounding the charge calculations. To account for this, a model error term (SOC_{error}) is added such that $SOC_{true} = SOC_{calc} + SOC_{error}$ where SOC_{calc} is the value coming from the PA analysis. Probabilistic distributions for each of these sources of uncertainty are presented in Eq. (3) below.

$$\begin{aligned} &(1-Ftech_i) \sim LN(-1,.14) \\ &SOC_{\mathrm{error}} \sim N(0,\ \ 0.05) \\ &eta \sim N(.2,\ 0.01) \\ &wing\ span \sim N(100,10) \\ &wing\ area \sim N(100,25) \end{aligned} \tag{3}$$

The introduction of uncertainty naturally complicates the optimization formulation. First, uncertainty may limit the engineer's ability to control design variables. In this problem, randomness in input parameters propagates through the analyses, resulting in output uncertainty. This affects both optimization objective and constraint functions. A reliability based design optimization (RBDO) formulation addresses these issues. In the RBDO formulation below, Eq. (4), design variables include deterministic input variables (i.e., those without uncertainty including *na*, *P_regen_fraction* and *wh2*). Additional uncertainty comes from random input variables not associated with design (e.g., F_i, SOC_{error}, *eta, wingspan,* and *wingarea*). The RBDO objective is to minimize *P_regen_fraction*. Finally, a probabilistic constraint accounts for the uncertainty in the state of charge (*SOC*) by requiring a 99% certainty that *SOC* is greater than zero. Due to the multidisciplinary nature of the problem, additional multidisciplinary constraints are also present. For the RBDO formulation, disciplinary response variables, like other output are also stochastic. They take on different values for every realization of random input parameters. However, the multidisciplinary constraints must be satisfied for any determination of system outputs (e.g., *SOC*) to be valid. (Note in the formulation, that $\boldsymbol{l}_{PA}$ and $\boldsymbol{l}_{PWS}$ now include random parameters.)

$$\text{Maximize } P\_regen\_fraction$$
$$\text{subject to } P(SOC > 0) \leq .01$$

$$\text{DesignVariables: } na,\ P\_regen\_fraction,\ wh2$$
$$\text{Random Parameters: } F_i, eta,\ wingspan,\ wing\ area,\ SOC_{error}$$

$$\left.\begin{array}{l} \boldsymbol{u}_{PA} - PA(\boldsymbol{s}, \boldsymbol{l}_{PA}, \boldsymbol{u}_{PWS}) = 0 \\ \boldsymbol{u}_{PWS} - PWS(\boldsymbol{s}, \boldsymbol{l}_{PWS}, \boldsymbol{u}_{PA}) = 0 \end{array}\right\} \text{Multidiscipline Feasibility Constraints}$$

Discipline Response Variables:

$$\boldsymbol{u}_{\text{PA}} = \begin{bmatrix} zfw\_out \\ \boldsymbol{W} \\ V\_loiter \\ drag\_loiter \\ P\_motor \\ eta\_prop \\ Power\_per\_cell \\ \text{charge/ discharge curve} \end{bmatrix} \text{ and } \boldsymbol{u}_{\text{PWS}} = \begin{bmatrix} zfw\_in \\ MP \\ nfc \\ Padjust \end{bmatrix} \tag{4}$$

Practical Implementation of Reliability-Based MDO

In order to solve Eq. (4), the RBDO-MDO methods in the previous chapter may be applied. Given the tight coupling of the UAV analyses and the fact that analytical gradients are not available, this problem presents a suitable real-world test of the methodology. Recall that the methods are based on combining one of two MDO strategies (integrated and simultaneous analysis and design) for analysis, one of two first order-reliability formulations of the optimization (direct and inverse FORM), and one of three RBDO strategies (nested, sequential, and single loop).

The twelve methods were discussed in detail in Chapter III; they are summarized in Chapter III, Table 1. Recall that the odd numbered methods are based on using an equivalent direct FORM constraint in lieu of the probability constraint, $P(SOC > 0) \leq .01$, as shown in Eq. (5). Here the probabilistic constraint is replaced by the FORM constraint, $\beta \geq \beta_{\text{target}}$ which is found by conducting a search for the most probable point, $\boldsymbol{\eta}^*$ representing the random local ($\boldsymbol{l}$) and shared($\boldsymbol{s}$) variables in standard normal space. Note that in order to evaluate the limit state (in this case, state of charge, $\boldsymbol{SOC}$), the mean value of the local and shared variables ($\boldsymbol{\mu}_s$ and $\boldsymbol{\mu}_l$, respectively) are needed as well as the MPP and the value of the disciplinary response variables ($\boldsymbol{u}$) at the MPP.

$$\text{Maximize } P\_regen\_fraction$$
$$\text{subject to } \beta \geq \beta_{\text{target}}$$

$$\text{where}$$
$$\beta : Min\beta = \|\eta^*\| \text{ such that } SOC(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}^*, \boldsymbol{u}(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}^*)) = 0 \tag{5}$$

$$\left.\begin{aligned} &\boldsymbol{u}_{PA} - PA(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}^*, \boldsymbol{u}_{PWS}) = 0 \\ &\boldsymbol{u}_{PWS} - PWS(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}^*, \boldsymbol{u}_{PA}) = 0 \end{aligned}\right\} \text{Multidisciplinary Feasibility Constraints}$$

Similarly for the even numbered methods, a deterministic equivalent constraint is used based on inverse FORM as given in Eq. (6).

$$\text{Maximize } P\_regen\_fraction$$
$$\text{subject to } g^* \geq 0$$

$$\text{where}$$
$$g^* : \text{Min } g = -\,SOC(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}', \boldsymbol{u}(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}')) \text{ s.t. } \|\eta'\| = \beta_{\text{target}} \tag{6}$$

$$\left.\begin{aligned} &\boldsymbol{u}_{PA} - PA(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}', \boldsymbol{u}_{PWS}) = 0 \\ &\boldsymbol{u}_{PWS} - PWS(\boldsymbol{\mu}_s, \boldsymbol{\mu}_l, \boldsymbol{\eta}', \boldsymbol{u}_{PA}) = 0 \end{aligned}\right\} \text{Multidisciplinary Feasibility Constraints}$$

For the first six methods based on fully-integrated analysis, the procedure follows that given in the last chapter exactly. The integrated analysis was adapted from the original design analysis provided; it is accomplished via Phoenix Integration's model integration software, Model Center (Phoenix Integration, 2006). Model Center executes the basic analyses and drives the iteration to satisfy the multidisciplinary feasibility constraints. A MATLAB optimization algorithm (Mathworks, 2006) based on sequential quadratic programming was chosen to perform the optimization. MATLAB and Model Center are easily linked through the COM interface provided by Model Center. Gradients for the integrated analysis are taken using a basic finite difference of each analysis; thus if an average of Z iterations are required to achieve multidisciplinary feasibility for the integrated system, a total of $(3+1)Z$ function calls are required to get a gradient with respect to the design variables and $(5+1)Z$ calls are required to get the gradient with respect to the random parameters.

Difficulty arises in trying to apply the last six methods involving simultaneous analysis and design. Recall from the last chapter that these methods use auxiliary disciplinary response variables to evaluate the limit state (in this case, *SOC*) without iterations to assure multidisciplinary feasibility. Instead, the multidisciplinary feasibility constraints are added to the probabilistic analysis formulation; this ensures that feasible discipline response variables are available for either the Most Probable Point (for direct form) or Performance Measure Approach Point (for inverse form) in order to evaluate the design optimization's probabilistic constraint. The problem arises from the nature of the integrated analysis, for which gradient based methods are ineffective in ensuring multidisciplinary feasibility. In fact, this can be seen from attempting to find feasible discipline response variables for a given design point. Using fixed point iteration, a feasible point is typically available within 5-7 iterations while a standard Newton approach takes well over 30. With this kind of difference, there is no value in applying gradient-based SAND approaches for MDO-RBDO since any potential computational savings would be already have been forfeited to the more effective fixed point iteration with fully-integrated methods. Rather, the last six methods are adapted to employ fixed point iteration simultaneously with reliability analysis. For the direct FORM methods (methods 7, 9, and 11), the Rackwitz-Fiessler FORM equation is employed on the distributed (i.e., uncoupled) system using the most recent estimate for the disciplinary response variables (***u***), updated the value for the disciplinary response variables with each iteration. This process is described in Fig. 4.

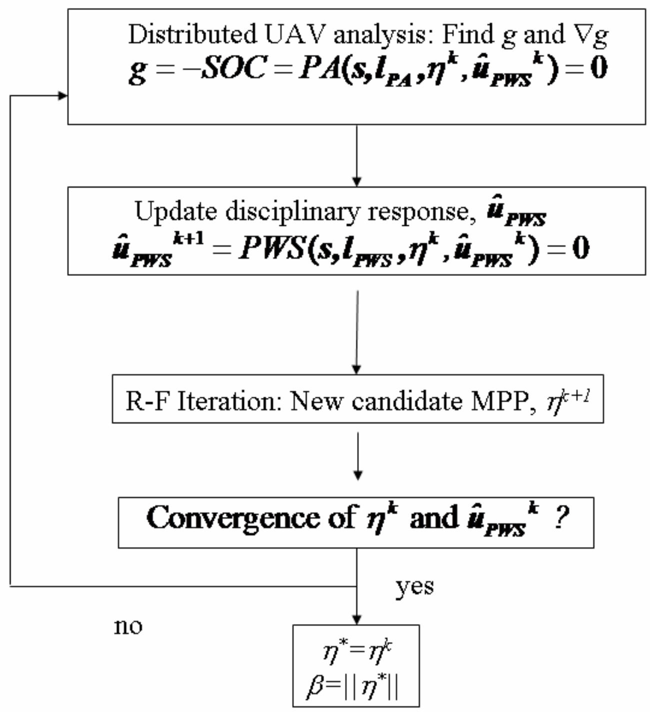


Figure 4. UAV Distributed Reliability Analysis with Fixed Point Iteration

For the inverse FORM methods, the process is similar, except that general step given by Eq. (9) in Chapter 3 is used to find the performance measure point. Note that these methods are heuristic. Fixed point iteration does not guarantee convergence even without the complications of probabilistic optimization. However, in practice, it can be effective for highly coupled systems.

Table 1. Design Results and Algorithm Performance

Multidisciplinary Analysis		**Fully-Integrated**		
RBDO Method		Solution Design Point [*na P_regen wh2*(ft^3)]	Number Optimization Iterations	Processing Time
Nested	1. Direct FORM	[50.6 **0.320** 3.22]	9	2 hr 17 min
	2. Inverse FORM	[50.2 **0.324** 3.24]	10	3 hr 33 min
Sequential	3. Direct FORM	[52.8 **0.325** 3.29]	8	33 min
	4. Inverse FORM	[50.2 **0.318** 3.20]	3	1 hr 3 min
Single Loop	5. Direct FORM	[50.0 **0.318** 3.72] * *Best β=2.2	10	Did not satisfy constraint
	6. Inverse FORM	[50.8 **0.320** 3.22]	10	43 min
Multidisciplinary Analysis		**Simultaneous Analysis and Design**		
RBDO Method		Solution Design Point [*na P_regen wh2*]	Number Optimization Iterations	Processing Time
Nested	7. Direct FORM	[50.1 **0.320** 3.52]*	12	> 4 hours *Did not satisfy MD constraints
	8. Inverse FORM	[50.3 **0.320** 3.20]*	10	> 3hours *Cycling evident with some start points
Sequential	9. Direct FORM	Did not converge		
	10. Inverse FORM	Did not converge		
Single Loop	11. Direct FORM	Does not converge		
	12. Inverse FORM	[50.3 **0.323** 3.17]* Best g^*=-.02	20	>3 hours Cycling during probabilistic analysis

Results

Each of the 12 RBDO-MDO methods from the previous chapter was applied to the UAV design problem to solve Eqs. (5) or (6). The best design solution and the processing time are recorded in Table 1. The processing time is a rough measure of computational effort which points to the relative efficiency of the methods (though this measure is obviously dependent on a number of other factors as well including processor speed). Similar to the analysis provided for the mathematical examples in the last chapter, the UAV design results may be dissected into the three algorithm choices: reliability analysis method, RBDO technique, and multidisciplinary analysis and optimization strategy. In this case, it is easiest to begin in reverse order. The fully-integrated methods worked much better than the SAND-based methods. Considering the fairly low computational effort required for the fully-integrated analysis (5-7 iterations on average), this is not entirely surprising. What is disconcerting, however, is that in most cases the SAND methods did not converge to any solution. Cycling, a typical problem with bi-level methods, was observed during probabilistic analysis. In other words, the candidate MPP or PMA point jumped from one extreme to the other rather than converging on a local optimum. A reduction in the number of random variables helped in some cases, but not in others. A few features of the UAV design hint toward the reason for this problem, though further analysis is needed to fully identify all the issues. First, the disciplinary analyses are not continuously differentiable. Discrete variables such as the number of fuel cells were modeled as continuous variables resulting in some noise in the analyses. Larger finite difference steps ($\Delta x = .01(x)$) were taken to attempt to compensate for this, but this could have had adverse effects on the optimization. Even more importantly, pre-optimization testing revealed that gradient based methods were ineffective in achieving a simultaneous solution to the disciplinary analyses (recall Newton's method required 30+ iterations just for multidisciplinary analysis). If this is the case for multidisciplinary analysis, it is most certainly a significant problem when reliability analysis is added. Thus methods 7-12 were implemented with the heuristic modification given in Fig. 4. A single fixed point iteration is used to estimate the auxiliary variables which is in turn used for the MPP (or PMA) search. If the estimate of the auxiliary variable is poor, significant error is introduced into the evaluation of the limit state and its gradient, hindering the MPP search.

Next, for fully-integrated methods, the most efficient RBDO strategy appeared to be sequential, followed by single loop (inverse FORM only), and finally nested methods. The performance of the inverse, single loop method matched that which would be expected based on the examples of the previous chapter. However, it is interesting to note that, unlike the mathematical examples of the last chapter, here sequential methods do offer a notable improvement over nested methods. On possible reason could be that no analytical derivatives were available for the UAV application. For this reason, a minimum of y (dimension of design variable vector) or z (dimension of random variable vector) disciplinary analyses were required for each iteration (optimization or reliability analysis respectively). Thus even a small improvement in the number of optimization or reliability analysis iterations is significant. Also, the lack of precise derivatives slowed down the MPP (or PMA) searches so that the number of complete sequential loops was in fact significantly smaller than the average number of reliability analysis iterations (i.e., $p < n$).

Finally, this application revealed slightly better performance using direct versus inverse FORM in terms of computational effort, though it is premature to draw definitive conclusions. Though the system is much more complex than the mathematical examples of the last chapter in terms of the number of disciplinary responses, the dimensionality of both the design variable vector and the random variable vector is still quite small (y = 3 and z = 5). Future work is still recommended to determine if inverse FORM methods may be superior for higher dimensioned problems.

One may also note that not all methods converged to the same optimal solution. However, given the noise in the system and the approximations used for finite difference gradients, the fact that the optimal power regeneration fractions were the same within 2 significant digits is considered acceptable.

In addition to the optimal design, a sensitivity analysis was conducted for the six methods which converged to a feasible solution. This sensitivity is a measure of the degree to which the uncertainty in the variable affects the uncertainty in the performance, in this case, state of charge *SOC*. The sensitivity factor is found by simply normalizing the derivative of the limit state with respect to the variable at the MPP. The most important revelation here was that the design was not sensitive to engine efficiency, *eta*, so this random variable could be eliminated from the optimization entirely. The technological advancement factor, *Ftech*, which had a high degree of uncertainty, and the model error, SOC_{error} had the highest sensitivities, though *wingarea* and *wingspan* were not insignificant.

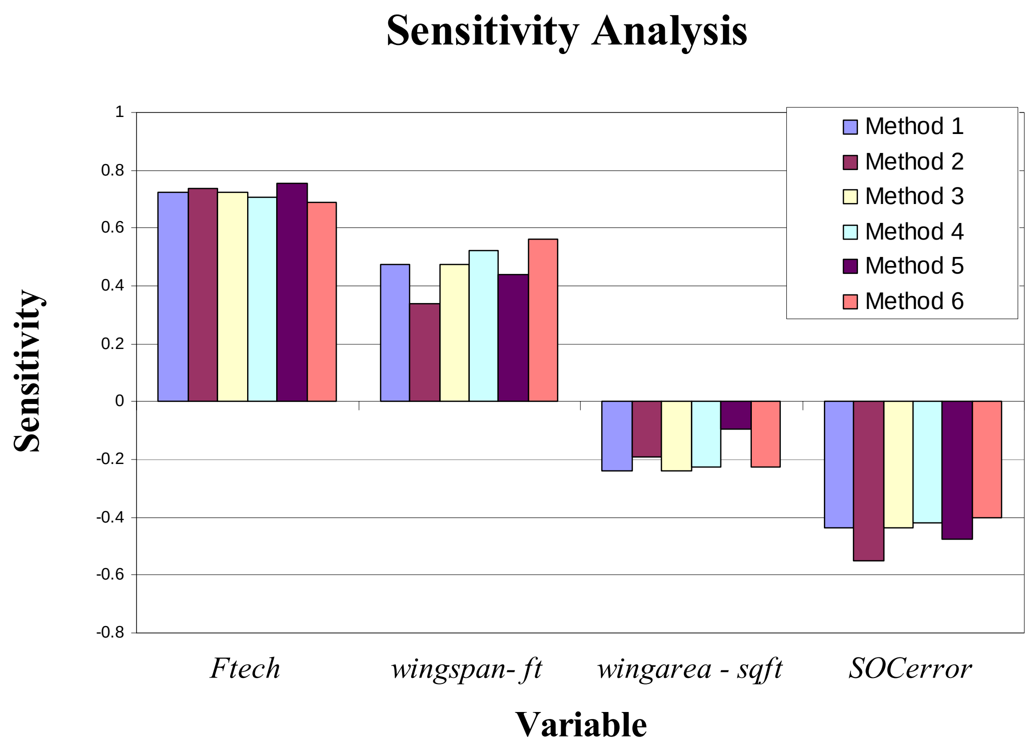


Figure 5. Sensitivity of Design to Random Variables

Conclusion

The purpose of this chapter was to demonstrate the value of reliability-based design optimization for a realistic design problem involving integrated analyses and to test the RBDO-MDO methods of the previous chapter for a real world application. The results show that reliability requirements can be accommodated for system design through multidisciplinary reliability-based design optimization. However, the performance of the twelve RBDO-MDO algorithms clearly reveals the weakness of SAND for certain problems. Furthermore, hybrid method provided by the algorithm given in Fig. 4 was not effective for this problem. It is recommended that designers first evaluate the effectiveness of Newton's method for solving the multidisciplinary analysis prior to attempting RBDO-MDO. If Newton's method does not outperform fixed-point iteration, only fully-integrated methods should be attempted. Sequential, direct FORM methods continue to indicate promise for future applications.

As mentioned in the previous chapter, use of any MDO-RBDO method is contingent upon clear definition of the design problem as well as a precise understanding of the interdependence of disciplines. In re-formulating the UAV design problem as an optimization, one of the integral steps was to clearly identify the design requirements, distinguishing legitimate equality constraints (e.g., for multidisciplinary feasibility) from performance requirements more appropriately treated as inequality constraints. For the UAV design, the key performance requirement was the requirement to maintain some residual state of charge after a 24-hour operation. In the optimization formulation, this was treated as an inequality constraint, while other equality constraints were maintained as analysis feasibility requirements (for example, requiring the input zero fuel weight from the performance analysis match the output calculation from the power, weights, and sizing analysis). In this way, the optimization algorithm negotiates the design space in consideration of system objective without unnecessary restriction. In addition, the performance requirement can then be the basis for a probabilistic constraint to assure system reliability. One can view this as distinguishing 'design' from 'analysis.' This is not a trivial point as many discipline codes merge design and analysis in a way that could impose undesired restrictions once integrated within a higher-level system design. In the following chapter, reliability-based design optimization is undertaken for a different application, the bi-level design of a reusable launch vehicle. This application will also demonstrate the importance of allowing system-level requirements to drive design at a subordinate (in this case at the component) level.

CHAPTER V

REUSABLE LAUNCH VEHICLE APPLICATION: INTEGRATING SYSTEM-LEVEL AND COMPONENT-LEVEL DESIGNS UNDER UNCERTAINTY

Introduction

The previous three chapters focused on probabilistic design via reliability-based design optimization (RBDO) of *multidisciplinary systems*. However, the applications have thus far required integration only at a single design level. Chapters V and VI take reliability-based *system* design a step further by providing two alternative strategies for integrating multiple design levels. These methods are applied to a bi-level design for a reusable launch vehicle which includes both the conceptual system-level design for vehicle geometry and the structural sizing of a component liquid hydrogen tank.

Chapters II and III demonstrated the 'triple loop' nature of system design problems that combine reliability analysis and optimization with iterative multidisciplinary analysis (recall Fig. 1 of Chapter III); these chapters introduced specific algorithms to mitigate this effect in order to improve computational efficiency. Mathematically, three characteristics distinguish the RBDO-MDO problem: (1) they are formulated as optimizations, (2) they include one or more constraints given in probabilistic terms, and (3) they require integration (and often iterative) of two or more distinct 'components' for system analysis. The formulation is blind, however, to the nature of the distinction between components needed for system analysis. *Multidisciplinary* analysis typically refers to the integration of distinct components based on developments in expertise along specific fields of study (e.g., structural, aerodynamic, economic, etc.), but ‘field of study’ need not be the distinguishing characteristic of the ‘disciplines’. The unmanned aerial vehicle problem of the last chapter, for example, showed an even wider application, treating a single analysis code and an iterative design code as highly coupled ‘disciplines’ in order to formulate an RBDO-MDO problem. This chapter takes the same approach, namely formulating an RBDO-MDO problem, which in this case incorporates a component-level design as an additional ‘discipline’ to be integrated with the system-level.

Systems engineering uses a design approach driven by top level requirements. At the higher levels of design, more of the system is considered with less detail. From this point a top-down design approach may be undertaken. As the design process continues, smaller components are designed to a greater level of detail. The highest (“system” or “conceptual”) level design provides the basis for design at the next level. Considering only the effect of the system-design on the component, however, could result in prematurely ‘pigeon-holing’ a system based on the initial conceptual design. This can be dangerous given that conceptual system assessment is typically quite approximate (i.e., low fidelity). Using higher fidelity models at the conceptual level is an alternative under active investigation but is very difficult to achieve. Thus, if the design process is to result in an efficient system, it must provide for integration between lower (e.g., component) and higher (e.g., system) level designs. Various models for system design (e.g., the System Engineering Vee, waterfall, and spiral models) depict this common theme of iterative feedback between design levels (Buede, 2000). Although the concept is prevalent in basic systems engineering theory, this inter-level communication is rarely automated and usually “ad hoc” at best. For design problems defined via optimization formulations, it is logical to use optimization to synthesize inter-level design levels as well. Only in this way can one ensure optimality at the system-level as well as compatibility of designs at lower levels.

This chapter develops a probabilistic optimization methodology for aerospace vehicle design that takes into account linkages between system-level and component-level design requirements. This methodology formalizes the inter-level iteration required by a systems engineering design approach. The system design considered optimizes the geometry of a re-usable launch vehicle (RLV) for minimum weight while satisfying aerodynamic constraints. The component design illustrated relates to the structural sizing of vehicle components, in this case a liquid hydrogen (LH_2) tank. The bi-level design is formulated as an RBDO-MDO problem, which is solved using sequential, inverse form reliability based design optimization with fully-integrated analysis (Method 4 from Chapter III).

The chapter is organized into six remaining sections. The first two sections set up an illustrative design problem, presenting the system-level and component-level reliability-based optimizations respectively. The next section describes how the system and component levels are coupled; this is followed by an integrated reliability-based optimization formulation which takes into account both system and component constraints. The final two sections discuss results and present conclusions.

System Design (RLV Geometry)

Establishing the rough geometry of the vehicle is a system-level analysis. At this level, it is necessary to approximate component contributions to the design in a low-fidelity, or non-detailed manner. In this “sample” case, weight-estimating relationships (WER) developed from vehicles already in the inventory are used for the conceptual sizing of new launch vehicles through the code, CONSIZ (Unal et al, 1998; Cerro et al, 2002). These WERs assess component contributions to the overall vehicle weight without getting into detailed analyses such as that required for assessing component structural performance. This analysis of the vehicle weight distribution is input into an initial aerodynamic performance assessment. In addition, it provides a conceptual starting point from which to base the more detailed design and analysis of components. The combination of weight prediction and aerodynamic performance assessment is the system-level design considered here.

Low fidelity second-order response surface models were developed for a deterministic sizing analysis of a wing-body, single stage-to-orbit vehicle (Unal et al, 1998). For this application, a launch vehicle is sized to deliver a 25,000 lb payload from the Kennedy Space Center to the International Space Station. The vehicle geometry, for illustration purposes, is shown in Fig. 1, and has a slender, round fuselage and a clipped delta wing. Elevons provide aerodynamic and pitch control. Vertical tip fins provide directional control and body flaps provide additional pitch control.

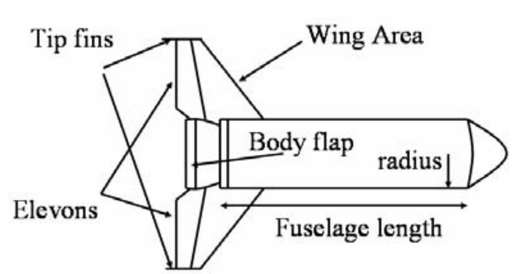


Figure 1. Illustrative Vehicle Geometry Concept

As a first step in the conceptual design, two analyses (weight prediction and aerodynamics) are considered in a constrained optimization problem. A vehicle geometry that minimizes mean dry weight is expected to minimize overall cost, so this is chosen as the objective function. For stability, the pitching moment (C_m) for the vehicle should be zero or extremely close to zero. In addition, C_m should decrease as the angle of attack increases. This is achieved by adjusting the control surfaces to trim the vehicle as the angle of attack is increased. Thus the aerodynamic analysis for pitching moment constrains the optimization. Additional constraints are placed on the lift-to-drag ratio for hypersonic flight (*L/D*), tail volume coefficient (*tvc*), and the ratio of landed weight to standard reference wing area and coefficient of lift ($W/S/C_L$). The hypersonic *L/D* is set to be greater than 1.2 to achieve a desired cross-range capability during entry. $W/S/C_L$ constraint limits the landing speed (227 corresponds to a landing speed of around 200 knots), and a maximum *tvc* value of 0.05 is set simply to limit the size of the tail fins.

The optimal vehicle design is determined by six design variables: fineness ratio (fuselage length / radius), wing area ratio (wing area / radius^2), tip fin area ratio (tip fin area / radius^2), body flap area ratio (body flap area / radius^2), ballast weight fraction (ballast weight/ vehicle weight), and mass ratio (gross lift-off weight/ burnout weight). For the aerodynamic part of the analysis, three additional variables are required to describe the adjustment of control surfaces in order to trim the vehicle: angle of attack, elevon deflection, and body flap deflection. The pitching moment constraint must hold during all flight conditions; nine flight scenarios (constructed with three velocity levels and three angles of attack) are used as a representative sample. The representative velocities (Mach 0.3, Mach 2, and Mach 10) were selected as those originally used in Unal, et al (1998) for which response surfaces were previously generated.

System uncertainty comes from various sources, modeled through probability density functions. For example, uncertainty in the geometry variables mentioned above may result from as-built conditions not matching with precision the design specifications made at this early conceptual level. Furthermore, uncertainties in operational performance lead to randomness in the control surface deflection variables. Given this input uncertainty, the output parameters such as empty weight; W_{empty}, pitching moment coefficients, C_m, and other significant aerodynamic ratios are also random variables. In the problem formulation, the pitching moment requirement is therefore provided as a probabilistic constraint based on an upper and lower limit state for each of the nine flight scenarios. The lower bound limit state is

$$g_{lower} = 0.01 + C_m, \tag{1}$$

and the upper bound limit state is

$$g_{upper} = 0.01 - C_m \tag{2}$$

The probability of failure is then defined as

$$\begin{aligned} P_f &= P(C_m \leq -0.01) + P(C_m \geq 0.01) \\ &= P(g_{lower} \leq 0) + P(g_{upper} \leq 0) \end{aligned} \tag{3}$$

Other constraints include first-order mean approximations for tail volume coefficient, hypersonic lift/drag and relative landed weight/lift constraints. Thus the system-level design may be defined by the following RBDO-MDO formulation:

$$\text{Minimize mean of } W_{\text{empty}} \tag{4}$$

$$\text{Subject to } P(|C_{m(i)}| \leq 0.01) \leq 0.1, \text{ i} = 1 \text{ to } 9$$

$$\text{mean of } tvc \leq 0.05$$

$$\text{mean of } W/S/C_L \leq 227$$

$$\text{mean of } L/D \geq 1.2$$

Component Design (LH$_2$ Tank Structural Sizing)

As mentioned in the previous section, the system-level design provides a basis for the more detailed design of individual components. In this case, the weight distribution of the RLV system provides input for inertial loads required for the structural design of individual components (Cerro et al, 2002; Mahadevan and Smith, 2003).

A launch vehicle is comprised of many components (Fig. 2). Each component must be designed to successfully perform its individual function, but must also integrate or 'fit' into the system as a whole. For the scope of this analysis, an LH$_2$ tank is considered. It is assumed to be a typical cylindrical tank with given end eccentricity, located at a fixed distance from the end of the vehicle. The tank is to be sized such that it is as light as possible but strong enough to resist stresses induced by inertial loads, internal pressures, and other forces.

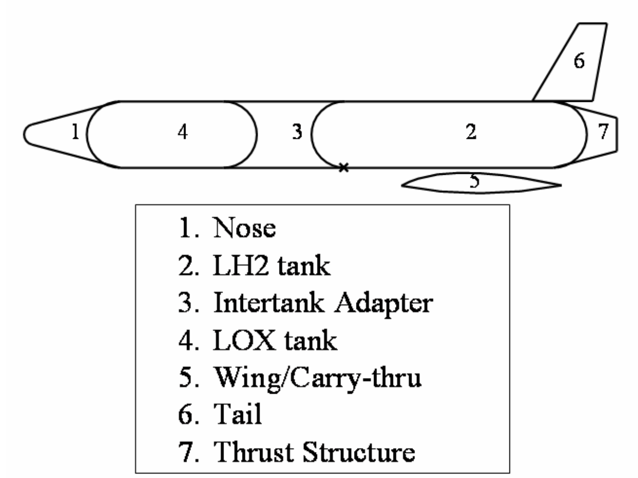


Figure 2. Launch Vehicle Components

The design goal for the liquid hydrogen tank then is to minimize the weight of the tank while meeting the requirements for fuel capacity and structural integrity. The fuel capacity requirement is determined by the system-level design (i.e., from weight estimating relationships used in the system-level weights analysis). At the component level, the fuel capacity is maintained by choosing the appropriate tank geometry. A deterministic optimization problem may be formulated to select the best design for the tank wall structure as

$$\text{Minimize Tank Weight} = \text{f}(R) \tag{5}$$

Subject to

$$R - S < 0 \text{ or } \frac{R}{S} - 1 \leq 0 \text{ (for all failure modes)}$$

where R is the tank resistance and S is the loading on the tank. Here R and S are generic symbols for resistance and loading, and can be tailored for different failure modes. The left-hand side of the above constraint is referred to as a limit state function in reliability analysis literature (Haldar and Mahadevan, 2000). The problem is re-formulated to consider the uncertainties in R and S.

$$\text{Minimize mean of tank weight} = \mathrm{f}\left(f_R(R)\right) \approx \mathrm{f}(\mu_R) \qquad (6)$$

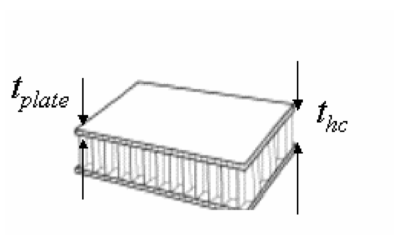
Subject to

$$P(R-S) < P_{\text{required}} \text{ (for all failure modes)}$$

This optimization formulation recognizes that the objective (tank weight) and constraints (failure limit states) are random variables. For well-defined optimization, objectives and constraints need to be selected from among the parameters that characterize the random distributions of these variables. In this case, the parameter mean tank weight is selected as the objective, and the probability of system failure is chosen as the constraint.

There are multiple modes of failure for the tank (i.e., Von Mises interaction failure, isotropic failure, panel buckling), multiple locations along the tank that could fail, and even multiple load cases (at various stages in the vehicle trajectory) that could cause failure. Each of these failure cases may be represented by a corresponding limit state. However, the overall reliability measure for the tank is the system failure probability, which synthesizes all of these modes. This system failure is represented by the union individual limit state failures. Several methods are available for approximating the union or intersection of several events (Ditlevsen, 1979; Hohenbichler and Rackwitz., 1983; Madsen et al, 1986; Gollwitzer and Rackwitz,, 1988; Xiao and Mahadevan, 1994). However, to simplify the optimization problem (for this "sample" problem), representative failure modes are given individual failure probability limits in lieu of a system failure constraint. Evaluating the structural failure criteria then involves four subtasks: (1) defining the system loading, *S*, based on information from the system-level analysis and the mission profile; (2) defining analytical models for various failure modes that incorporate the loading model and resistance, *R*, in terms of design variables; (3) quantifying the uncertainty in the inputs to the failure model; and (4) formulating and solving the resulting reliability-based design optimization.

For the first subtask, system load calculations are based on a simple beam model in this chapter, for the sake of illustration. For the second subtask, a multi-mode failure model of the system is considered. This model synthesizes three failure modes for a honeycomb sandwich wall tank, consisting of 40 individually designed panels (Fig. 3). The honeycomb sandwich consists of top and bottom plates of Aluminum, AL2024 and Hexcell 1/8"-5052-.0015 for the sandwich material. Design of the panels must specify the thickness of the plates and sandwich. For the tank walls, the significant failure modes are: exceeding isotropic strength in the transverse direction, exceeding Von Mises strength, and honeycomb buckling. Three limit state functions (g_{ISO}, g_{VM}, and g_{HCB} respectively) are defined such that $g_i < 0$ indicates failure by a particular mode *i*. To facilitate probabilistic optimization, response surfaces for each failure mode was developed from a design of experiments using commercial structural sizing software.



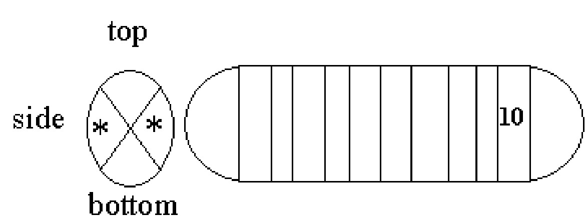


Figure 3. Segmented Honeycomb-Wall Tank

The third subtask requires modeling system uncertainty. As seen from the first subtask, loading is a function of several variables. Plate thickness (t_{plate}) is the design variable, and honeycomb thickness (t_{hc}) is an additional resistance variable. All of these have a degree of uncertainty that affect the structural integrity of the component (i.e., whether or not the LH_2 tank satisfies the three failure criteria). The variables are summarized in Table 1. The first 6 variables in Table 1 are determined by the mission profile for the launch vehicle. They vary along the flight trajectory and include two reaction locations (*R1* and *R2*) representing support locations at lift-off, aerodynamic lift points during flight, or wheel locations during landing. Other mission variables are the fuel percentage, horizontal and vertical components of acceleration, and the liquid oxygen to LH_2 mixture ratio. The *system* variables are relevant geometry parameters and component weight predictions obtained from the RLV system design.

Table 1: LH_2 Tank Sizing Variables

Parameter	Origin	Mean	Cov	Description
R1	mission	350	0.1	Location of first reaction point
R2	mission	2000	0.1	Location of second reaction point
%_fuel	mission	0.9	0.1	Percent of fuel remaining in tank
ax	mission	1	0.1	axial acceleration
ay	mission	1	0.1	normal acceleration
mixratio	mission	0.2	0.1	ratio of lox weight to lh2 weight
radius	system			RLV & tank radius
fuel wt	system			total fuel weight (lh2 and lox)
t plate	component	design var	0.1	top and bottom plate thickness
t hc	component	0.1	0.1	honeycomb sandwich thickness
oal	system			overall length
wstruct	system			distributed load along entire RLV
wwing	system			distributed load along wing

Finally, for the fourth task, the RBDO formulation below is given for the structural sizing problem.

$$\text{Minimize mean of } t_{\text{plate}} \tag{7}$$

$$\text{Subject to}$$

$$P(g_{\text{VM}} \le 0) \le P_{\text{VM acceptable}}$$

$$P(g_{\text{ISO}} \le 0) \le P_{\text{ISO acceptable}}$$

$$P(g_{\text{HCB}} \le 0) \le P_{\text{HCB acceptable}}$$

Data Coupling between System and Component

As apparent from Table 1, the component-level design relies on input from the system design. For example, the tank geometry is constrained by the vehicle geometry (the tank radius must be smaller than the vehicle radius) and by the volume of fuel needed for the mission (i.e., propulsion weight). The loads placed on the tank are a function of both vehicle geometry (radius and length) and weight distribution (modeled as uniform distributions for major components). This data flow represents the decomposition phase of design.

After component design is completed, a more accurate estimate of the tank weight is available from Eq. (8) as

$$\sum_{\text{all panels}} (\text{panel length} * \text{panel width}) * (\rho_{\text{plate}} * t_{\text{plate}} + \rho_{\text{hc}} * t_{\text{hc}}), \tag{8}$$

where panel length and width are calculations performed during the structural sizing analysis. The “refined” tank weight may be fed back into the system design to verify if the system-level requirements are still met. Recall that the system design analysis initially accounts for the component weight contributions through the weight estimating relationships (WER). One option is to replace the LH_2 WER with the weight from the component-level analysis as a constant. However, we chose instead to adjust a reduction factor (included as part of the WER) so that the tank weight will adjust as system-level design changes are made. This reduction factor is denoted $rf_{\text{tank weight}}$ and is updated according to the following formula:

$$rf_{\text{tank weight}} = 1 - \frac{\text{tank weight from structural sizing}}{\text{baseline tank weight from Weights analysis}} \tag{9}$$

where the baseline tank weight is given by a response surface of the LH_2 tank weight (prior to applying the reduction factor) from the software code CONSIZ (Unal et al, 1998; Cerro et al, 2002).

When desiring a true "optimal" design, a single pass of information from system to component and back is inadequate. Instead an iterative process is needed to converge on optimal solutions for both the system and component designs. Perhaps the most obvious iteration strategy is to use a brute force fixed-point iteration method; in other words to simply repeat the system–component–system design cycle and hope for ultimate convergence so that neither design changes in subsequent cycles. This idea is depicted in Fig. 4.

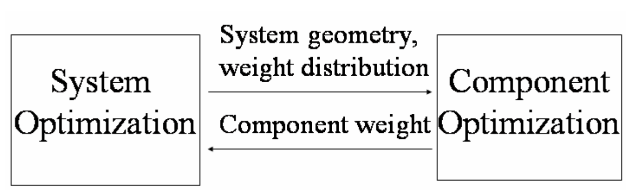


Figure 4. Fixed-point Iteration Between System and Component-level Optimizations

This bi-level optimization is a common strategy for design; it does not require inter-level data flow during optimization and preserves a degree of autonomy for component-level designers. However, this strategy may not be able to find a converged solution to the bi-level system with a reasonable amount of computational effort if at all. As more components are added, finding a feasible solution will become even more difficult.

An alternate approach is to integrate the two optimizations through an expanded MDO-RBDO formulation which treats the component design as an additional disciplinary analysis. To understand how this may be done, it is helpful first to map out the data flow for each design level. Figs. 5a and 5b provide such a mapping for the system and component level optimizations respectively.

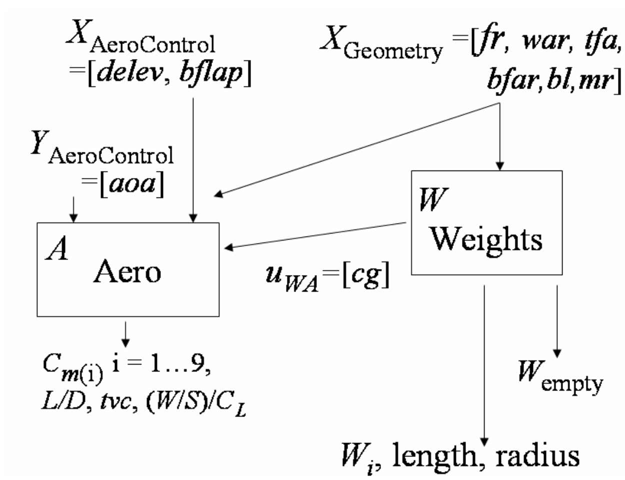


Figure 5a. System Design Optimization Data Flow

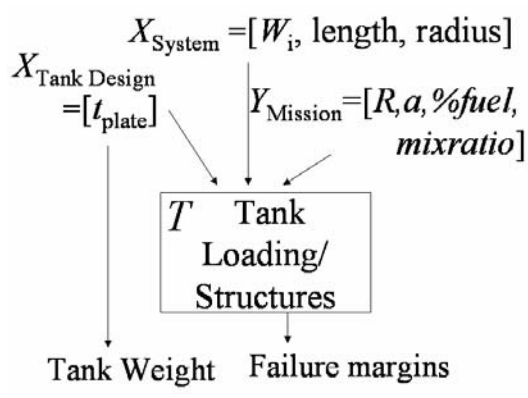


Figure 5b. LH_2 Tank Design Optimization Data Flow

(Note that in Fig. 5, design variables are denoted by *X*, while *Y* denotes other uncertain input variables.) Figs. 5a and 5b reveal several issues. First, from the two-discipline system analysis in Fig. 5a, we notice that the quantities of interest for the component tank analysis, *T*, come from the weights analysis, *W*; the aerodynamics analysis, *A* is needed only to evaluate the system-level constraints. Similarly, during system updating, the tank weight is directly relevant only to the weights analysis, *W,* but affects the aerodynamics discipline through the centers of gravity passed as a state (or coupling) variable from *W*. In addition, it is evident that to truly couple the system and component analyses, the weights analysis needs modification to make the LH_2 tank weight reduction factor an explicit input. This requires a new design of experiments to generate new weight response surfaces incorporating the additional input. Finally, the multidisciplinary system of Fig. 6 for the system-component coupling is proposed. Note in this figure that the tank design variables ($X_{\text{TankDesign}}$) from Fig. 5b (plate thickness and honeycomb thickness) have been replaced by the tank weight reduction factor ($rf_{\text{tank weight}}$). This exchange can easily be made since the reduction factor is uniquely determined by the tank design (i.e., plate thickness for given honeycomb thickness) using Eqs. (8) and (9).

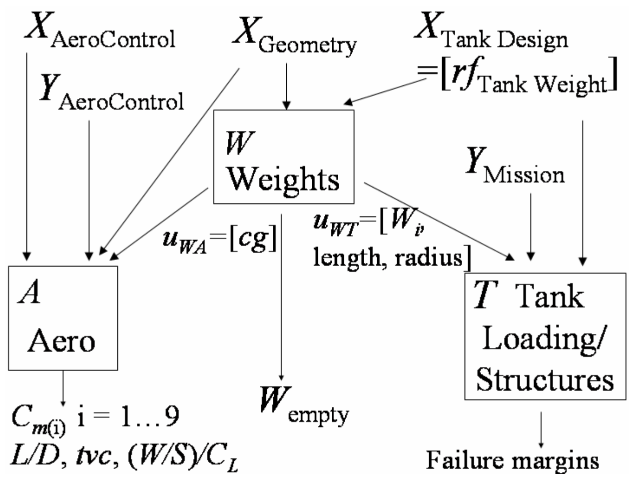


Figure 6. Integrated Multidisciplinary System

Probabilistic MDO Formulation of RLV System/Tank Design

The influences of uncertainty in the combined design have to be treated carefully. Uncertainty in the geometry propagates through the weights analysis to the other disciplines through the intermediate state variables (e.g., *cg*, the weight distribution, length, radius, and tank weight). These uncertainties combine with uncertainty in the Aero Control and Mission variables resulting in uncertainty in the aerodynamic constraints as well as in the structural failure analysis. With this uncertainty propagation in mind, an RBDO-MDO formulation for the system given in Fig. 6 follows:

$$\underset{\mu_{XGeomery},\ \mu_{rf},\ \mu_{XaeroControl}}{\text{Minimize Mean of } W_{\text{empty}}} \tag{10}$$

subject to

$$P(|C_{m(i)}| \leq 0.01) \leq P_{\text{acceptable}} \text{ for } i = 1\ldots9$$

$$P(g_{\mathrm{VM}} \leq 0) \leq P_{\mathrm{acceptable}}$$

$$P(g_{\mathrm{ISO}} \leq 0) \leq P_{\mathrm{acceptable}}$$

$$P(g_{\mathrm{HCB}} \leq 0) \leq P_{\mathrm{acceptable}}$$

$$\text{mean of } tvc \leq 0.05$$

$$\text{mean of } W/S/C_L \leq 227$$

$$\text{mean of } L/D \geq 1.2$$

$$\text{where } C_{m(i)} = \mathrm{A}(X_{\mathrm{Geometry}}, X_{\mathrm{AeroControl}}, u_{WA,} Y_{\mathrm{AeroControl}}),\ i = 1\ldots9$$

$$\text{and} \quad g_{\mathrm{j}} = T(u_{WT}, Y_{\mathrm{Mission}}),\ j = \mathrm{VM, ISO, HCB}$$

The mean values (denoted by μ) of the input variables ($X_{\mathrm{Geometry,}}$ $X_{\mathrm{AeroControl}}$, and $rf_{\mathrm{tank\ Weight}}$) are the design variables. The first order mean approximation for empty weight is the objective just as in the system-level analysis. Probabilistic pitching moment constraints are also used as in the system-level analysis; they are functions of geometry inputs, aerodynamic control inputs, the coupling variable, u_{WA} (i.e., center of gravity) from the weight analysis, and the random parameters ($Y_{\mathrm{AeroControl}}$). Similarly, first order mean values for *tvc*, $W/S/C_L$, and *L/D* are given as constraints. The probabilistic constraints for structural failure are the same as those given in Eq. (7), specifically the probability of three significant modes of failure dependent on output from the weights analysis (u_{WT}) and the random parameter (Y_{Mission}). The structural sizing objective (i.e., minimize tank plate thickness) disappears from the formulation. However, since the plate thickness directly affects the vehicle weight, minimizing the overall objective (i.e., vehicle empty weight) will also ensure minimal tank plate thickness.

Conveniently, the formulation in Eq. (10) does not have feedback coupling (the aerodynamic and structural analyses depend on data flow from the weights analysis, but the weights analysis does not require input from the other disciplines.). Thus, fully-integrated multidisciplinary analysis may be achieved with a single evaluation of each of the disciplinary analyses therefore there is no benefit to a simultaneous analysis and design (SAND) approach. (It is important to realize that the absence of multidisciplinary analysis iteration is only a feature of this particular problem, and may not necessarily be the case for system to component integration problems in general.) Given that the formulation above includes twelve probabilistic constraints, the fully-integrated sequential RBDO method using inverse FORM (Method 4) was chosen as the solution algorithm based on this method's stability and efficiency when tested on the example problems of Chapter III as well as when applied to the UAV design of Chapter IV.

In accordance with MDO-RBDO Method 4, the deterministic optimization sub-problem for Eq. (10) is given by Eq. (11); Eq. (12) provides an example of the inverse FORM reliability analysis for a single pitching moment constraint.

$$\underset{\mu_{X\text{Geomery}},\ \mu_{rf},\ \mu_{X\text{aeroControl}}}{\text{Minimize}} \text{ Mean of } W_{\text{empty}} \approx W(\mu_{X\text{Geomery}}, \mu_{rf}) \tag{11}$$

$$\text{subject to}$$
$$|C_{m(i)}| = W - A(\mu, \eta^{k-1(i)}) \geq 0 \text{ for } i = 1...9$$
$$g_{VM} = W - T(\mu, \eta^{k-1(10)}) \geq 0$$
$$g_{ISO} = W - T(\mu, \eta^{k-1(11)}) \geq 0$$
$$g_{HCB} = W - T(\mu, \eta^{k-1(12)}) \geq 0$$
$$\text{mean of } tvc \leq 0.05$$
$$\text{mean of } W/S/C_L \leq 227$$
$$\text{mean of } L/D \geq 1.2$$

$$\underset{\eta}{\text{Min}}\ |C_{\text{m(i)}}| = W\text{-}A(\mu^k_{X(\text{Geometry})}, \mu^k_{X(\text{AeroControl})}, \mu^k_{\text{rf}}, \mu_{Y(\text{AeroControl})}, \eta) \tag{12}$$

$$\text{subject to}$$

$$\beta = \|\eta\|_2 = -\Phi^{-1}(0.1)$$

Here μ represents the mean values for all random variables (i.e., $\mu_{X\text{Geometry}}$, $\mu_{XA\text{eroControl}}$, μ_{rf}, $\mu_{Y\text{Mission}}$ and $\mu_{Y\text{AeroControl}}$). *W-A* represents the sequence of analyses, weights followed by aerodynamic analysis, while *W-T* represents weights analysis following by structural analysis. The parametric constraints (mean of *tvc*, mean of $W/S/C_L$, and mean of *L/D*) are already in deterministic form so appear exactly as in the original formulation. Each of the probabilistic constraints has been replaced by a deterministic equivalent in accordance with sequential, inverse FORM RBDO. These constraints are functions of the parametric design quantities (mean values of X_{Geometry}, $X_{\text{AeroControl}}$, and $rf_{\text{tank Weight}}$) and the stochastic components of all random variables, η^k.

Following the optimization, an inverse FORM analysis is required for each probabilistic constraint to determine the PMA point, $\eta^{k(i)}$. Eq. (22) provides the search formulation for a single pitching moment constraint.

$$\underset{\eta}{\text{Min}} \; | C_{\text{m(i)}} | = W\text{-}A(\mu^k_{X(\text{Geometry})}, \mu^k_{X(\text{AeroControl})}, \mu^k_{\text{rf}}, \mu_{Y(\text{AeroControl})}, \eta) \tag{22}$$

subject to

$$\beta = \|\eta\|_2 = -\Phi^{-1}(0.1)$$

The resulting random realization η then becomes $\eta^{k(i)}$ for the next optimization. Note that each constraint is associated with its own $\eta^{k(i)}$ since $\eta^{k(i)}$ represents the current (k^{th})estimate of the PMA point for the particular constraint, *i*.

Results and Discussion

The optimization, Eq. (10), was solved using a Matlab routine which implements the reliability-based optimization (as outlined above) with a sequential quadratic programming algorithm from their Optimization Toolbox (Mathworks, 2003). A converged solution was obtained in 4 iterations. For comparison, RBDO-MDO Method 1 (fully-integrated nested RBDO approach based on direct FORM) was attempted but was not able to converge to a solution. The results are given in Table 2 for two different reliability constraints: a 10% probability of constraint failure and a 0.0013 probability of constraint failure (corresponding to target reliability indices of 1.28 and 3 respectively).

Table 2: Optimization Results

P_f β_{target}		0.10 1.28	0.0013 3.00
Bounds	Optimal Design		
[0, 0.9]	$rf_{\text{tank weight}}$	.033	0.00
[4, 7]	*fr*	7.00	7.00
[10, 20]	*war*	15.99	16.839
[.05, 3.0]	*tfar*	0.50	0.96
[0, 0]	*bfar*	0.00	0.00
[0, 0.4]	*bl*	0.0033	0.0013
[7.5, 8.25]	*mr*	7.74	7.76
	$\mu_{\text{Empty Weight}}$ (lb)	202,180	212,800
Computational Effort			
Decoupled RBDO Iterations		4	4
Optimization Function Evaluations		15,349	13,932
Probabilistic Analysis Function Evaluations		10,772	10,358

As expected, the higher reliability requirement results in a larger mean vehicle weight (about a 5% increase in mean weight for a slightly less than 100 fold improvement in reliability). For both reliability levels, the two active constraints are the ninth pitching moment constraint (corresponding to a maximum angle of attack at hypersonic speed) and the isotropic strength constraint. A post-optimization sensitivity analysis (based on relative partial derivatives at the design point) reveals that the most significant variables for empty weight are the fineness and wing area ratios. These two variables are also the most significant for the pitching moment constraints. However, the upper bound constraint for the mean of the fineness ratio is also active, limiting its contribution to improve the optimal weight. The probabilistic constraint for isotropic strength failure is dominated by the tank weight reduction factor ($rf_{\text{tank weight}}$).

Note that the function evaluations for each RBDO phase (i.e., the optimization phase and the probabilistic analysis phase) include evaluations required for finite difference approximations of the gradient. In the optimization phase, there are 15 constraints and twenty-two design variables, so a minimum of 331 function evaluations are required to approximate the Jacobian during each iteration of the optimizer. In this case, each optimization phase requires approximately 4000 function evaluations in an average of 12 iterations. (Note that a deterministic safety-factor based formulation of the original problem would be also be expected to require about 4000 function evaluations using finite differences to approximate the gradient.) In the probabilistic analysis phase, there are 38 random variables so a minimum of 39 function evaluations is required for every probabilistic analysis loop; this is required for each of the twelve probabilistic constraints. The importance of derivative calculations in large-scale optimization problems is well documented and these results only reinforce their significance. In this example, there would be considerable value in identifying inactive constraints so that those calculations could be avoided. This was not done, but in hindsight could have resulted more than seven-fold improvement in computational effort given that only two of the fifteen constraints were active at the optimum. Even so, the total computational effort for the decoupled RBDO (e.g., 26,121 function evaluations for a 10% failure probability) is under seven times that for deterministic optimization (roughly 4000 function evaluations).

Another significant observation is that the total number of evaluations does not increase as the required failure probability is decreased. This is an advantage of using an analytical approach (i.e., first order reliability analysis) to evaluating the probabilistic constraint as opposed to Monte Carlo simulation-based methods. However, accuracy of the reliability estimate could be compromised, especially for highly non-linear constraints. Therefore, the final design should be checked with Monte Carlo Simulation.

Conclusion

Design by decomposition is a fairly common and practical strategy for complex engineering. However, some degree of integration is required to ensure the multi-level designs are compatible. This is a special challenge when the effects of uncertainty are considered. The process outlined in this chapter presents a strategy for coupling design levels as a multidisciplinary optimization under uncertainty.

The example application demonstrates some obvious advantages and drawbacks for this approach. First, by using reliability based design optimization (RBDO), reliability requirements may be explicitly enforced during design. A deterministic factor of safety design, on the other hand, does not provide a quantitative measure of reliability.

The RBDO approach also allows engineers to see the affects of varying reliability requirements on design optimality, which is extremely useful in making informed trade-off decisions. By using a multidisciplinary optimization to couple design levels, the uncertainty information also passes formally between system and component level designs. This approach prevents low-fidelity system-level analyses from unduly restricting future component level design decisions. The obvious drawback for the methodology is the increase in computational effort over deterministic methods. However, decoupled RBDO methods reduce this liability significantly.

Incorporating the design of additional components would require additional probabilistic constraints and additional design variables linking component requirements to the system-level objectives (e.g., reduction factors for weights from each component). The MDO problem complexity and the computational effort required to solve it will increase proportionally. However, this approach is likely less difficult than attempting to integrate the individual component designs directly with one another on a single level. Another added complexity would be to consider additional component design variables (e.g., tank properties other than plate thickness). In this case, it might not be possible to use a variable such as the tank weight reduction factor to link the system and component weight analyses. Instead, a component-level optimization could be used for the structural sizing analysis of the tank. Finally, the issue of how to handle system reliability constraints (such as a system failure defined by the union of several failures) in conjunction with efficient reliability-based optimization needs to be addressed.

In short, integrating system and component designs into an MDO-RBDO formulation is a promising design choice if (1) it is possible to clearly map interactions between levels (2) the designer is willing to invest significant computational effort in achieving integration. Further research is needed on larger problems to evaluate how quickly computational effort increases with design problem characteristics such as number of component analyses, dimension of design space, dimension of random space, and number of constraints.

CHAPTER VI

REUSABLE LAUNCH VEHICLE APPLICATION: MODEL ERROR REFINEMENT FOR SYSTEM-LEVEL DESIGN

Introduction

In the previous chapter, reliability-based design for optimal geometry of a conceptual reusable launch vehicle (RLV) given aerodynamic constraints was integrated with a component-level design for a liquid hydrogen tank (constrained by structural integrity requirements). The two design levels, that of the conceptual "system" (or overall geometry) and that of the component tank, were coupled through the tank weight and iteration was needed to integrate the two levels. This is typical of a systems design process, which, as it progresses, moves from conceptual, lower-fidelity, less computationally intensive analysis to more detailed, higher fidelity, more intensive analyses. In the last chapter, the tank design was treated as an additional discipline and multidisciplinary analysis methods were used to integrate it with the system design.

In this chapter, the same application is studied from the perspective of model error; the system design includes a conceptual, low-fidelity weights analysis with significant model uncertainty, while the tank design provides a more rigorous analysis for the weight of one component with significantly less uncertainty. Evaluating model error of disciplinary analyses as a metric for model uncertainty provides two important benefits. The first is as a metric for selecting appropriate disciplinary models to integrate into multidisciplinary analysis at a given design stage. The second benefit is as the basis for an alternative methodology for iterating between design levels; rather than fully integrating the system and component analyses, this method uses model uncertainty to leverage the component design to refine the system analysis.

As discussed in the previous chapter, a systems design process progresses from the top down. At higher levels, design concepts are broad in scope (e.g., they encompass the entire physical system) and have limited detail; as the design progresses, detail increases and scope decreases (e.g., from system, sub-system, component, etc.). Another important characteristic of this progression concerns the trade-off between model error and effort (or expense) of the analysis. At the highest design level, since less detail is required, it is typical to achieve fast, inexpensive analysis at the expense of increased model error or uncertainty. Conversely, as the design progresses, higher fidelity analyses are needed to assure accurate assessment of system performance. This progression is evident in the RLV-tank application. At the conceptual system level, a weights analysis for the entire vehicle is used (so the scope of the analysis is very broad), and the design detail achieved is limited (in this case just basic geometric parameters for the vehicle as a whole). At the same time, the analysis is based on simple parametric equations developed from historical vehicles; it is fast but there is a great deal of uncertainty regarding its accuracy. In other words, the conceptual system analysis has a high degree of model error. As the design progresses to the liquid hydrogen tank, the scope is reduced (i.e., a single component versus the RLV as a whole) and design detail is increased (i.e., to specific tank dimensions, location, materials, etc). Meanwhile, a more rigorous analysis is required (in this case, structural sizing), one for which there should be less uncertainty.

One fundamental characteristic of the design process is that it requires iteration to ensure communication between levels in both directions. Again considering the RLV-tank progression, the tank design requires information from the system (e.g., an overall weight profile overall dimensions, etc.) while the system design must assume (initially) information about the tank (e.g., tank weight). The tank design will likely invalidate the initial assumptions used in the system design so iteration is necessary to synthesize the two. The most typical manner in which this bi-directional communication between design levels is accomplished is through fixed-point iteration. For example, the RLV geometry/ tank-sizing design might be coupled as shown in Fig. 1.

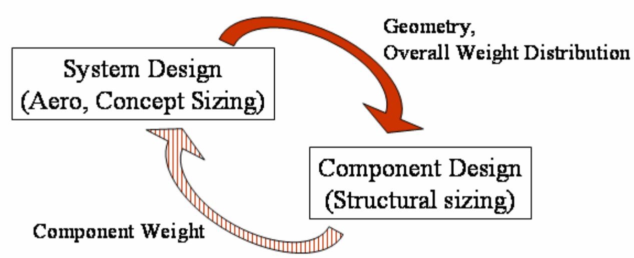


Figure 1. Coupling of RLV System and Component Designs

However, there are major drawbacks to this approach including significant computational effort (i.e., repeated optimizations) and the fact that fixed-point iteration may not yield a solution. In the previous chapter, an alternative procedure was used to iterate between design levels. This required re-mapping the data flow for a combined, multidisciplinary reliability-based optimization as depicted in Fig. 2. Here two optimizations (system and component-level) merge into one by combining constraints (aerodynamic and structural) and elevating a coupling variable, the tank weight. (In this case the tank weight is uniquely mapped to a reduction factor, $rf_{\text{Tank Weight}}$ that is an input into the conceptual weights model, W.) The disadvantage to this approach is that the system optimization could easily become intractable as components (e.g., liquid oxygen tank, wings, thrust structure, etc.) and accompanying performance constraints are added.

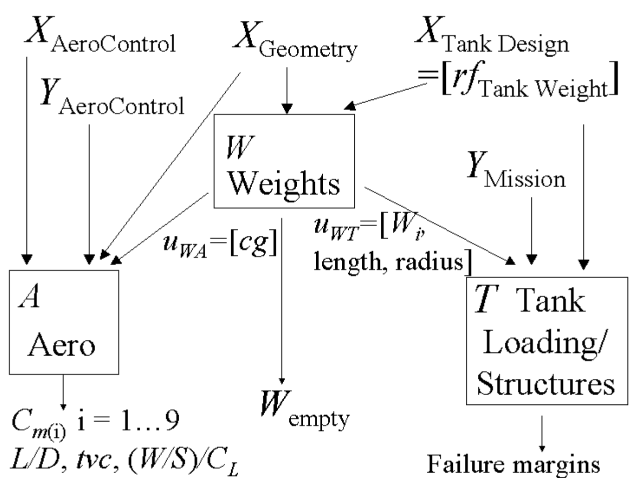


Figure 2. Integrated System-Component Multidisciplinary Analysis

This chapter proposes an alternative methodology for communicating information across design levels. This method maintains autonomy between the system optimization and the component design but takes a sample of component designs in order to characterize the model error of the system analysis. For the RLV application, the model error of concern results from the tank weight prediction in the conceptual weights analysis, *W*. The component design of the tank provides a more accurate prediction of tank weight. A comparison of the two predictions for a given system design provides the model error. This error propagates through the system analysis and affects the overall assessment of system weight as well as aerodynamic performance. Thus, model error is a significant random parameter to be included in the system optimization and one that links the system and component designs. In addition, the sensitivity of the system design constraints to the uncertainty associated with this error defines the effect of disciplinary model error on a multidisciplinary system and provides a useful metric for model selection.

The following section introduces a brief background on model error assessment which has been significantly studied for the purpose of comparing experimental data with results from computational analysis. This is followed by a more detailed description of how model error will be assessed and subsequently used to link design levels. This methodology is then applied to the RLV geometry and tank design problem of the previous chapter with a discussion of results. The chapter concludes with a summary and overall assessment of the methodology.

Model Error Assessment

Computational models are prevalent throughout the design process as a means to predict system performance (in order to adjust the system design to assure desired performance). As introduced in Chapter I, there exists some degree of uncertainty regarding how well these models predict true system behavior arising from a number of sources. First, as a physical model is developed to represent the true physical system, uncertainty is introduced through assumptions regarding the system itself and its operating environment. Eventually, the physical model is reduced to a mathematical model (most often a partial differential equation) which typically may only be solved through a discretized computational model. As these abstractions are made, further uncertainty is introduced.

Collectively, the sources of uncertainties in the model predictions may be described as *model error*, or simply the difference between the model prediction and actual performance of the system. For some performance measure, Y, Eq. (1) describes this relationship where ε_m represents the collective effect of all sources of model uncertainty:

$$Y_{\text{actual}} = Y_{\text{model}} + \varepsilon_{\text{m}} \tag{1}$$

Since true performance (Y_{actual}) is random (it is characterized by natural variability), model error (ε_m) is also a random variable. Oberkampf et al. (2002) provide a detailed taxonomy of model errors based on their source of uncertainty. Methods are available to quantify some sources of error. Many techniques in the literature exist for quantifying discretization error, e.g., error arising from the choice in mesh size for finite element computational approximations (e.g., Richardson, 1977). Errors associated with mathematical approximations (e.g., response surfaces) and probabilistic analysis methods (such as Monte Carlo analysis, FORM, etc.) are also well known.

Much research has been directed at quantifying individual sources of uncertainty for the purpose of model refinement to reduce error or model selection in order to minimize error. For a detailed review see Rebba, 2005 (Chapter IV). Difficulty arises, however, from aggregating all types of model error which are not necessarily additive. Rebba, et al (2006a) provide a method for quantifying model error based on a sample of experimental results and established methods for determining numerical errors. After characterizing the random model error variable ε_{mf} with a probability density function, Mahadevan and Rebba (2006b) subsequently perform reliability-based design optimization using this variable as a source of uncertainty.

Assessing Model Error

This study uses established methods for quantifying model error and suggests this error term as a metric for disciplinary model selection. In other words, model error provides a basis for selecting disciplinary models for integration within a multidisciplinary system analysis as well as a means to communicate design information across levels within the design process. Mahadevan and Rebba (2006a) combine two relationships, one using experimental error and one using the combined effects of model error in order to obtain the relationship in Eq. (2). (They distinguish between *model form* error - that arising from assumptions required to develop a mathematical model from the physical/conceptual model, and *numerical* error – errors arising from the progression from mathematical to computational model.)

$$Y_{\text{true}} = Y_{\text{obs}} + \varepsilon_{\text{exp}} = Y_{\text{model}} + \varepsilon_{\text{mf}} + \varepsilon_{\text{num}} \tag{2}$$

This results in an expression for obtaining model error from experimental data, where ε_{obs} is simply the difference between observed performance, Y_{obs} and the model prediction, Y_{model}.

$$\varepsilon_{\text{mf}} = \varepsilon_{\text{obs}} - \varepsilon_{\text{num}} + \varepsilon_{\text{exp}} \tag{3}$$

Each of the errors on the right hand side of Eq. (3) are random variables. A comparison of experimental and model results provides a sample ε_{obs}, extrapolation techniques are used to characterize ε_{num}, and experimental results may be used find statistics of ε_{exp}. Equation (3) is then used to obtain a sample for ε_{mf} and a bootstrapping technique developed by Efron and Tibshirani (1993) is used to interpolate a smooth probability density function for ε_{mf}.

Early in the design process, experimental data is not available. However, there is typically an abundance of disciplinary model choices, each with varying degrees of model uncertainty (for a specific disciplinary analysis, for example). Results from more detailed analysis with reduced total model error (including both model and numerical errors) may be used in lieu of experimentation to determine the model error for a less detailed, conceptual analysis as in Eq. (4) where $\varepsilon_{concept}$ and ε_{detail} are model errors for the conceptual and detailed analyses respectively and $Y_{concept}$ and Y_{detail} are the performance predictions from each model.

$$\begin{aligned} Y_{true} &= Y_{concept} + \varepsilon_{concept} = Y_{detail} + \varepsilon_{detail} \\ \therefore \varepsilon_{concept} &\equiv Y_{detail} - Y_{concept} + \varepsilon_{detail} \end{aligned} \tag{4}$$

An initial probability density function for the model error for the detailed model, ε_{detail} may be assumed. In the absence of additional information (which would be revealed as the design progresses), a normal distribution with zero mean and small standard deviation is assumed. A random sample of input variables for both models is selected in order to obtain a set of $Y_{concept}$ and Y_{detail}; this is combined with a random sample for ε_{detail} to obtain a sample of $\varepsilon_{concept}$ in accordance with Eq. (4). Efron and Tibshirani's bootstrapping technique is then used to provide a smooth probability density function. At this point the conceptual model error can be propagated through conceptual multidisciplinary analysis, optimization, and design. In this way, the detailed (and computationally intensive) disciplinary analysis is used to calibrate the conceptual model but need not be directly integrated with other disciplines or optimization (as in the previous chapter), which could be intractable due to the computational effort required.

Model Selection using Sensitivity to Model Error

During the top design level, conceptual analysis tools are common. Analyses at this level would have a large uncertainty associated with model error, $\varepsilon_{\text{concept}}$. This uncertainty, once quantified, could play an important role in model selection and refinement during the various stages of design. When the model uncertainty for a particular conceptual disciplinary analysis has a minor effect on the uncertainty of system performance, there is little cause to expend additional resources to upgrade to a more detailed analysis. Conversely, if a system is very sensitive to the uncertainty of a given disciplinary model, increases in analysis detail and fidelity will significantly reduce the uncertainty of system performance. For example, the equation below gives the sensitivity of failure probability to model error, ε.

$$\frac{\partial P_f}{\partial \varepsilon_{mf}} = \frac{\partial}{\partial \varepsilon_{mf}} \Phi(-\beta) = \phi(-\beta) \frac{\sigma_\varepsilon \ \partial g / \partial \varepsilon}{\sqrt{\sum \sigma_i^2 (\partial g / \partial x_i)^2 + \sigma_\varepsilon^2 (\partial g / \partial \varepsilon\ )^2}} \tag{5}$$

Here g is a failure limit state, β is the first order reliability index, σ_ε is the standard deviation of the model error, x_i are other random variables, and σ_i are their respective standard deviations. (Note, this is the same sensitivity factor plotted in Fig. 5 of Chapter IV for the UAV application. In that case, the state of charge error variable represented model error.) In order to evaluate Eq. (5), statistics of model error, ε are needed; these statistics may be derived from the sample generated by the methodology described in the previous section.

Given recent advances in computational power and resources, there is a temptation to continually improve models when in fact this may have very little effect on the overall reliability of an analysis given other sources of uncertainty. However, in comparing $\frac{\partial P_f}{\partial \varepsilon_{mf}}$ for various model choices, engineers can make an informed decision on when it is worthwhile to upgrade to more detailed models.

Tank Weight Estimation Models

The previous chapter treated the integration of the reusable launch vehicle system geometry design and a component tank design. The design progression moved from the conceptual system design to a more detailed, but reduced scope component design. The two designs were coupled through the overall weight distribution and the tank weight. In providing an alternative to fixed point iteration between the design levels, Chapter V combined the two in a single multidisciplinary optimization, treating the structural analysis of the component design as an additional discipline (Fig. 2). However, this chapter treats the structural sizing of the LH_2 tank not as a new discipline but as a more detailed analysis model for component weight estimation than that contained within the conceptual weights model, *W*. *W* uses parametric equations to estimate component weights based on legacy vehicles with technology improvement assumptions. These equations profile a weight distribution for the vehicle, enabling calculations for center of gravity as well as overall sizing measures such as length and radius. The tank design provides a better estimate of the tank weight based on structural integrity requirements and material properties. In fact, similar analyses for the other components could replace ALL the parametric weight calculations in *W*. However, note that in order to accomplish the tank design, the overall weight profile is needed to assess loads. Without the conceptual information provided by *W*, this would require the tank design be directly coupled to the design of all component designs, which would make finding a solution intractable. Instead, one may continue to use the conceptual design, an extremely efficient way to obtain an overall geometry and weight distribution, while leveraging the information from the tank design selectively.

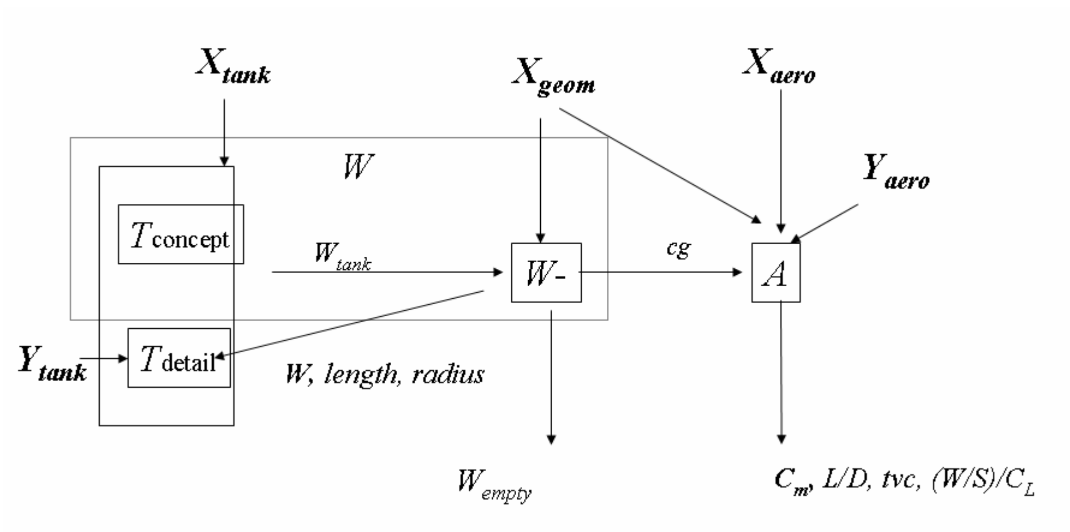


Figure 3. Variable Fidelity Analysis

In Fig. 3, an analysis system for the RLV geometry/ tank sizing problem is given with two alternative models for tank weight estimation. The original weights module, *W*, is decomposed into the conceptual calculation for the tank weight, T_{concept} and the balance of the calculations in the module denoted *W*-. Thus the conceptual design is the original optimization of the RLV geometry based on the conceptual weights model and aerodynamic analysis. The detailed calculation for the tank weight, T_{detail}, is the structural sizing analysis; it solves the optimization problem constrained by structural failure probability (i.e., Eq. (7) of the previous chapter). Note that a combined detailed analysis would result in feedback (i.e., tank weight is an output of T_{detail} and an input to W- while the weight distribution, length and radius are outputs of W- and inputs to T_{detail}) while the conceptual analysis does not, so solving the conceptual system optimization requires much less effort.

Both the conceptual and detailed component analyses for tank weight have model error. A probability density for the conceptual model error, $\varepsilon_{\text{tankWt}}$, is obtained in accordance with the methodology for assessing model error presented earlier. Both tank weight analyses (T_{concept} and T_{detail}) were implemented for a sample of 20 input values ($\boldsymbol{X}_{\text{tank}}$ and $\boldsymbol{X}_{\text{geom}}$). Note that the structural sizing analysis, T_{detail} includes a probabilistic optimization in itself (i.e., minimizing tank weight subject to an acceptable probability of structural failure) which accounts for random parameters $\boldsymbol{Y}_{\text{tank}}$ associated only with the high fidelity analysis. Thus there is no variability in tank weight predictions for a given set of input values ($\boldsymbol{X}_{\text{tank}}$ and $\boldsymbol{X}_{\text{geom}}$) and model error for the high fidelity analysis is neglected. The bootstrapping technique was used to generate a probability density for conceptual model error, $\varepsilon_{\text{tankWt}}$, given in Fig. 4.

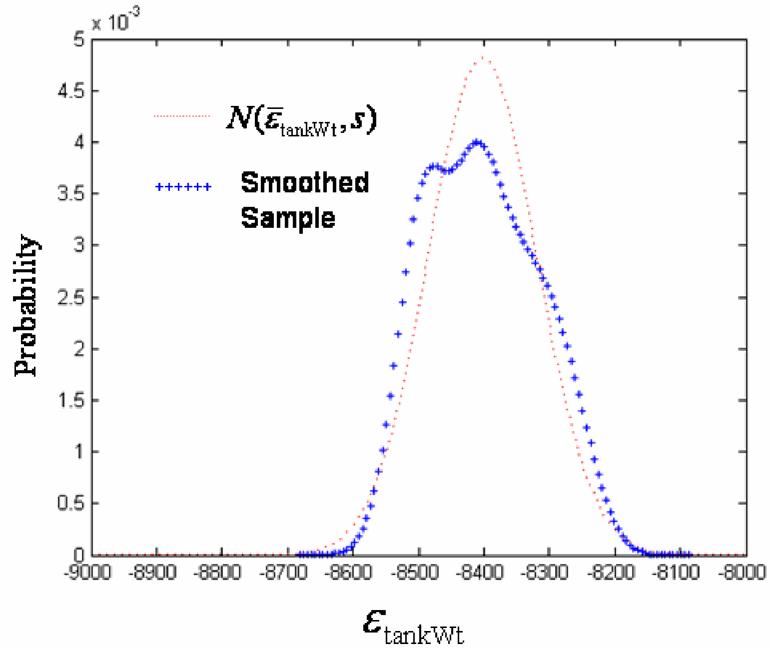


Figure 4. Probability Density for Low Fidelity Model Error, $\varepsilon_{\text{tankWt}}$

At this point, now that statistics for conceptual model error for tank weight are available, a sensitivity analysis for system failure probability is possible. Recall from the previous chapter, that the conceptual RLV design problem is constrained primarily by pitching moment failure probability (i.e., Eq. (4) of Chapter V). The sensitivity of pitching moment failure to tank weight model error is calculated according to Eq. (5) and normalized to compare with sensitivity to other sources of uncertainty. Results are plotted in Fig. 5, giving relative sensitivities to pitching moment failure for the hypersonic, maximum angle of attack flight profile. Although tank weight model error is significant, it is not the dominant source of uncertainty with respect to pitching moment failure probability.

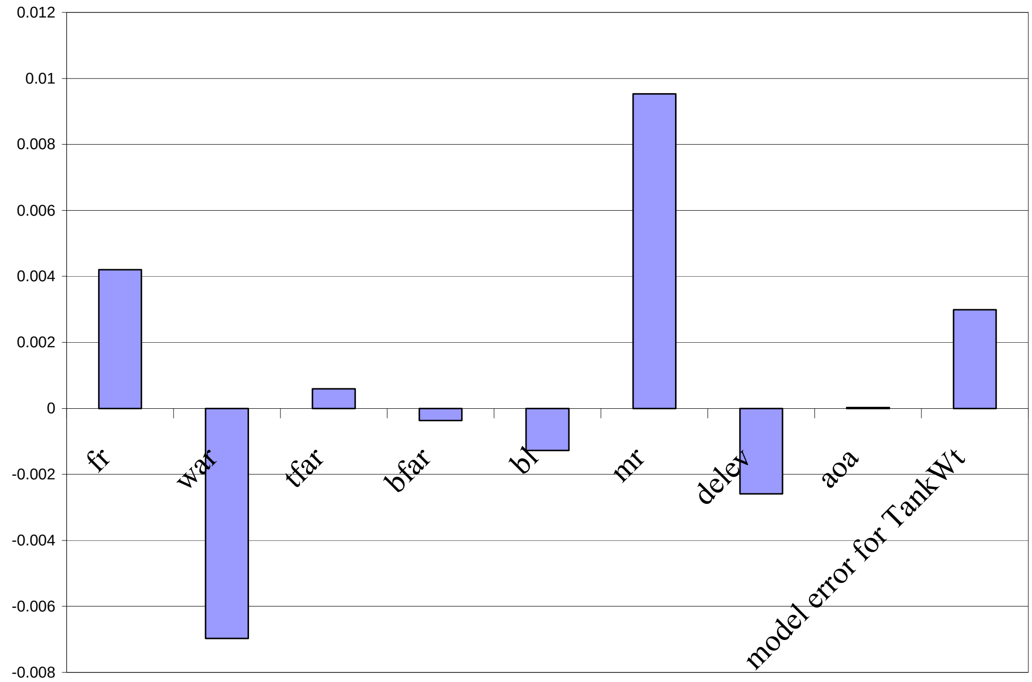


Figure 5. Comparison of Failure Sensitivity to Various Sources of Uncertainty: Pitching Moment for Hypersonic, Maximum Angle of Attack

As the design process progresses, an increase in level of detail should correspond to a decrease in the uncertainty of system performance. Conveniently, an 'upgrade' in system analysis models often accomplishes both ends. For example, transitioning from the conceptual parametric equation model for tank weight to the structural sizing model provides additional detail (i.e., tank geometry) and reduces the predicted probability of pitching moment failure. However, considering a system of several disciplinary analysis models, the improvement in system reliability from improving a particular disciplinary analysis may not be significant to warrant the additional computational effort. The system sensitivity to model error, Eq. (5), is therefore an obvious metric for selecting which disciplinary models to upgrade at a particular stage in design to achieve the desired reduction in performance uncertainty.

Reliability-based Design Optimization Including Model Error

Reliability-based design optimization of the conceptual system, as formulated in Eq. (6) was performed using the fully-integrated, sequential, inverse form RBDO method (i.e., Method 4) as presented in previous chapters. In this case, tank weight model error (characterized in Fig. 4) contributes to the uncertainty in pitching moment.

$$\text{Minimize mean of } W_{\text{empty}} \tag{6}$$

$$\text{Subject to } P(|C_{m(i)}| \leq 0.01) \leq 0.1, \text{i} = 1 \text{ to } 9$$

$$\text{mean of } tvc \leq 0.05$$

$$\text{mean of } W/S/C_L \leq 227$$

$$\text{mean of } L/D \geq 1.2$$

Here W_{empty} is the total empty weight of the RLV, $C_{m(i)}$ is the pitching moment coefficient at one of the nine flight scenarios (subsonic, supersonic, and hypersonic flight at minimum, nominal, and maximum angles of attack), *tvc* if the tail volume coefficient, $W/S/C_L$ is the relative landed weight to coefficient of lift ratio, and *L/D* is the ratio of lift to drag. Also recall from the previous chapter that the six geometric design variables include mean values for the fuselage fineness ratio, *fr*, wing area ratio, *war*, tip fin area ratio, *tfar*, body flap area ratio, *bfar*, ballast fraction, *bl*, and mass ratio, *mr*.

Results are shown in Table 1. The optimal empty weight of the vehicle is 198 kips, similar to that obtained in the previous chapter (202 kips) though there are some differences in the optimal geometry. Note that the total number of function evaluations is similar to that from the integrated design in the previous chapter (Chapter V, Table 2). However, only 20 evaluations were required for the detailed structural analysis in order to obtain 20 samples for the component design.

Table 1: Optimization Results

P_f β_{target}		0.0013 3.00
Bounds	Optimal Geometry	
[4, 7]	*fr*	6.75
[10, 20]	*war*	15.58
[.05, 3.0]	*tfar*	0.5
[0, 0]	*bfar*	0.00
[0, 0.4]	*bl*	0.006
[7.5, 8.25]	*mr*	6.82
	$\mu_{\text{Empty Weight}}$ (lb)	192,400
RBDO Iterations		6
Optimization Function Evaluations		17,462
Probabilistic Analysis Function Evaluations		2106

Integrating Model Errors from Multiple Disciplines

In the above analysis, the RLV component tank design uses a detailed disciplinary analysis model to characterize model error for a less detailed, conceptual model. This model error variable may then be used, first to assess the importance of disciplinary model selection on the performance of a multidisciplinary system (through system sensitivity to model error), and secondly to calibrate the system design through reliability-based design optimization. The real advantage to this approach is seen as the next level of design is expanded to include other disciplines. Consider a design hierarchy for the RLV system as shown in Fig. 6.

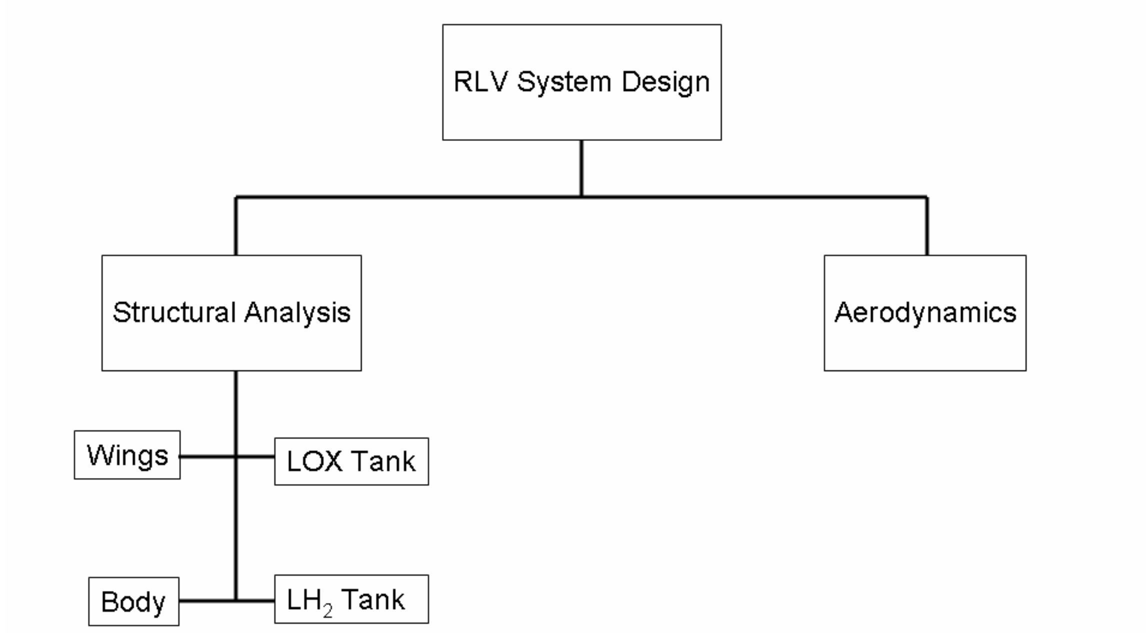


Figure 6. Multidisciplinary Design Hierarchy for Reusable Launch Vehicle

The conceptual design presented in this and the preceding chapter integrates two primary disciplines: aerodynamics and structural analysis (i.e., weights and sizing). Further, as has been shown, as the design progresses and additional detail is needed, individual components may be analyzed separately. Using the methodology shown in this chapter, a limited number of detailed disciplinary analyses may be performed independently (i.e., not in the context of the multidisciplinary system) in order to characterize model errors associated with each of the discipline/component analysis. This would yield for example, up to five model error terms for Fig. 5: ε_{LH2_WT}, ε_{LOX_WT}, ε_{WING_WT}, ε_{BODY_WT}, and ε_{AERO}. A sensitivity analysis would reveal which of these errors have the greatest affect on system performance (i.e., probability of pitching moment failure). This would enable engineers to assess the benefit to cost ratio for upgrading disciplinary models used in the system-level analysis as the design progresses.

For example, at the current conceptual level, the aerodynamic analysis is somewhat more rigorous (and thus reliable) than the weights analysis (based on empirical parametric equations); thus one would expect ε_{AERO} to have both a smaller mean and standard deviation than the other sources of disciplinary model error. However, the pitching moment is also more sensitive to the aerodynamic analysis in general. Assume for illustration purposes, that these two factors balance and that a probabilistic sensitivity analysis, Eq. (5), reveals that the probability of pitching moment failure is moderately sensitive to errors in aerodynamic analysis and the sizing of tanks and wings but is fairly insensitive to body weight errors. However, the increase in computational effort that would result from choosing an aerodynamic analysis model to reduce ε_{AERO} is significant compared to alternative models available to improve the tank and wing weight predictions. The next phase of design should therefore include more rigorous weights analysis, may not yet need the higher fidelity aerodynamic analysis, and would probably not require an upgrade to the body design until later in the design process.

A quick study of Fig. 5 also reveals the advantages of not integrating higher fidelity disciplinary analyses for reliability-based design optimization. The previous chapter demonstrated that the computational effort for MDO-RBDO for the bi-level design was not insignificant despite methods developed to improve efficiency. With the addition of more component designs, the complexity of the optimization increases: there are more design variables, more random parameters, and additional constraints. However, RBDO incorporating disciplinary model errors provides an alternative methodology that, though it sacrifices full integration (of the detailed analyses) in order to keep the system design tractable, provides for conceptual system integration calibrated with valuable information from the detailed design analyses.

Conclusion

This chapter extends recent research in model error quantification and application to reliability-based design optimization for use in integrating multiple levels of design. Throughout the design process, engineers select models to analyze system performance, making trade-offs between effort and detail. At higher, more conceptual design levels, the fidelity and detail of individual, disciplinary analyses are typically low. However, the scope of system analysis at this stage is significant, involving the integration of multiple disciplinary models. Once quantified, model error may be propagated through multidisciplinary analysis and optimization routines using probabilistic analysis.

These concepts have been applied to the integration of a conceptual geometry design and component tank design. In this case, the communication between component (detailed) design and system (conceptual) analysis is made through the updating of model error statistics. Thus the communication between design levels is much less stringent than for MDO-RBDO as used in the previous chapter. No agreement between levels is ever required. In addition, this method maintains the advantage of using conceptual analysis at the system level, i.e., at reduced computational effort and complexity. Finally, assessing model error as a stochastic input provides an effective means for measuring the importance of disciplinary model error on the conceptual system design. This has significance for discipline model selection, pointing to areas where reducing uncertainty will have the greatest pay off in terms of the reliability of the system analysis.

There are, however, a few limitations to this approach. First, the method presumes the 'higher fidelity' detailed model is actually more accurate than the conceptual model. It requires knowledge (or a good guess) about the model error associated with the detailed model. In some situations, this may not be the case or else it may be unknown. (One may see Rebba et al., 2006 for hypothesis testing based methods to compare model quality). Second, the bootstrapping technique is an approximation of the true characterization of the conceptual model error; additional accuracy may be achieved but at the cost of additional detailed analysis. In addition, the error is dependent on the design which is not known a priori. Finally, this method only accounts for the presence of model error; it offers no means to reduce the error other than to incorporate the detailed analysis in the RBDO design formulation. Thus a sensitivity analysis is recommended at the onset decide what level of fidelity is required for each disciplinary model during conceptual design

CHAPTER VII

CONCLUSION

In providing reliable design of complex systems under uncertainty, it is critical that the design process incorporate methods to account for uncertainty and ensure meeting reliability goals throughout all stages of design. In this dissertation, this communication of uncertainty has been addressed on two fronts. The first is for the integration of multiple disciplinary analyses at a single level. To this end, efficient methods were presented in order to address concerns about the computational effort required for reliability analysis and optimization of complex systems. On the second front, two alternative strategies were developed for the communication of uncertainty across two design levels (as distinguished by the scope, detail, and fidelity of the performance analysis).

A first step in reliability-based system design is reliability-based analysis. Chapter II provided two specific algorithms for reliability-analysis of multidisciplinary systems. The Partial FOSM method is a low effort, low fidelity method particularly suitable for problems for which the sensitivity of system failure to intermediate disciplinary response variables is small relative to other sources of uncertainty. It is also ideal for systems for which the effort for a disciplinary analysis of interest is significantly lower than that required for multidisciplinary systems. Communication between disciplines is required during the first (i.e., FOSM) phase of this methodology to achieve multidisciplinary feasibility, but only the mean (where interdisciplinary agreement is more likely to be achieved). Limitations to this method include the assumption that discipline responses are linear and normal; errors associated with these assumptions will propagate according to sensitivity of the failure probability to these discipline responses. An alternative algorithm, for distributed multi-constraint FORM was also provided. This algorithm provides a step based on the distributed (MDO) formulation for FORM provided by Du and Chen (2002). Multi-constraint FORM requires interdisciplinary communication more often that Partial FOSM and requires agreement at more extreme values (the most probable point). However, its performance for a limited example demonstrates the potential to reduce overall computational effort over equivalent methods that employ either fully-integrated FORM or use SQP to solve the distributed formulation. Limitations to this method include the assumption of a linear limit state and the fact that it is based on Newton-Raphson methods which do not have proven convergence but are known to either diverge or cycle for certain starting points. Furthermore, this algorithm will only provide an improvement over fully-integrated FORM methods if gradient-based methods are at least as effective as fixed point iteration in achieving multidisciplinary feasibility. It is recommended to employ this method as a first choice algorithm and defer to fully-integrated FORM with SQP if it does not converge after some minimal number of cycles (10, for example.)

In Chapters III and IV, multidisciplinary reliability analysis was extended to multidisciplinary optimization under uncertainty using algorithms developed by combining existing MDO and RBDO techniques using theory due to Chiralaksanakul, and Mahadevan (2005). Further study is needed to fully define the classes of problems suitable to each method. However, based on limited early results, a few inferences are drawn. First, fully-integrated, single loop, inverse FORM (Method 6) appears to be promising as a first attempt algorithm for systems with nearly linear limit states. This method has the most potential for significant computational savings, although its success is highly dependent on starting point especially for non-linear limit states. Second, sequential, fully-integrated, direct FORM (Method 7) appears promising as a solid overall algorithm which performs well in many situations (it was the best algorithm for the real world UAV application). As with any fully-integrated algorithm, use of this method presumes that multidisciplinary integration is tractable. Finally, for systems in which multidisciplinary integration is expensive and gradient-based methods are effective in achieving multidisciplinary feasibility, simultaneous analysis and design algorithms (7-10) show promise in reducing the overall effort by requiring interdisciplinary 'agreement' only at the design solution.

In summary, the methods developed in the first part of this dissertation all require a clearly defined design problem with interdisciplinary relationships that are well defined. Both the UAV Power System design in chapter IV and the RLV/LH_2 Tank design in Chapter V demonstrate that this is no trivial task. However, methods differ in the conditions under which interdisciplinary agreement must be achieved. Fully-integrated methods benefit from frequent agreement during both the design process (optimization) and reliability analysis while distributed methods postpone agreement until the design is finalized.

The second half of this work took a different direction, addressing the incorporation of design under uncertainty across design levels. The ideas were motivated by the relationship between a conceptual reusable launch vehicle design and that of one of its major components, a liquid hydrogen tank. Two alternatives to fixed point iteration between designs were presented. Chapter V integrated the two designs within a single reliability-based optimization. In Chapter VI, the same problem was addressed from a different perspective. There, the two designs were distinguished by their level of detail as well as by their respective disciplinary model errors.: A sample of designs at both levels provided a means to quantify model error for the RLV design, and the system sensitivity to model error was presented as a valuable metric for selecting disciplinary models at various stages of design. Furthermore, this was incorporated into the reliability-based design optimization providing a conceptual design linked to the detailed design through the model error variables. The integrated bi-level RBDO method of Chapter V appears promising for designs for which (1) close interaction between design levels is both possible and desired, (2) the design process can 'afford' the additional computational effort, and (3) the detailed design is needed to reduce otherwise unacceptable uncertainty associated with the conceptual design. The model error propagation method might be more suitable for system-component integration when it is difficult or impossible to achieve inter-level agreement and close integration is not required.

Future Research Needs

Short term research needs include deeper analysis into the performance of MDO-RBDO algorithms in order to (1) determine the applicability of these methods to large scale problems and (2) fully characterize the system properties suitable to particular methods. Characteristics to be studied could include the conditioning of the limit state and/or disciplinary analyses (continuity, concavity, etc.), the effect of dimensionality of the design variable vector and random vector, the number of probabilistic constraints, and the number of disciplines. In addition, a specific algorithm for combining inverse FORM and SAND is needed to improve performance of these methods.

In the area of multidisciplinary analysis under uncertainty, this dissertation examined the two most basic strategies for optimization, fully-integrated optimization and analysis and simultaneous analysis and design with six reliability-based design optimization algorithms. The methods proposed, though providing improvements in efficiency can nevertheless be computationally expensive as problem complexity increases. Numerous other methods exist, particularly for multidisciplinary optimization, which could be exploited for using in design under uncertainty in the development of new algorithms. Another important direction for research regards incorporating discrete design variables. Many real world applications have discrete design choices (material choice, for example). Accommodation of discrete design variables would significantly expand the applicability of these methods. Incorporating system reliability constraints would be yet another noteworthy addition.

This research examined the integration of a conceptual system design with the design of a single component under uncertainty. It would be worthwhile to extend these concepts (from both chapters V and VI) to the coupling of a conceptual design to a number of component designs to examine the cumulative effect on computational effort required. This would provide a better measure of which methods are truly viable for the design of real systems.

Finally, this dissertation has presented concepts and methods for implementation of multidisciplinary RBDO within the design process. However, it has focused on a relatively narrow area, namely design parameter optimization under uncertainty. Another important area worthy of future study would include a probabilistic approach for the requirements flow down of reliability goals. Engineers would greatly benefit from future research to consolidate methods for incorporating stochastic uncertainty throughout the entire design cycle in a systematic manner that mirrors an accepted systems engineering model such as the Systems Engineering Vee Model (Forsberg and Mooz, 1992).

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